Advanced Topic - Signaling games

1. Theory

• Model: (assume pure strategies to simplify)

- Sender (S) is informed about type θ and chooses a_s at stage 1;

- Receiver (R) is uninformed about θ and has a prior belief over θ represented by the probability distribution $p(\theta)$; R chooses a_r at stage 2, after observing a_s;

- Payoffs are $u_s(a_s, a_r, \theta)$ and $u_r(a_s, a_r, \theta)$ respectively for S and R;

• Solving the game:

- At stage 2, R observes a_s, and updates his belief. The posterior is a function $\pi(\theta|a_s)$ obtained by guessing that S chooses a strategy at state 1 (guess must coincide with actual strategy at equilibrium), and posterior is computed using Bayes rule:

$$\pi(\theta|a_s) = \frac{p(\theta)Prob(a_s|\theta)}{\sum_{\theta'} p(\theta')Prob(a_s|\theta')}$$

where $Prob(a_s|\theta)$ is the probability that S chooses a_S if his true type is θ.

- At stage 2, R chooses a_r such that

$$a_r^*(a_s) = \text{argmax}_{a_r} \sum_{\theta} [\pi(\theta|a_s)\, u_r(a_s, a_r, \theta)]$$

- At stage 1, S anticipates how R will play at stage 2, and chooses a_s such that:

$$a_s^*(\theta) = \text{argmax}_{a_s}\, u_s(a_s, a_r^*(a_s), \theta)$$

• Equilibrium concept: Perfect Bayesian equilibrium. It consists of strategies $a_s^*(\theta)$ and $a_r^*(a_s)$ and posterior beliefs $\pi(\theta|a_s)$ such that the strategies satisfy the two previous optimality conditions (called perfection conditions) and the beliefs satisfy the Bayesian rationality condition above.

• There are generally MANY equilibria in those games. A separating equilibrium is such that the type is fully revealed at equilibrium. It is necessary to specify out-of-equilibrium beliefs that sustain this equilibrium.

2. Example of Buyer-Seller game

A seller has private information about the quality of a car. It can be Good or Bad with probabilities q and $1-q$ respectively. The seller can put it on sale and if so, the price is p. The buyer does not know the quality, he observes the price and decides whether to buy or not. The qualities are valued as follows by the two players. For the buyer, good is valued G, and bad is valued B. For the seller, good is valued g and bad is valued b. Let $G > g$ and $B > b$.

• At stage 2, the buyer observes the price and decides to accept or not. He realizes that the seller offers the good quality cars only if the price exceeds g.

- If $p < g$: the buyer knows it is a bad quality car. Then, the revised beliefs are $\pi(Good|p < g) = 0$ and $\pi(Bad|p < g) = 1$. The buyer buys it if and only if $B - p \geq 0$.

- If $p > g$: the car can be either good or bad and the buyer may guess that both cars can be on the market. No information is revealed. Given the buyer does not observe the quality, he is willing to pay at most $qG + (1-q)B$. If $qG + (1-q)B < g$, there is no such equilibrium $p < qG + (1-q)B$ and $p > g$.

• At stage 1, the seller sets the price.

- conditional on setting $p < g$, the buyer accepts at most at price B. Only bad cars can be sold at price B. The seller gets $B - b$.

- conditional on setting $p > g$, the buyer is willing to pay at most $qG + (1-q)B$ provided $qG + (1-q)B > g$. Trade occurs at price $p = qG + (1-q)B$. The seller gets $qG + (1-q)B - b$ if the car is bad and $qG + (1-q)B - g$ if the car is good.

Overall, if $qG + (1-q)B \leq g$, only bad cars are put on sale and the seller chooses $p = B$. If $qG + (1-q)B \geq g$, it is profitable to sell both types of cars. The buyer knows it but given he cannot disentangle between good and bad cars, the price is $p = qG + (1-q)B$.

In that example, the seller cannot signal the quality of the car.