MATH 445 ASSIGNMENT 8
Fall 2009
Prof. Alexander

Due Monday October 26.

Kreyszig:
12.5 p. 560 #2, 4, 5, 7, 10, 14, 20, 25, 28, 31, 32

Some hints:

In #4, L is not given so your answer will depend on L.

In #10, the idea is that after a "long time", the rate of temperature change, $\partial u/\partial t$, becomes 0. (Actually it approaches 0 as a limit, but we ignore this distinction.) When it is 0,what does the heat equation say?

In #20, you have to determine what the given initial conditions on u say about $F(x)$ and $G(t)$ when you are finding eigenfunctions. Also, even though the left end is kept at 0 degrees, the initial temperature is taken as U_0 throughout the bar. Physically what happens is that the temperature immediately drops after time 0 in a small neighborhood of the left end so that the temperature becomes a continuous function, coming down to 0 at the left end.

In #25, the point is to find the choice of $w(t)$ for which this works. By "works" I mean that if you take $u(x,t)$ that's a solution of the usual heat equation (1) and multiply it by the right $w(t)$, the result $v(x,t) = u(x,t)w(t)$ will satisfy the convection PDE. You need to substitute the formula $v(x,t) = u(x,t)w(t)$ into the convection PDE to figure out what equation $w(t)$ needs to satisfy.

Some even-numbered solutions:

(4) $.00702L^2$ (10) $u_I = U_1 + (U_2 - U_1)x/L$

(14) $u = \frac{\pi}{2} - \frac{4}{\pi}\left((\cos x)e^{-t} + \frac{1}{9}(\cos 3x)e^{-9t} + \frac{1}{25}(\cos 5x)e^{-25t} + ...\right)$

(20) $u = \sum_{n \text{ odd}} \frac{4U_0}{n\pi} \sin\frac{n\pi x}{2L} \exp\left\{-\left(\frac{n\pi c}{2L}\right)^2 t\right\}$

(28) $u = \frac{880}{\pi}\sum_{n=1}^{\infty} \frac{1}{(2n+1)\sinh(2n+1)\pi} \sin\frac{(2n+1)\pi x}{20} \sinh\frac{(2n+1)\pi y}{20}$

(32) $u = \frac{4U_1}{\pi}\sum_{n=1}^{\infty} \frac{1}{2n-1} \sin\frac{(2n-1)\pi x}{24} \frac{\sinh[(2n-1)\pi y/24]}{\sinh(2n-1)\pi} + \frac{4U_0}{\pi}\sum_{n=1}^{\infty} \frac{1}{2n-1} \sin\frac{(2n-1)\pi x}{24} \frac{\sinh[(2n-1)\pi(1-y/24)]}{\sinh(2n-1)\pi}$