The welfare cost of inflation under imperfect insurance

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This paper examines the welfare cost of inflation in an economy where agents hold money in order to smooth consumption in the face of income variability for which there is no insurance. The findings indicate that, for the type of economies considered here, the area under the empirical money demand curve is a poor measure of the welfare cost of inflation. This area accounts for only one-fourth of the true costs in the case of 5% inflation.

1. Introduction

Beginning with the work of Bailey (1956), welfare costs of anticipated inflation were analyzed by computing the triangle under the demand for money curve. Using this approach, Lucas (1981) and Fischer (1981) found the welfare cost of 10% inflation to be 0.45% and 0.30% of GNP, respectively. These analyses focused on the role of money as a medium of exchange and inflation was shown to generate a loss since it caused an increase in transaction costs due to agents economizing on holdings of real cash balances. Hansen and Cooley (1989) utilized a different approach. They considered a one-sector stochastic optimal growth model with the cash-in-advance constraint and estimated the welfare cost of inflation by comparing steady states of two economies that display different money supply paths. In their

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environment welfare costs were due to agents substituting leisure for consumption in the face of a positive inflation and causing output to be lower. They report a welfare cost of 0.4% of GNP.

A key feature in the above studies is that they adopt the transaction-based approach to motivate the demand for money and analyze economies populated with identical agents. The purpose of this study is to examine an economy where liquid assets are used to facilitate consumption smoothing in the presence of idiosyncratic risk among heterogeneous agents. This structure is in the permanent income tradition as analyzed by Bewley (1977), Clarida (1985), or Schechtman (1976), where agents hold liquid assets to self-insure against fluctuations in their incomes. The question that is addressed here is whether the cost estimates in these economies where money is held for consumption smoothing purposes differ significantly from the ones obtained by using transaction-based approaches, and whether the traditional measure of estimating the area of the triangle under the money demand function is an appropriate measure for finding the welfare cost of inflation in such economies. The findings indicate that the traditional measure underestimates the welfare cost of inflation by a factor of four in case of 5% inflation and by a factor of three in case of 10% inflation in these economies where liquid assets are used for consumption smoothing.

In the economies studied agents face a stochastic work option and work whenever this option is available. On the other hand, if the option is not available, agents are unemployed and receive a much lower compensation through household production. If there were a full set of Arrow-Debreu contingent claims markets, agents would be identical by insuring against individual income variability and attain a constant consumption stream. However, in this study all the insurance technologies are precluded exogenously, in line with Scheinkman and Weiss (1986) or Kehoe, Levine, and Woodford (1989), in order to motivate the demand for money. Indeed, in the economies studied the only technology that the consumer has an access to is a storage technology; borrowing is not allowed.¹ Thus, agents accumulate money during the periods when they work to provide for consumption during the periods when they are unable to work. This is the precautionary demand for money discussed by Leland (1968) or Dreze and Modigliani (1972). In this environment, at a given point in time, agents are heterogeneous with respect to their money holdings and employment status.

In order to examine the welfare cost of inflation, economies with constant rates of inflation are generated through lump-sum transfers of money to the agents by the government. The average utility of agents in these economies

¹Such restrictions can be described as liquidity constraints. Liquidity constraints have proven to be important for some issues. For example, see Scheinkman and Weiss (1986) and Grossman and Weiss (1983).
are compared with the average utility of agents in an economy where no inflation is observed. The average rate of unemployment and the average level of consumption and income are kept the same for the economies with and without inflation. This enables us to isolate the effect of a fluctuating consumption stream on the welfare of the consumer. In particular, the annual real income supplement that would be necessary to make an individual in economies with 10% and 5% inflation rates as well off as an individual in an identical economy but with no inflation is computed. In this setting, welfare costs arise since agents’ ability to smooth out consumption is hindered by inflation.

The monetary arrangements set up here do not necessarily reflect those currently employed in U.S. In particular, this environment abstracts from the institutional developments that have taken place in late 1970’s that facilitated the use of interest-bearing assets for consumption smoothing purposes. The abstractions used here are better suited for countries that rely upon a monetary system with legal restrictions or perhaps for the monetary system in U.S. before the financial deregulation. It is important to note that what is crucial for the purposes of consumption smoothing is the real return on liquid assets, and a real return of \(-5\%\) has been experienced even in the U.S. economy.

