BOUNDS ON THE MINIMUM ENERGY-PER-BIT FOR BURSTY TRAFFIC IN DIAMOND NETWORKS

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Motivation

- Wireless Sensor Network
- Detection of sporadic events
  - e.g., earthquake, intruder, malfunctioning device
- Nodes use multi-hopping
Motivation

• What is the energy-optimal communication scheme?

• What is the energy-optimal subset of relays?
Network Model

- **Diamond Network**

- **Discrete-time, AWGN channel model**

\[
Y_{r_i}[t] = \sqrt{g_i} X_s[t] + Z_{r_i}[t], \quad i = 1, 2
\]

\[
Y_D[t] = \sqrt{h_1} X_{r_1}[t] + \sqrt{h_2} X_{r_2}[t] + Z_D[t]
\]

\[Z_{r_1}, Z_{r_2}, Z_D \overset{iid}{\sim} \mathcal{N}(0, N_0)\]
Bursty Communication Model

- Asynchronous communication model from [1,2]

- $B$ bits available at time $T \sim \text{Unif}\{1, 2, \ldots, A\}$
- Destination must decode within a delay $d$
- We are interested in the minimum energy-per-bit

[Tchamkerten, Chandar and Wornell “Communication Under Strong Asynchronism”]

[Chandar, Tchamkerten and Tse “Asynchronous Capacity per Unit Cost”]
Asymptotic Regime

- Analyze asymptotic regime when $B \to \infty$ [1,2]

- Delay $d(B)$ must be subexponential in $B$:

  \[ \log \frac{d(B)}{B} \to 0 \]

- \[ A = 2^{\beta B} \sim 1 \text{ year} \]

- \[ T \leq d(B) \sim 5 \text{ minutes} \]

- e.g., Tsunami detector

[2] Chandar, Tchamkerten and Tse “Asynchronous Capacity per Unit Cost”
Achievable Energy-Per-Bit

- Energy-per-bit $e_b$ is achievable if there exists a sequence of codes $\{C_B\}$, where code $C_B$ has $2^B$ codewords, such that

$$
\left(1\right) \quad \frac{E\left[\mathcal{E}(C_B)\right]}{B} \to e_b, \text{ as } B \to \infty
$$

$$
\left(2\right) \quad \text{Pr}_{A}\left[\text{error}\left(C_B\right)\right] \to 0, \text{ as } B \to \infty
$$

$\mathcal{E}(C_B) = \sum_{t=1}^{T} \left( X_s^2[t] + X_{r_1}^2[t] + X_{r_2}^2[t] \right)$

- Incorrectly decode $B$ bits
- Late decoding: $\tau > T + d(B)$
“Separation-Based” Schemes

- Perform tasks of \textit{synchronization} and \textit{communication} separately

- Training sequence used for synchronization

- Use a known scheme for communication (DF, BAF)
  - Finding optimal scheme is still an open problem

\[ \begin{array}{c}
S \quad \sqrt{g_1} \quad r_1 \quad \sqrt{h_1} \\
\quad \sqrt{g_2} \quad r_2 \quad \sqrt{h_2} \\
D
\end{array} \]

How well do separation-based schemes perform?
"Separation-Based" Schemes

Theorem: If $g_1 = g_2$, separation-based schemes achieve energy-per-bit $e_b$ satisfying

$$\frac{e_b}{e_b^{\text{min}}} \leq \frac{1 + \beta}{\frac{1}{2} + \beta}.$$

$T \sim \text{Unif}\{1, 2, \ldots, 2^\beta B\}$

Separation-based schemes are nearly optimal in the highly asynchronous regime $\beta \gg 1$. 
Energy-per-bit achieved by a separation-based scheme

- By using a separation based scheme with
  - Single pilot for synchronization
  - Decode-and-forward for communication

we achieve energy-per-bit

\[
\left( \frac{1+\beta}{g} + \frac{1+\beta}{h_1 + h_2} \right) 2N_0 \ln 2
\]
Outer Bounding Technique: Relay Synchronization

- Should the relays be synchronized?

