Vagueness at every order

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Principles:

1. $\Delta^* p \to \Delta\Delta^* p$.
2. $\neg\Delta^* p \to \Delta\neg\Delta^* p$.

(1) and (2) give us the unacceptable:

PRECISION: $\Delta\Delta^* p \vee \Delta\neg\Delta^* p$.

by LEM and reasoning by cases (in conditional form, i.e., globally and locally valid.)

Reasons to deny PRECISION:

1. 'is a Δ^* child' is Soritesable in the same way 'is a child is'.
2. If it were precise what barrier would there be to my finding out how many nanoseconds my Δ^* childhood lasted, in the same way there is no barrier to my finding out how many nanoseconds in a year.
3. Assertions like 'my Δ^* childhood lasted 12987587298739 nanoseconds' are evidently bad, but if it is not because of their vaguess, then what?

1 Denying Precision

(1) follows from distributivity:

DIST $\bigwedge_{i<\omega} \Delta\phi_i \to \Delta \bigwedge_{i<\omega} \phi_i$.

(2) follows from

B: $p \to \Delta\neg\Delta\neg p$

B$^{n\prime}$: $p \to \Delta\neg\Delta^n\neg p$

B^n: $p \to \Delta(q \to \phi_n)$ where $\phi_1 := \neg\Delta\neg p$; $\phi_{n+1} := \neg\Delta\neg(q \wedge \phi_n)$

B*: $\Delta(p \to \Delta p) \to (\neg p \to \Delta\neg p)$

Frame conditions

B: Reflexive frames, if Rxy then Ryx

$\mathsf{B}^{n\prime}$: 'Everywhere you can get to you can get back from in less than n steps'. If Rxy then for some $z_1, \ldots, z_n$, Ryz_n, …, Rz_1x.

B^n: 'Everywhere you can get to you can get back from in less than n steps each of which you can initially get to'. If Rxy then for some $z_1, \ldots, z_n$, Ryz_n, Rz_nz_{n-1},…, Rz_1x and Rxz_1, …, Rxz_n.

B^*: 'Everywhere you can get to you can get back from in **finitely many** steps each of which you can initially get to'. If Rxy then for some n and $z_1, \ldots, z_n$, Ryz_n, Rz_nz_{n-1},…, Rz_1x and Rxz_1, …, Rxz_n.

In the lattice of modal logics KTB^* is the infimum of KTB^n.

KTB^* imposes a second order condition on it's frames. It is not compact or strongly complete, but it is characterised by the frames with the unbounded backtrack property.

2 Models

Definition 2.0.1. *A v-frame is a triple $\langle W, d(\cdot,\cdot), f(\cdot)\rangle$ where $\langle W, d\rangle$ is a metric space, and $f : W \to \mathbb{R}^+$ obeys the following:*

(A) $\forall w, v \in W, |f(w) - f(v)| \leq d(w, v)$

A formula of propositional modal logic is valid on a v-frame $\langle W, d, f\rangle$ iff it is valid on the Kripke frame $\langle W, R\rangle$ where Rxy iff $d(x, y) \leq f(x)$.

Fact: The modal logic of v-frames is KT. In particular you can refute B^n for each n and B^*

Fact: The modal logic of v-frames based on R^n in which $f(x) > 0$ contains KTB^*, but refutes each B^n.