Vagueness and Thought

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Abstract

According to orthodoxy the study of vagueness belongs to the domain of the philosophy of language. To solve these paradoxes we need to investigate the nature of words like ‘heap’ and ‘bald’.

In this book I criticise linguistic explanations of the state of ignorance we find ourselves in when confronted with borderline cases and develop a classical non-linguistic theory of vagueness in its stead. The view places the study of vagueness squarely in epistemological terms, situating it within a theory of rational propositional attitudes. The resulting view is applied to a number of problems in the philosophy of vagueness.
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Chapter 1

The Sorites Paradox

Consider the following valid piece of reasoning:

1. Anyone with 100,000,000 cents (a millionaire) is rich.
2. For any \( n \) if a person with \( n + 1 \) cents is rich a person with \( n \) cents is rich.
3. Therefore anyone with 1 cent is rich.

This argument can be formalised and logically proven from the rule of modus ponens (from \( A \) and ‘if \( A \) then \( B \)’ infer \( B \)) and universal instantiation (from ‘everything is \( F \)’ infer ‘\( a \) is \( F \)’)—both of which are principles accepted by pretty much everyone who has participated in this debate. Anyone who accepts the premises of this argument and closes their beliefs under modus ponens and universal instantiation will therefore accept the conclusion. Yet the premises are seemingly acceptable and the conclusion, seemingly, is not.

When I present this paradox to students for the first time, or when I am explaining what I do to friends and family, I commonly receive the following dismissive response:

There is no paradox here, it all just depends on how you define the word ‘rich’.

I am sure many reading this are familiar with this response. How exactly does this remark solve the paradox? Does it recommend accepting the conclusion, and if not which premise does it recommend denying? After a little thought some will elaborate as follows:

Once you have said what you mean by ‘rich’ it will become clear which premise to deny. If you stipulate ‘rich’ to mean having more than 20,000,000 cents (200,000$) then the second premise fails, but there is nothing puzzling about this—read this way the second premise entails ‘if a person with 20,000,001 cents has more than 20,000,000 cents a person with 20,000,000 cents has more than 20,000,000 cents’ and this is quite clearly false.

Of course, there is nothing particularly special about the second premise. You might also add that if you stipulate ‘rich’ to mean having more than a trillion trillion cents then nobody is rich and the first premise fails. If you stipulate that ‘rich’ applies to everything then the conclusion holds. Whatever one takes ‘rich’ to mean there is no paradox; on some readings the premises aren’t acceptable (they don’t even seem acceptable) and on others the conclusion isn’t unacceptable (and doesn’t even seem unacceptable.)

\(^1\)A number of simplifying assumptions have been made to make the argument below easier to parse. For example, I have assumed that whether someone is rich depends on how many cents they have. Of course this assumption is wildly unrealistic— one can have wealth in other currencies and assets that are not monetary.

\(^2\)I shall idealize throughout by assuming that whether you are rich depends only on how many cents you have (I ignore other assets one might have, wealth in different currencies, etc.)
I hope that everybody agrees that this ‘solution’ is not satisfactory. Whether someone is rich or not does not depend on what you have decided to stipulate the word ‘rich’ to mean. There is a quite straightforward empirical test one can perform at home to demonstrate this: log into your bank account, stipulate away, and observe that you do not become one bit richer. Your balance will remain exactly the same – if you were rich before you will remain rich and if you weren’t you will remain that way too. This is a get-rich-quick scheme that will not work.

To think that you have solved the sorites paradox by saying something about the word ‘rich’ is wrong headed – the word ‘rich’ and the way it is used has nothing intrinsically to do with being rich. There is, of course, another sorites paradox not about rich people but about people to whom the word ‘rich’ applies when it is being stipulated that ‘rich’ means such and such. However deflating this paradox does very little to address the original.

This is the explanation I give to my students, and for what it is worth, it is the explanation I gave before I endorsed the views defended in this book. It is also, I hope you’ll agree, good old common sense.

It should be extremely surprising, then, to discover that almost all contemporary accounts of vagueness commit something like the conflation noted above. Invariably the conflation is less blatant, but it is there nonetheless. According to these theorists, vagueness – the phenomenon responsible for the sorites paradox – is just a feature of the way that linguistic communities use words like ‘rich’. By one popular account (but by no means the only one satisfying this description) the use of the word ‘rich’ by English speakers is not specific enough to allow it to latch on to a single property and to consequently draw a single boundary. On each way of drawing that boundary it is sharp – a single cent can take you over the boundary – but the use of the word ‘rich’ in English doesn’t determine which cent that is because it doesn’t determine a single sharp meaning with which to draw the boundary.

