(1)(a) Let \( z = x + iy \). Squaring the equality gives the following sequence of equivalent equalities:

\[
|z - (3 + 4i)|^2 = 4|z|^2 \\
(x - 3)^2 + (y - 4)^2 = 4(x^2 + y^2) \\
0 = 3x^2 + 3y^2 + 6x + 8y - 25 \\
0 = x^2 + y^2 + 2x + \frac{8}{3}y - \frac{25}{3} \\
0 = (x + 1)^2 + (y + \frac{4}{3})^2 - 1 - \frac{16}{9} - \frac{25}{3} \\
(x + 1)^2 + (y + \frac{4}{3})^2 = \frac{100}{9}.
\]

Here the next-to-last equality is obtained from the previous one by completing the square. Thus the locus is a circle with center \((-1, -\frac{4}{3})\) and radius \(\frac{10}{3}\).

(2)(a) \(|e^i| = e\), so \(\log|e^i| = 1\), and \(\text{Arg } e^i = \frac{\pi}{2}\). Hence \(\text{Log}(e^i) = 1 + \frac{\pi}{2}i\) and \(\log(e^i) = 1 + (\frac{\pi}{2} + 2n\pi)i, n \in \mathbb{Z}\).

(b) Using (a), and using \(e^{i\pi} = -1\),

\[
(e^i)^{\pi i} = e^{\pi i \log(e^i)} = e^{\pi i(1+i(\frac{\pi}{2}+2n\pi))} = e^{i\pi - \pi^2(\frac{1}{2}+2n)} = -e^{-\pi^2(\frac{1}{2}+2n)}, \ n \in \mathbb{Z}.
\]

(3) Parametrize: \(\gamma(t) = 2 - i + it, \ 0 \leq t \leq 4\). Then \(|\gamma(t)|^2 = 2^2 + (t - 1)^2 = t^2 - 2t + 5\) and \(\gamma'(t) = i\), so

\[
\int_\gamma |z|^2 \, dz = \int_0^4 (t^2 - 2t + 5)i \, dt = i \left( \frac{t^3}{3} - t^2 + 5t \right) \bigg|_0^4 = i \left( \frac{64}{3} - 16 + 20 \right) = \frac{76}{3}i.
\]
(4) The image is a “quarter annulus” in the lower right quadrant: \( \{z : 1 < |z| < e^3, -\frac{\pi}{2} < \Arg z < 0\} \).

(5)(a) Let \( \gamma : [a, b] \to \mathbb{C} \) be a curve from \( p \) to \( q \). We use the Chain Rule, \( \frac{d}{dt} F(\gamma(t)) = F'(\gamma(t)) \gamma'(t) \), to conclude

\[
\int_\gamma F'(z) \, dz = \int_a^b F'(\gamma(t)) \gamma'(t) \, dt
= \int_a^b \frac{d}{dt} F(\gamma(t)) \, dt
= F(\gamma(b)) - F(\gamma(a))
= F(q) - F(p).
\]

(b) Every polynomial \( f(z) \) has an antiderivative, since each term \( a_k z^k \) has antiderivative \( a_k \frac{z^{k+1}}{k+1} \), so \( f(z) = F'(z) \) for some \( F \). By (a),

\[
\int_\gamma f(z) \, dz = \int_\gamma F'(z) \, dz = F(2-i) - F(1+i),
\]

which is the same for all \( \gamma \). So no, the integral cannot depend on the path.

(6)(a) I must apologize—this problem is not correct as stated! (It was OK until I changed it and put in the \( \cos z \)...) It will be treated as extra credit if you made a plausible try.

(b) If you assume (a) (even though it’s false) then (b) can be done as follows. For all \( z \in \gamma_R \) we have

\[
\left| \frac{f(z)}{z^3} \right| = \left| \frac{f(z)}{|z|^3} \right| \leq \frac{2|z|}{|z|^3} = \frac{2}{|z|^2} = \frac{2}{R^2},
\]

where the inequality here follows from (a). \( \gamma_R \) has length \( 2\pi R \) so

\[
\left| \int_{\gamma_R} \frac{f(z)}{z^3} \, dz \right| \leq \frac{2}{R^2} \cdot 2\pi R = \frac{4\pi}{R} \to 0.
\]