Noise threshold overview: Upper bounds, Existence proofs, Lower eds, Simulations

Threshold upper bounds

Åkerlund, Ben-Or, Impagliazzo, Naum 96110:8

for more than log depth (ANN), need fresh source of ancillas

\[ I(\rho) = n \times S(\rho) \rightarrow (1 - \frac{4}{3}p) I(\rho) \]

indeq depolarizing\( (\rho) \)

(Observable) need full parallelism, no global noise or growing noise with system

Aharonov, Ben-Or 922

for fan-in 2 gates, if \( \frac{4}{3}p > 0.97 \) \( \Rightarrow \) log depth

Raiborov 0210130

if \( \frac{4}{3}p > 1 - \frac{1}{e} \) for fan-in 2 gates \( \Rightarrow \) log depth

branching process

Kempe, Regev, Ungar, & Wolf 0802.1464

ditto for \( k = 3 \), \( \frac{4}{3}p > 35.7% \) (for CNOT and \( \frac{4}{3}p > 29.7\% \))

Pauli decomposition \( \rho = \sum_{p} \frac{1}{2} (\sigma_p (\rho_p)) \rho \)

\[ E(\rho) = \sum_{\rho_p} E_p(\rho_p) \]

Heuristic bounds

Bruss et al 98: capacity of depolarizing channel to transmit qo. information

\( \leq 0 \) for depol rate \( \frac{4}{3}p \geq \frac{1}{2} \).

Plemai & Venni: 0810.4540 (also Knill): bounds for particular magic-stokes distillation-based schemes.

Bounds allowing classical control

Harrow & Necham 0301018

multi-bit gates separability preserving \( \Rightarrow \) classically unnuisible (any depth)

CNOT + indep. depolarizing noise \( \frac{4}{3}p > 0.74 \)

CNOT + adversarial inop noise \( p \geq 0.5 \)

magic-stokes/BeKenman-Knill bounds
Existence proofs:

1. Leakage errors $10^7, 11^7, 12^7, 13^7, \ldots$
   - Computational leaks
   - Teleportation eliminates leakage (free with Knill-type error correction)
   - Fault-tolerant against X errors, but error on leaks is undefined

2. Locality constraints
   - Models: grid of qubits, or grid plus paths
   - Level-2 swap, implemented by level-1 swaps, with level-1 error correction (uses level-0 swaps)

Is this fault-tolerant?

No.

Possible solution 1: use higher-distance code

Possible solution 2: use aux qubits

Possible solution 3: change the model to allow paths
3. All unitary control

   Issue:

   data ➡ extract syndrome ➡ unitary application ➡ classical data ➡ syndrome reverse

   (Note: corrections need to be applied before any non-Clifford gate)

   not fault-tolerant to classical errors

   Trivial solution: repeat full circuit $n$ times, determining the correction for each data qubit independently.

   (Fault-tolerant classical control, using the repetition code, phase errors irrelevant)

4. Further extensions

   Knill-style computation + error correction

   - no erases
   - different techniques can apply $R^P$
     also simulations

Thresholds for non-Markovian noise

$H = H_s + H_o + H_{sb}$

1. if $s_i$ only interacts touching data qubits

   $\varepsilon = \max \| H_{sb}(t) \|_{tr}$

2. for noise coupling all data qubits, decaying in space

   $\varepsilon_i^3 = \max \sum_j \| H_{ij}(t) \|_{tr}$

   single molecule can break two gates (not strictly fault tolerant)

   - norms are not measurable, may be $\propto$

   e.g. both a harmonic oscillator $H = a^+a$

   $10\uparrow, 11\uparrow, 1\downarrow, \ldots$, $a^+|1\downarrow\rangle = |1\uparrow\rangle \cdot \frac{\sqrt{2}}{\sqrt{3}}$

   intuition whether both can reach high energy space

   nuclear [Alicki] typical coupling $2rt_{\text{rad}}(a+a^*)/\hbar$

   partial progress [Wig & Preskill, 0810.4953]
Lower bounds for specific fault-tolerance schemes, noise models
deplingering noise \{ \} \rightarrow adverse noise at level $L$
biased noise Pauli
postsctrction + Fibonacci schemes

Simulations
Steane 0207119
QIP-style simulations & estimates

Nature

Knill 0402171, 0404104, 0410199
R
AP 0409.5063