Even with such a preliminary sketch of the view, it is hard not to think that it is ignoring the moral we drew from our discussion of the naïve response to the paradox – that one cannot solve the sorites paradox just by saying something about the word ‘rich’. Even if we could convince the entire English speaking population to use the word ‘rich’ differently, doing so would not make you any richer or poorer. The account therefore does very little to explain why we find it hard to imagine that one person could be rich while another person possessing one less cent isn’t. The most it does is explain why it’s hard to imagine that the word ‘rich’ could apply in English to one person without applying to someone with one less cent. This would be fine if we had asked about the variant sorites involving people to whom ‘rich’ applies in English, but once again we have done little to address the paradox we started with which was about rich people. (Indeed, some semantic indecision theorists are even quite explicit about the centrality of the variant sorites. For example McGee and McLaughlin, after proving there must be sharp cut-off concerning what looks red to someone, they write ‘How do we get from the thesis that, for some n the nth tile looks red to you but the n + 1th tile does not to the metatheoretical conclusion that the concept expressed by the phrase ‘looks red to you’ has a sharp boundary? This, it seems to us, is the crux of the sorites problem.’ (McGee and McLaughlin, [90]).)

To see why such accounts are explanatorily unsatisfactory note that one doesn’t need to be familiar with the English language to appreciate the original paradox. A monolingual Chinese speaker can easily see the conflict between saying that millionaires are rich and that small amounts of money cannot make the difference between being rich and not rich. Like us they can also feel the intuitive pull of both claims. However they will be utterly baffled by an attempt to explain this in terms of the conditions under which the English produce tokens of the word ‘rich’, which is to them just an unintelligible sound.

These preliminary points, I think, will leave many people unconvinced. After all, if there’s one thing that almost everyone agrees on – whether you’re an epistemicist or supervaluationist or something else – it’s that vagueness has something to do with the way we
use language. And if it isn’t linguistic it must be either metaphysical or purely epistemic, and neither of these options seem particularly promising. The primary aim of this book is to outline an account of vagueness that isn’t fundamentally a linguistic phenomenon or, indeed, a metaphysical or purely epistemic phenomenon (indeed, this seems to be a false trilemma.) In the next chapter I attempt to put pressure on linguistic explanations of the state of ignorance we find ourselves in when confronted with borderline cases.

Rather than treat the study of vagueness as a branch of the philosophy of language, the alternative view places the study of vagueness squarely in epistemological terms – here vagueness is characterised by its role within a theory of rational propositional attitudes; specifically belief and desire (see chapters 4-6.)

The approach I recommend here, whilst endorsing classical logic, ends up departing fairly substantially from orthodox treatments of vagueness. It rejects, for example the traditional formalism for dealing with vagueness that invokes possible worlds and precisifications (see chapter 3) and proposes an alternative formalism involving certain kinds of function (‘automorphisms’) that preserve a certain class of rational attitudes (see chapter 7).

1.1 Responding to the Sorites

We began this chapter with a paradox. Any solution worth its salt must at minimum say which premise to reject. I reject the second premise; I deny that for any \( n \), if a person with \( n + 1 \) cents is rich, so is a person with \( n \) cents.

Let us say that number (of cents) is a cut-off for the property of being rich if the corresponding conditional is false. This definition generalises:

**Definition** An element, \( x \), of a sorites sequence for \( F \) is an interior point if and only if, if \( x \) is \( F \) then \( x' \) (\( x \)'s successor in the sequence) is \( F \).

**Definition** An element, \( x \), of a sorites sequence for \( F \) is a boundary, or cut-off point if and only if it’s not an interior point. I.e. It is not the case that if \( x \) is \( F \) then \( x' \) is \( F \).

I also think that properties, like being rich, have cut-off points. If one assumes classical logic this fact can be inferred, via contraposition, from the validity of our initial argument. Thus, from the fact that anyone with 100,000,000 cents is rich, and the fact that someone with 0 cents is not rich it follows that for some \( n \), it’s not the case that someone with \( n \) cents is rich if someone with \( n + 1 \) is. That is, the property of being rich has a cut-off point.

This means there is a number, \( n \), such that the person with \( n + 1 \) cents is rich, whilst the person with \( n \) cents is not: a single cent can make the difference between being rich and not being rich.\(^4\) If that number is 159,927,821 cents, for example, it follows that people with 159,927,821 cents are rich while people 159,927,820 cents are not rich.

Although this might sound wild, it is important to stress at this point that this conclusion was derived in classical logic only from the claim that millionaires are rich and the claim that people who have nothing are not rich. This conclusion is therefore common to anyone who accepts classical reasoning and these two premises. The existence of cut-off points is not just the domain of epistemic theories of vagueness, it is quite general.

Throughout the rest of this book I shall assume that the reader is familiar with, and is at least able to get into the mindset of someone who accepts this result; unless they do so they will get very little out of this book. It will be worth our while, therefore, to briefly

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\(^3\)In this broad respect the approach matches the approach of Field [44] and Schiffer [110] who focus on the interaction between vagueness and degrees of belief.