Tools for computing equilibria for monetary economies with heterogeneous agents are limited. The approach taken in this research is to discretize the economy and use numerical methods to compute the equilibrium for the approximate economies. The statistical properties of the equilibrium stochastic process are determined and examined for these approximate economies.

The paper is organized as follows: The second section is a presentation of the economies studied. The calibration of the model and the computation method is described in section 3. In section 4 the results are discussed and the statistical properties of the equilibrium are examined. Section 5 concludes.

2. Structure of the economies

The economy consists of many infinitely-lived individuals who are different at a point in time only in their real cash balances and employment opportunities. They maximize:

\[
E \sum_{t=0}^{\infty} \beta^t U(c_t),
\]

where \(0 < \beta < 1\) is their subjective time discount factor and \(c_t\) is their consumption of the nondurable good in period \(t\). The utility function is twice
continuously differentiable, and increasing and concave in $c_t$, and has the following form:

$$U(c_t) = \frac{(c_t^{1-\sigma} - 1)}{(1-\sigma)}, \quad \sigma > 0.$$  \hfill (2)

Agents are endowed with one indivisible unit of time in each period and face an employment opportunity which is independent across agents. The employment state, $s$, is assumed to follow a first-order Markov process with two possible states, $s = u$ and $s = e$, that stand for unemployed and employed, respectively. Transition probabilities for this process are given by

$$\chi = (s, s') = \Pr\{s_{t+1} = s', s_t = s\} \quad \text{for} \quad s, s' \in S = \{e, u\},$$  \hfill (3)

where, for example, $\Pr(s_{t+1} = e/s_t = u) = \chi(u, e)$ is the probability of being employed at $t + 1$ conditional on the agent being unemployed at period $t$.

If the employed state $s = e$, occurs, an agent produces $y$ units of the consumption good using the time allocation. In the unemployed state $s = u$, an agent produces $\theta y$ units of consumption good through household production, where $0 < \theta < 1$.

Individuals enter each period with individual nominal money balances equal to $m_{t-1}$ that are carried over from the previous period. These balances are augmented with a lump-sum transfer equal to $T_{t-1}$, where $G_{t-1}$ is the per-capita nominal money supply at time $t - 1$ and $\tau$ is the constant growth rate of money supply. The money supply follows the law of motion

$$m_t = (1 + \tau) m_{t-1}.$$  \hfill (4)

Let $p_t$ be the price of money in terms of the consumption good at time $t$. Then an individual’s budget constraint can be written as

$$c_t + p_t m_t = p_t m_{t-1} + y_t + \tau p_t \tilde{M}_{t-1},$$  \hfill (5)

where

$$y_t = \begin{cases}  y & \text{if} \quad s = e, \\ \theta y & \text{if} \quad s = u, \quad 0 < \theta < 1. \end{cases}$$  \hfill (6)

If we define, $1/(1 + \Pi_t) = p_t/p_{t-1}$, where $\Pi_t$ is the inflation rate and $m_t = p_t \tilde{m}_t$ as the real money balances, we can rewrite the budget equation as

$$c_t + m_t = \frac{1}{(1 + \Pi_t)} m_{t-1} + y_t + \left(\frac{\tau}{(1 + \Pi_t)}\right) M_{t-1},$$  \hfill (7)

where $M_{t-1}$ is the real per capita money stock and $y_t$ is given by (6).
In this environment borrowing is not allowed; $m_t$ is required to be nonnegative. Since event-contingent insurance is not permitted, individuals can insure only through holdings of liquid assets. In equilibrium they will accumulate cash balances during the periods when they work to provide for consumption during the periods when they are unemployed.

The maximization problem faced by an individual in this economy can be represented as a dynamic programming problem where $m = m_{t-1}$ and $s = s_t$ are the state variables and $m' = m_t$ is the decision variable.

