MATH 445 ASSIGNMENT 12
Fall 2009
Prof. Alexander
Due Friday December 4.

Kreyszig:
13.3 p. 617 #2, 4, 8, 9, 14, 16, 18, 21, 24
13.4 p. 623 #1, 2, 5, 6, 7, 8, 14, 19
13.5 p. 626 #3, 6, 11, 18, 19
14.1 p. 645 #1, 2, 3, 4, 10, 11, 13, 19, 20, 21, 22
Problems (1) and (2) below.

Problems on 14.2 and probably 14.3 will be suggested next week, as this material will be on the final, but they are not to turn in.

Problems based on the relation $\Delta f \approx f'(z_0)\Delta z$:

Example 1. Suppose that a curve passes through the point $2-i$ at an angle of $\pi/4$ with the positive x-axis. At what angle does the image of this curve pass through the image of $2-i$, for the mapping $w = z^3$? Solution: The derivative of $f(z) = z^3$ is $3z^2$, so the derivative at z_0 is $3z_0^2 = 3(2-i)^2 = 9-12i$, which in polar coordinates is $f'(z_0) = 15e^{i\theta}$ for $\theta = \arctan(-4/3)$. Therefore the mapping will rotate the curve (counterclockwise) by the angle $\arctan(-4/3)$, meaning that the image of the image of the curve passes through the image of $2-i$ at angle $\frac{\pi}{4} + \arctan(-\frac{4}{3})$.

Example 2. For the mapping in Example 1, about how far is the image of $1.97 - .99i$ from the image of $2-i$? Solution: $\Delta z = (1.97 - .99i) - (2-i) = -.03 + .01i$ so $|\Delta z| = \sqrt{.03^2 + .01^2} = \sqrt{10}/100$. Since $f'(z_0) = 15e^{i\theta}$, we have $|f'(z_0)| = 15$ and so

$$|\Delta f| \approx |f'(z_0)||\Delta z| = 15\sqrt{10}/100 \approx .474$$

is the approximate distance from $2-i$.

(1) For the mapping $w = f(z) = z^2 - 3z$ and the point $z_0 = 1+i$:
(a) At what angle does the image of the diagonal $y = x$ pass through $f(z_0)$?
(b) About how far is $f(.99 + 1.02i)$ from $f(z_0)$?

(2) For the mapping $w = f(z) = e^{z^2}$ and the point $z_0 = 2i$:
(a) Suppose a curve passes through z_0 at an angle of $\pi/6$ with the horizontal axis. At what angle does the image of this curve pass through the image $f(z_0)$?
(b) About how far is $f(.001 + .998i)$ from $f(z_0)$?

Some even-numbered solutions:

13.3 (14) Re $f = \frac{1-x}{(1-x)^2+y^2}$, Im $f = \frac{y}{(1-x)^2+y^2}$; $1.6 + 0.8i$ (24) $\frac{2iz}{(z+i)^3}$

13.4 (2) No (6) No (8) Yes (14) Yes, $f(z) = 1/z$.

13.5 (6) -23.1, 23.1 (18) $\frac{\ln 4}{3} \pm \frac{2n\pi}{3} i, n = 0, 1, 2, ...$

14.1 (10) $z(t) = 1 + i + (3 - 3i)t, 0 \leq t \leq 1.$ (20) $\frac{1}{2} + \frac{2}{3}i$ (22) $1 - \cosh 2$

HINTS:

13.4 #1, 2, 5, 6, 7, 8: Some of these may be easier if you express the function in polar form.