

MATH 425b ASSIGNMENT 5
SPRING 2008
Prof. Alexander
Due Friday March 13.

Chapter 9 #6, 7, 8, 13, 27 and:

(I) The *conorm* of a linear transformation $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is defined by

$$\mathbf{m}(T) = \inf \left\{ \frac{|T\mathbf{x}|}{|\mathbf{x}|} : \mathbf{x} \neq 0 \right\}.$$

Let U be the closed unit ball in \mathbb{R}^n .

(a) Show that the norm and conorm of T are the radii of the smallest ball that contains $T(U)$ and the largest ball contained in $T(U)$, respectively.

(b) Show that if $T : \mathbb{R}^n \rightarrow \mathbb{R}^n$ has conorm $\mathbf{m}(T) > 0$, then T is invertible.

(II)(a) Suppose $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is linear. Show that T is uniformly continuous.

(b) In contrast, let (a, b) be an interval and define $T : \mathcal{C}'(a, b) \rightarrow \mathcal{C}(a, b)$ by $T(f) = f'$. (Here $\mathcal{C}'(a, b)$ is the space of differentiable functions on (a, b) .) Show that T is linear but not continuous. Here the distance in both spaces is given by the sup norm.

HINTS:

(6) To show D_1f and D_2f exist at $(0, 0)$, you can't just differentiate the formula (which is only valid for $(x, y) \neq (0, 0)$.) Use the definition instead. To show lack of continuity, consider limits approaching $(0, 0)$ along a line, say $x = y$ or $x = 0$ or $y = 0$ or $x = -y$ or...

To give yourself a better geometric understanding of f , write it in polar coordinates: $f(r, \theta) = \frac{1}{2} \sin 2\theta$.

(8) It's enough to show each entry of the matrix of $f'(x)$ is 0.

(27) As with #6, you can't just differentiate the formula at $(x, y) = (0, 0)$. To show lack of continuity at $(0, 0)$, consider approaching $(0, 0)$ along line $y = cx$. Do all c give the same limit?

(II)(b) Continuity would mean $f_n \rightarrow f$ implies $T(f_n) \rightarrow T(f)$.