Second Degree Price Discrimination

Model

- One firm sells to consumers with heterogeneous types
- Type θ of each customer not observed by firm
- Two possible types $\theta \in \{\theta_1, \theta_2\}$ with $\theta_1 < \theta_2$
- Outside option normalized to 0
- Population size: N
- Proportion of type θ_1: λ
- Cost of production of quantity q is $c \cdot q$
- Firm induces auto-selection by offering a suitably chosen menu of pairs quantity-price: (q_1, T_1) for θ_1 and (q_2, T_2) for θ_2.

Suitably chosen means that

(i) type θ_i is better-off if he buys rather than not

$$\theta_1 V(q_1) - T_1 \geqslant 0$$
$$\theta_2 V(q_2) - T_2 \geqslant 0$$

constraints called **individual rationality** constraints.

(ii) type θ_i prefers his pair

$$\theta_1 V(q_1) - T_1 \geqslant \theta_1 V(q_2) - T_2$$
$$\theta_2 V(q_2) - T_2 \geqslant \theta_2 V(q_1) - T_1$$

constraints called **incentive compatibility** constraints.

Problem of the monopolist

$$\max\ \pi(q_1, q_2) = N\lambda[T_1 - cq_1] + N(1-\lambda)[T_2 - cq_2]$$

$$\begin{aligned}\text{s.t.} \quad & \theta_1 V(q_1) - T_1 \geqslant 0 \\ & \theta_2 V(q_2) - T_2 \geqslant 0 \\ & \theta_1 V(q_1) - T_1 \geqslant \theta_1 V(q_2) - T_2 \\ & \theta_2 V(q_2) - T_2 \geqslant \theta_2 V(q_1) - T_1\end{aligned}$$

General procedure:

Step 1: study the constraints & characterize T_1 and T_2

Step 2: plug back T_1 and T_2 into objective –

We obtain a relaxed problem

Step 3: Take the first order conditions

⇒ Characterize the solution of the relaxed problem

⇒ Check the remaining constraints ex-post

(they will always be satisfied in this class)

Step 1

The first incentive compatibility constraint rewrites as

$$T_2 \geqslant \theta_1(V(q_2) - V(q_1)) + T_1$$

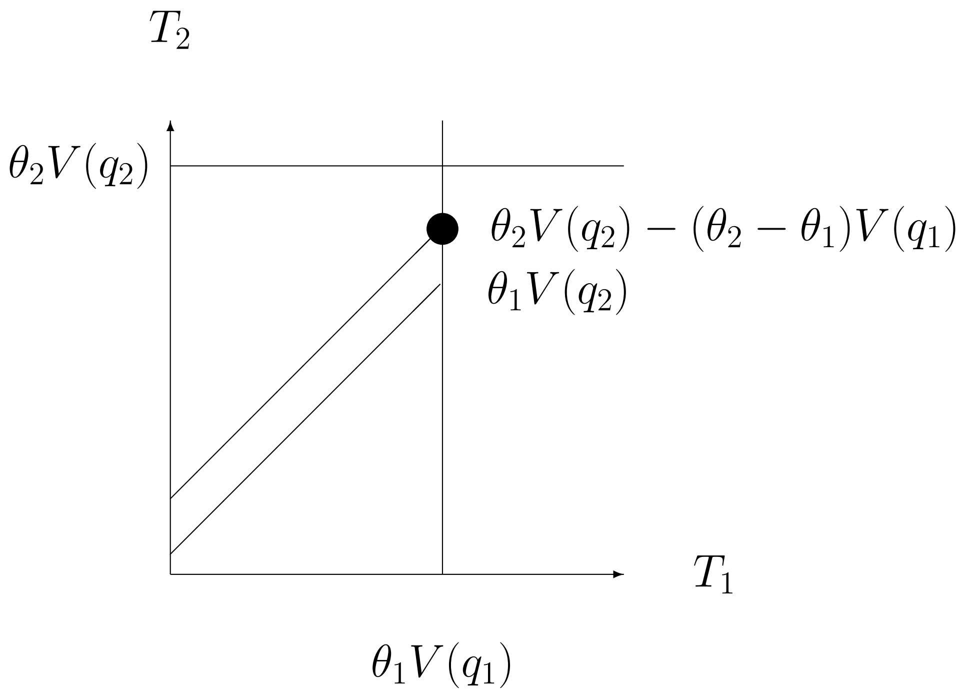
T_2 must be <u>above</u> the line with equation $\theta_1(V(q_2) - V(q_1)) + T_1$ (this is a function of T_1 that takes value $\theta_1 V(q_2)$ at $T_1 = \theta_1 V(q_1)$ and $\theta_1(V(q_2) - V(q_1))$ at $T_1 = 0$).