The optimality equation for this problem is

$$V(m, s) = \max \left\{ U(c) + \beta \sum_{s'} \chi(s, s')V(m', s') \right\}, \quad (8)$$

where maximization is over $m'$ and is subject to the constraint (7) and $m' \geq 0$.

A stationary competitive equilibrium for this economy consists of a sequence of prices of money in terms of consumption goods $\{p_t\}$, a set of decision rules $m'(m, s)$ and $c(m, s)$, and an invariant distribution, $\lambda(m, s)$, that is a measure of agents of type $(m, s)$ such that:

(i) Given the sequence of prices, the individual's decision rules solve (8).

(ii) The goods market clears. That is,

$$\sum_{m, s} \lambda(m, s)c(m, s) = \sum_{m, s} \lambda(m, s)y(s). \quad (9)$$

(iii) The money market clears. That is,

$$\sum_{m, s} \lambda(m, s)m(m, s) = M, \quad (10)$$

where $M$ is the real per-capita money stock.

(iv) $\lambda(m, s)$ is a stationary distribution,

$$\lambda(m', s') = \sum_{s} \sum_{m \in \Omega(m', s)} \chi(s, s')\lambda(m, s), \quad (11)$$

where $\Omega(m', s) = \{m: m' = m'(m, s)\}$.

The last condition indicates that we are examining the stationary equilibria where the distribution of agents with respect to $(m, s)$ stay constant over time. The procedure to obtain this invariant distribution is explained in the following section.
3. Computation of the equilibrium

For the economies to be fully specified it is necessary to choose the invariant transition probabilities for $\chi$ and specific parameter values for the subjective time discount factor, $\beta$; the coefficient of risk aversion, $\sigma$; and the income of an unemployed individual relative to the income of an employed person, $\theta$. The following values are taken directly from İmrohoroğlu (1989), where they were chosen so that certain key statistics for the model economies would match those for the U.S. economy: $\beta = 0.995$, $\sigma = 1.5$, $\theta = 1/4$.

The time period for the model is selected to be six weeks and the income of an employed individual is normalized to be one. In order to compute the welfare costs of 10% and 5% inflation, three economies that differ only in their money supply growth rates are considered: $\tau = 0.0125$, $\tau = 0.0063$, and $\tau = 0.0$ per period, indicating economies with steady state inflation rates of 10%, 5%, and 0%, respectively. For a given inflation rate, the equilibrium real money balances will be constant between periods since there is no aggregate risk.

The transition probabilities for $\chi$ are selected so that the average duration of unemployment, $D_u$, and the rate of employment, $N$, are\(^2\)

$$D_u = 12 \text{ weeks} \quad (2 \text{ model periods}),$$

$$N = 0.92.$$  \hspace{1cm} (12) \hspace{1cm} (13)

Given a choice for average duration, $D_u$, the proportion of people who stay unemployed can be computed easily. In general the average duration of any state $s$ is $(1 - \chi(s, s))^{-1}$. This can be seen by the following example. Let $n_{i,t+1}$ be the number of people unemployed for $i$ periods at $t + 1$ and $\phi$ be the proportion of people who stay unemployed. Then

$$n_{i,t+1} = \phi n_{i,t},$$

and

$$D_u = \sum_i in_i / \sum_i n_i = 1/(1 - \phi).$$  \hspace{1cm} (14)

Thus, for our economy the proportion of the people who stay unemployed, $\chi(u, u)$, is given by $\chi(u, u) = 1 - 1/D_u$. Since the rows of the transition matrix sum to one, there is only one more transition probability that needs to be selected. $\chi(e, e)$ is selected so that given $\chi(u, u)$, the fraction employed is

\(^2\)Unemployment rate in our model economy does not correspond directly to that which the Bureau of Labor Statistics measures. For the purposes of the model, there is no distinction between in and out of the labor force.
0.92. With these parameter values, the transition probabilities matrix is

\[
\begin{bmatrix}
0.9565 & 0.0435 \\
0.5000 & 0.5000
\end{bmatrix}.
\]

(15)

The values for the risk aversion parameter and the time discount rate are always the same for economies with and without inflation.

