MATH 445 ASSIGNMENT 7
Fall 2009
Prof. Alexander

Due Monday October 19.

Kreyszig:
12.3 p. 548 #16, 17
12.4 p. 552 #3, 4, 5, 7, 13, 14, 15

In #5, the difference from before is that now the derivative $u_x(L,t)$ is 0 for all t, instead of the position $u(L,t)$ being 0 for all t. This means that when you look for solutions $u(x,t) = F(x)G(t)$ by solving the equations (5) and (6) p. 541, the boundary conditions on F and G will be different. You need to imitate what was done in section 12.3, but using these different boundary conditions.

In #7 (which we are doing in place of the graphing we skipped in Assignment 6), make a reasonably careful sketch of the graph of the odd periodic extension of f. You don't have to actually plot many points if you think about what the function should look like—where are its minima and maxima in $[0,1]$, and what are its values at these points—and use the fact it is a cosine wave on $[0,1]$, translated upward. You can get graphs of $f(x+t)$ and $f(x-t)$ by translating the graph of f horizontally, and then graph the average (which is $u(x,t) = \frac{1}{2}(f(x+t) + f(x-t)))$ by drawing a curve halfway between these two graphs. I suggest using different colors for the different graphs, since they will all be plotted together! Do this just for $t = 1/4, t = 1/2, t = 3/4$, to get an idea how the string actually moves. You can do this using software if you prefer, but beware of the fact that the given formula for $f(x)$ only applies for x between 0 and 1; outside that interval, you need the formula for the odd periodic extension.

In #13, just identify the type. (But do the whole problem in #14, 15.)

Some even-numbered solutions:

(16) Solution is given by (12) p. 543 (same as the string), with B_n still given by (14) p. 543, but now B_n^* is given by

$$B_n^* = \frac{2L}{c(n\pi)^2} \int_0^L g(x) \sin \frac{n\pi x}{L}\, dx.$$

(4) $20.21n$

(14) $u(x,y) = f_1(x+y) + f_2(2x-y)$