The second incentive compatibility constraint rewrites as

$$T_2 \leqslant \theta_2(V(q_2) - V(q_1)) + T_1$$

T_2 must be <u>below</u> the line with equation $\theta_2(V(q_2) - V(q_1)) + T_1$ (this is a function of T_1 that takes value $\theta_2 V(q_2) - V(q_1)(\theta_2 - \theta_1)$ at $T_1 = \theta_1 V(q_1)$ and $\theta_2(V(q_2) - V(q_1))$ at $T_1 = 0$).

- Values of T_2 exist if the line $\theta_2(V(q_2) - V(q_1)) + T_1$ is above the line $\theta_1(V(q_2) - V(q_1)) + T_1$. Therefore, we must have $(\theta_2 - \theta_1)(V(q_2) - V(q_1)) \geqslant 0$, that is $q_2 \geqslant q_1$.
- Using a graph, we see that the highest payments that satisfy all constraints (provided $q_2 \geqslant q_1$) are $T_1 = \theta_1 V(q_1)$ and $T_2 = \theta_2 V(q_2) - (\theta_2 - \theta_1)V(q_1)$.


T_2
$\theta_2 V(q_2)$
$\theta_2 V(q_2) - (\theta_2 - \theta_1) V(q_1)$
$\theta_1 V(q_2)$
T_1
$\theta_1 V(q_1)$

Step 2

We plug the values of T_1 and T_2 into the objective.

The problem becomes

$$\max_{q_1,q_2} N\lambda(\theta_1 V(q_1)-cq_1)+N(1-\lambda)(\theta_2 V(q_2)-(\theta_2-\theta_1)V(q_1)-cq_2)$$

$$\text{s.t.} \quad q_1 \leqslant q_2$$

The relaxed problem is:

$$\max_{q_1,q_2} N\lambda(\theta_1 V(q_1)-cq_1)+N(1-\lambda)(\theta_2 V(q_2)-(\theta_2-\theta_1)V(q_1)-cq_2)$$

Step 3

The first order conditions with respect to q_1 and q_2 are

$$N\lambda(\theta_1 V'(q_1) - c) - N(1-\lambda)(\theta_2 - \theta_1)V'(q_1) = 0$$

$$\theta_2 V'(q_2) - c = 0$$

and the optimal quantities are therefore q_1^{**} and q_2^{**} such that

$$\theta_1 V'(q_1^{**}) = c + \frac{N(1-\lambda)}{N\lambda}(\theta_2 - \theta_1)V'(q_1^{**})$$

$$\theta_2 V'(q_2^{**}) = c$$

Recall, under perfect discrimination, we have

$$\theta_1 V'(q_1^*) = c, \qquad \theta_2 V'(q_2^*) = c$$

$$T_1^* = \theta_1 V(q_1^*) \qquad T_2^* = \theta_2 V(q_2^*)$$

Overall, under second degree price discrimination

(i) type θ_2 buys $q_2^{**} = q_2^*$ at price $T_2^{**} = \theta_2 V(q_2^{**}) - (\theta_2 - \theta_1)V(q_1^{**})$ (and gets a rebate)

(ii) type θ_1 buys $q_1^{**} < q_1^*$ at price $T_1^{**} = \theta_1 V(q_1^{**})$ (and gets no surplus).

(iii) Production is $q_1^{**} + q_2^{**} < q_1^* + q_2^*$.

(iv) Profit is smaller than under perfect discrimination.

(v) Cons' surplus is higher than under perfect discrimination.

Last, $q_1^{**} < q_1^* < q_2^* = q_2^{**}$, the remaining constraint is satisfied.