Tools for computing the equilibria for the above economies are limited. An approach that is proving to be very useful to the analysis of models with nonconvexities or models with inequality constraints is to study their properties directly by using numerical methods. This approach has also been used by Diaz-Gimenez and Prescott (1989) and Aiyagari and Gertler (1989) in their analysis of economies with heterogeneous agents.

The first step is to discretize the state space and the control space. The state space consists of real money balances, \( m \), and the employment state, \( s \). The latter is a discrete variable that takes two values. The former is discretized by choosing the maximum level of cash holdings that an agent is permitted to hold to be roughly equal to the average annual per-capita income if the employed state continues for a year, and then utilizing a grid of 301 points with increments of 0.027 in money holdings. The sensitivity of the results to the limit on maximum asset holdings and to the tightness of the grid is examined by increasing the limit to 10 and keeping the grid at 301, and also by widening the grid to 251 while keeping the limit on savings at 8. These changes had a negligible effect on the results. The total number of possible states for the individual is then 602, the number of employment states times the number of the states for real cash holdings. At each point in time the number of possible outcomes is finite, never exceeding 301. Consequently the problem is a finite state, discounted dynamic program.

The optimal value function and the decision rule for cash holdings are obtained by method of successive approximations. The basic approach is to start with the initial approximation, \( V_0(m, s) \), compute the next approximation of the value function, and continue this process until the sequence of value functions converge. We have found that the sequence of decision rules converged after 120 iterations.

After the decision rules are found, the \( 602 \times 602 \) equilibrium state transition probability matrix is checked for ergodicity. Given that the Markov chain has a unique ergodic set, there exists a unique invariant distribution with probabilities \( \lambda(m, s) \), for the equilibrium Markov process governing an individual’s state of employment prospects and real cash balances. This invariant distribution specifies the limits of the fractions of the time a particular individual is in these various cash balances-employment states as

\[3\text{The proof of the ergodicity of the state transition matrix is given in the appendix.} \]
the sample period goes to infinity. Moreover, for an economy without aggregate uncertainty, as the one examined in this study, this distribution is also the same as the distribution of people's states at a given point in time, given the independence of processes over individuals.

The invariant distribution is obtained in the following way. Let \( \lambda_t(m, s) \) be the fraction of the time an individual attains a particular state \((m, s)\). The probability that state \((m', s')\) occurs, given the last period's state \((m, s)\) and the decision rule \(m' = m'(m, s)\) is \(\lambda_{t+1}(m', s')\). Thus to compute the invariant distribution, an initial guess for \(\lambda_0(m, s)\) is chosen and the following iteration is carried out:

\[
\lambda_{t+1}(m', s') = \sum_s \sum_{m \in \Omega(m', s)} \chi(s, s') \lambda_t(m, s),
\]

where

\[
\Omega(m', s) = \{ m : m' = m'(m, s) \}.
\]

The ergodicity of the Markov process and the absence of cyclically moving subsets guarantees that this sequence of recursively defined distributions converges to a unique invariant distribution \(\lambda(m, s)\), from any initial distribution.

The statistical properties of the equilibrium Markov process are computed using the invariant distribution \(\lambda(m, s)\). Since the law of large numbers holds, the sample average of any function \(f(m, s)\) converges to the expected value of \(f\) with respect to this invariant distribution. For example, if \(m(m, s)\) is the equilibrium money holdings, then average money holdings, \(\bar{m}\), is

\[
\bar{m} = \sum_{m, s} m(m, s) \lambda(m, s).
\]

The same results are obtained by creating long time series by using Monte Carlo methods. In this study individual time series that consist of 500,000 periods are generated and average utility, consumption, income, and money holdings are found for the economies analyzed.