- What does it mean for a relay to be synchronized?
Outer Bounding Technique: Relay Synchronization

**Definition:** A sequence of codes $\{C_B\}$ synchronizes relay $i$ if

$$\lim_{B \to \infty} \frac{H(T|Y_i)}{B} = 0.$$ 

**Theorem:** If $g_1 = g_2$, any sequence of codes $\{C_B\}$ achieving a finite energy-per-bit $e_b$, must synchronize both relays.

Energy-optimal schemes synchronize both relays.
Proof Idea

- Each relay can build a list decoder for $T$ based on its transmit signals.

- Can show that, for some $\alpha$, $\Pr(T \notin \hat{T}_{r_1}) \to 0$, as $B \to \infty$. 

![Diagram of a network with relays and nodes labeled with $X_{r_1}[t]$, $X_{r_2}[t]$, $S$, $D$, and $O(B)$, with $T_{r_1}$ and $A$. The diagram includes symbols for $\sqrt{g}$ and $\sqrt{h_1}$, $\sqrt{h_2}$, and $d$. The diagram also shows $X_{r_1}^2[t]$ with a bar graph indicating $\geq \alpha B$.]

1  $t_1$ $t_2$ $t_3$ $A$
Proof Idea

- It can be shown that

\[
\frac{H(T|Y_{r_i})}{B} \leq \frac{H(T|\hat{T}_{r_i})}{B} \leq 1 + \log |\hat{T}_{r_i}| + \beta B \Pr(T \notin \hat{T}_{r_i}) \rightarrow 0
\]

since \( |\hat{T}_{r_i}| = O(B) \) and \( \Pr(T \notin \hat{T}_{r_i}) \rightarrow 0 \)

- Thus, both relays must be synchronized
Outer Bound from Relay Synchronization

- Take a sequence of codes $\{C_B\}$

- $T \sim \text{Unif}\{1, 2, \ldots, 2^{\beta B}\}$

- Code $C_B$ “transmits” an extra $\beta B$ bits to each relay and the destination
Outer Bound from Relay Synchronization

- We focus on each hop separately

- Energy-per-bit used by the source
  \[ \geq \left( \frac{1}{2} + \beta \right) \frac{2N_0 \ln 2}{g} \]

- Energy-per-bit used by the relays
  \[ \geq (1 + \beta) \frac{2N_0 \ln 2}{h_1 + h_2} \]
Theorem: If \( g_1 = g_2 \), the minimum energy-per-bit of the diamond network with bursty traffic satisfies

\[
\left( \frac{1 + \beta}{g} + \frac{1 + \beta}{h_1 + h_2} \right) 2N_0 \ln 2 \geq e_b^{\text{min}} \geq \left( \frac{1/2 + \beta}{g} + \frac{1 + \beta}{h_1 + h_2} \right) 2N_0 \ln 2.
\]

- Separation-based scheme
- Outer bound from relay synchronization

The ratio between upper and lower bound is at most

\[
\frac{1 + \beta}{\frac{1}{2} + \beta} \to 1 \quad \text{as} \quad \beta \to \infty
\]
The Asymmetric Case

- What if $g_1 \neq g_2$?

- Should both relays be synchronized?
The Asymmetric Case

- Can assume WLOG \( g_1 \geq g_2 \)

**Theorem:** There are optimal codes such that either
a) Both relays are synchronized, or
b) Relay 1 is synchronized and relay 2 remains silent

It is optimal to synchronize any relay that is used

- Can also find upper and lower bounds on the minimum energy-per-bit satisfying
\[
\frac{\text{upper bound}}{\text{lower bound}} \leq 2 \quad \text{and} \quad \frac{\text{upper bound}}{\text{lower bound}} \to 1, \quad \text{as } \beta \to \infty
\]
The Asymmetric Case

- We can approximately characterize which subset of relays is optimal.
Conclusion

- Bursty traffic in wireless sensor networks
  - Approximately characterized the minimum energy-per-bit
  - Characterized the optimal relay selection for most channel gain values