(1)(a)(13 points) For \( A = \begin{bmatrix} -2 & 2 & 0 \\ 7 & 4 & -4 \\ 1 & -2 & -3 \end{bmatrix} \), find \( \det(A) \) by using elementary row operations.

(b)(5 points) For \( B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \), what is \( \det(B) \)? HINT: Almost no calculations are necessary.

(c)(11 points) Find \( A + B \) and use the cofactor method to find \( \det(A + B) \). Does \( \det(A + B) = \det(A) + \det(B) \)?

(2)(a)(15 points) Solve the following system of equations, or show that no solution exists:
\[
\begin{align*}
-2x_1 + x_2 + 4x_3 &= 3 \\
3x_1 + 5x_2 - 7x_3 &= -1 \\
x_1 + 6x_2 + 2x_3 &= 2
\end{align*}
\]

(b)(4 points) Is the matrix of coefficients \( A = \begin{bmatrix} -2 & 1 & 4 \\ 3 & 5 & -7 \\ 1 & 6 & 2 \end{bmatrix} \) in (a) nonsingular? How do you know? HINT: No further calculations are necessary if you did (a).

(c)(7 points) Suppose we change the right side of the first equation in (a) from 3 to another value \( c \). Are there values of \( c \) that change whether the system is consistent, compared to what you found in (a)? How do you know?

(3)(a)(11 points) Suppose an operation is linear, and the outputs produced are vectors in \( \mathbb{R}^2 \). Suppose some input \( x \) produces output \( \begin{bmatrix} 1 \\ -2 \end{bmatrix} \) and some other input \( y \) produces output \( \begin{bmatrix} 3 \\ 5 \end{bmatrix} \). If the input is \( 2x - y \), what will the output be? What input produces an output of \( \begin{bmatrix} 2 \\ -4 \end{bmatrix} \)?

(b)(12 points) Let \( V = \{ \text{all } 2 \times 2 \text{ matrices with real entries} \} \) and \( S = \{ A \in V : a_{11} = a_{22} \} \). Determine whether \( S \) is a subspace of \( V \).
(4)(22 points) Use elementary row operations to find the inverse of \( A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix} \). HINT: All entries of the inverse are integers.