Our computational procedure determines whether an equilibrium exists. The first step of the procedure is to solve the agents problem given the inflation rate, \(\Pi\), and an initial price level. Notice that at the steady state, inflation rate will equal the growth rate of money supply. Thus we only need to choose the initial price level in order to start the process. The next step is to compute average consumption, output, and money holdings using the invariant distribution. The third step is to check whether the commodity market clears at this initial price, if not a new initial price level is chosen and the above process is repeated until market clearing in the commodity market is obtained.
4. Findings

Table 1 represents some of the statistical properties of the economies with 10%, 5%, and 0% inflation rates, respectively. Average consumption and income are the same in these economies, however, average money holdings are significantly lower in economies with higher inflation rates. This is due to agents economizing on cash balances. As it was discussed earlier, in these economies with imperfect insurance, inflation hinders the ability to self-insure. Consequently, the volatility of consumption (as measured by the standard deviation of consumption) is larger and the average utility is smaller in economies with larger inflation rates.

Given the information in table 1, there are several measures that can be used to estimate the welfare cost of inflation. The first method is to use the traditional approach and estimate the area of the triangle under the money demand function.

4.1. Welfare triangle

In Lucas (1981), the welfare cost of inflation is expressed as a fraction $C$ of real national income:

$$ C = \frac{1}{b v} \left[ 1 - (1 + b \Pi) e^{-b \Pi} \right] \approx \frac{1}{2} \frac{b}{v} \Pi^2. \tag{18} $$

where $v$ is the velocity of real cash balances at zero inflation rate and $b$ is the semi-interest elasticity of the money demand function.

Using the above equation and selecting $b$ to be five and $v$ to be four, Lucas estimates the welfare cost to be 0.45% of GNP. Fischer (1981) obtains
a cost estimate of 0.30%, however, he selects the semi-elasticity, $b$, to be 2.5.\(^4\)

The information in table 1 can be used to construct the money demand curve for this economy. Real money balances go down from 2.267 to 1.110 as the inflation rate is increased from 0% to 10%, which yields the semi-interest elasticity of the money demand function as 3.43. Velocity is found by dividing the annual income for this economy, which is equal to average income per period, 0.940, times the number of periods in a year, 8, by the amount of cash holdings in an economy with zero inflation, 2.267. This calculation yields $v$ equal to 3.32. Both the semi-interest elasticity and the velocity found for this economy are reasonably close to the ones selected for the U.S. economy by Lucas and Fischer.

Using the equation given in (18), the area under the money demand curve is computed and the welfare cost of inflation for this economy is found to be 0.41% of GNP.

Similar calculations can be made to estimate the welfare cost of five percent inflation. The semi-interest elasticity of the money demand curve in that region turn out to be 4.16. The area under the demand curve yields a cost estimate of 0.14% of GNP which is almost the same as Lucas' 0.13% for this inflation rate.

Given that the model economy generates a money demand curve with properties similar to the ones used by Lucas and Fischer we can utilize it to answer the following question. Is it appropriate to use the traditional approach of measuring the triangle under the money demand function to estimate the welfare cost of anticipated inflation in economies with imperfect insurance of the idiosyncratic risk? In order to answer this question welfare costs will be computed by comparing the steady state average utilities.

4.2. Comparison of steady state average utilities

Given the information in table 1 and the invariant distribution $A(m, s)$, the welfare cost of inflation is computed by finding the increase in average income that is necessary to compensate the individual for the loss of utility due to 10% inflation. The level of income that will leave the individual indifferent between the economies with 0% and 10% inflation is found by searching over higher levels of income, $y$, in the economy with 10% inflation. This approach explicitly takes into account the changes in welfare that are due to changes in volatility of consumption.

For the parameter values specified in section 3, the welfare cost of inflation is 1.07% of total GNP. That is, an increase in average income of 1.07% every

\(^4\)The cost estimates are measures of compensating variation. Fischer notes that the difference between the compensated and uncompensated elasticities is less than 0.01 in case of unitary income elasticity.
period is needed to compensate the individual for the loss of utility due to 10% inflation. This cost estimate is about three times larger than the one found by using the traditional measure and it is quite large as deadweight losses go.

Similar calculations are made to find the welfare cost of 5% inflation. For this economy the welfare cost is found to be 0.57% of GNP. The difference between this cost estimate and the one found by using the welfare triangle is by a factor of four. In fact, the difference between the two measures is more pronounced at lower inflation rates. This is due to the fact that the volatility of consumption in these economies approaches to the volatility of income as the inflation rate approaches to infinity. The fact that the traditional measure accounts for only one-fourth of the true costs in the economy with 5% inflation is noteworthy since it is easier to motivate the use of money for consumption smoothing purposes at lower inflation rates than at higher inflation rates.

The behavior of real cash balances at the steady state, for different risk aversion parameters is also examined. Increasing the risk aversion coefficient from 1.5 to 5.0 increased the average cash holdings from 2.267 to 5.107 in the 0% inflation economy and from 1.110 to 3.254 in the 10% inflation economy. Comparison of steady state utilities yielded the welfare cost of inflation to be 1.18% of income. Costs computed by estimating the area under the money demand function were 0.65% of income.

The sensitivity of these results to the specification of the subjective time discount factor, $\beta$, is also examined. For $\sigma = 1.5$, the welfare cost of 10% inflation with $\beta = 0.9925$ and $\beta = 0.9975$ which imply steady state annual subjective time discount rates of 6% and 2%, respectively, are computed. The corresponding cost estimates found by examining the average utilities are 1.04% and 1.14% of income. For these economies the traditional approach yields estimates of 0.30% and 0.64% of income, respectively.

Overall, the above results indicate that it may not be appropriate to use the traditional approach to estimate the welfare cost of anticipated inflation in countries where the institutional arrangements are such that individuals have to hold non-interest-bearing assets to facilitate consumption smoothing. This conclusion is especially valid at low inflation rates where it is easier to motivate the precautionary demand for money.

The reason for the discrepancy between the two measures can be explained as follows. In order for the traditional approach to yield a good approximation to the true costs, the marginal rate of substitution across individuals need to be identical. This assumption is satisfied in transaction-based approaches to money demand. Indeed, Cooley and Hansen (1989) compare the steady state average utilities in a cash-in-advance model of money and still obtain a cost estimate similar to Lucas'. However, in the economics studied here, the marginal rate of substitution is not equal across
individuals. This is due to the fact that agents are not allowed to perfectly
insure against the idiosyncratic risk. At a given point in time the restrictions
on asset holdings will be binding for some individuals and not for some
others, causing their marginal rates of substitution to differ.

These findings suggest that models in which money is held for consumption
smoothing purposes may have different policy implications for some issues
than models that motivate the use of money through transaction costs.

5. Concluding remarks

In this paper simple pure currency general equilibrium models are studied
in order to assess the welfare costs of inflation in economies where money is
used to facilitate consumption smoothing in the presence of idiosyncratic
risk. Several economies that differ only in their money supply paths are
considered. It is shown that the model economies constructed here generate
a money demand function that is very close to that obtained by Lucas (1981)
and Fischer (1981). The findings suggest that, for the type of economies
considered here, the area under the empirical money demand curve is a poor
measure of the welfare cost of inflation. This area accounts for only one-fourth
of the true costs in case of 5% inflation and one-third of the true costs in
case of 10% inflation.

Appendix

Definition. An ergodic set $E$ is a set of states such that if $x, y \in E$, then for
some $n$, $P^n(x, y) > 0$.

For the economy with inflation the ergodic set is $E = \{x \in X : m < 1.39\}$
where $x = (m, s)$ and $s = e$.

Proof. The equilibrium law of motion for cash holdings shows that, starting
from any state $x$, there is a positive probability of reaching the level of cash
holdings $m = 1.39$ in a finite number of periods, given all $\chi = (s, s')$ are
positive. Further, if $m_0 > 1.39$, no point with $m > 1.39$ can be reached with
positive probability. Therefore, states with $m > 1.39$ are transient. In other
words

\[ P^n(x_0, x) = 0 \quad \text{for all } n \quad \text{if} \quad m > 1.39, \]
\[ P^n(x_0, x) > 0 \quad \text{for some } n \quad \text{and for all} \quad m < 1.39. \]

This result follows from having $\chi(s, s') > 0$ for all $s$ and $0 < \Delta f_s(m)/\Delta m < 1$
for each $s$. □
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