\(^4\)Again, I am assuming classical logic here. The non-classical logician who distinguishes between \( \neg(Fx \rightarrow Fx') \) and \( Fx \land \neg Fx' \) simply has two different notions of a cut-off point. A non-classical logician attempting to avoid sharp cut-off points must therefore be careful to ensure that they avoid both kinds of cut-off point.
say a bit about the alternatives. The alternatives can, broadly speaking, be divided into two kinds: (i) those that reject classical logic, and (ii) those that reject the first premise or accept the conclusion – i.e. those who maintain that either nobody or everybody is rich. Since the dialectic will be pretty symmetrical I clump both those options into the same category. I shall now argue that these alternatives are even worse than the result that a single cent can make the difference between being rich and not rich.

1.1.1 Weakening Classical Logic

A perhaps noteworthy aspect of the result that there is a sharp cut-off between the rich and non-rich people is that it is non-constructive. Classical logic tells you there must be some sharp cut-off, but it doesn’t tell you which. How is it that you can know the existential claim that there is a cut-off point, without knowing any of the instances? You know because you know the last guy isn’t rich and the first guy is, and classical logic guarantees that there must be a cut-off point even if there is no particular point that it guarantees is the cut-off. The possibility of this kind of situation is a feature distinctive of classical logic, and there are alternative logics that do not have this.

A related feature of classical logic that you might also find puzzling is that it commits you to the law of excluded middle – ‘either \( A \) or it’s not the case that \( A \)’. In the particular case at hand this entails that everyone is either rich or not rich. This is surprising, for let us suppose that Harry has that borderline amount of money in which it is neither clear that he’s rich nor clear that he’s not rich. The law of excluded tells us that even here, either Harry is rich or he isn’t. Of course, logic alone doesn’t tell us which, but what’s surprising is that logic alone delivers disjunctions whose disjuncts seem to be in principle unknowable.

All of this might suggest that it is the non-constructive nature of classical logic that is responsible for this paradox, particularly the law of excluded middle which seems to play a special role in the derivation of the sorites paradoxes. However, one might argue that the instances of the law of excluded middle responsible for the paradoxes don’t seem to be particularly plausible. To prove that there’s a cut-off for richness one will inevitably have to appeal to instances of LEM that have borderline disjuncts, and these don’t seem to have much independent appeal.

Indeed, if the dispute just boiled down to the plausibility of the law of excluded middle I think that non-classical responses to the paradoxes would have the clear upper hand. The loss of this non-obvious theorem of classical logic is, to my mind, of little importance when compared to the puzzling phenomena of cut-off points.

The idea that the law of excluded middle is the culprit is supported by the fact that its presence suffices for a derivation of the existence of cut-off points, against some relatively natural background logic. Given the role of the law of excluded middle in the non-constructive proof of a cut-off point the most obvious alternative to classical logic to consider would be intuitionistic logic. Not only does this logic fail to have LEM as a theorem, it has the more general property that a disjunction is only provable if one of the disjuncts is, and an existential formula is provable only if one of its instances is, making it suitable for constructive reasoning in general.

Unfortunately, although excluded middle is one way to prove the existence of a cut-off point it is not the only way. For example, it’s been known for a while that one can in a certain sense prove the existence of a cut-off point in intuitionistic logic.\(^5\) This derivation makes no use of the law of excluded middle since it is not present in intuitionistic logic.

Thus the assumptions we must relinquish to avoid the existence of sharp cut-off points must

\(^5\)Although this fact depends somewhat how one formalises the claim that there are cutoff points. One can prove the negated conjunction ‘it’s not the case that: 1 is an interior point and 2 is an interior point and ...’. One can also prove that it’s not the case that there are no cut-off points. Although the intuitionist might be able to make their peace with these conclusions, other formulations of the existence of sharp cutoff points can arise if we add natural principles about truth and justification. See Williamson [134]; see also Read and Wright [104] for more discussion.
include more than just the assumption that everyone is either rich or not rich. What, then, must we give up in addition to excluded middle?

There are no doubt many intuitive principles that must be given up, but let me focus on a couple that I think are particularly striking:

CS. \((A \rightarrow B) \land (B \rightarrow C) \rightarrow (A \rightarrow C)\)

PMP. \((A \land (A \rightarrow B)) \rightarrow B\)

These are sometimes called conjunctive syllogism and pseudo modus ponens respectively. In both cases one can prove from these principles, along with some relatively modest background logic, that there is a sharp cut-off point. For the proof, and a more detailed description of the background logic, see appendix 9.1.6

The kinds of instances of the first principle that we need to appeal to can be interpreted as follows. Imagine you have two people, Alice and Bob, with \(n\) and \(n + 1\) cents respectively. Then to avoid sharp cut-offs the non-classical logician must reject instances of PMP like the following:

(1) If Alice is rich and, moreover, Bob is rich if Alice is rich, then Bob is rich.

Unlike the claim that either Alice is rich or she isn’t, the denial of this principle invites the incredulous stare.7 If Bob is rich if Alice is rich, and moreover Alice is rich, how on earth could Bob fail to be rich? To deny (1) borders on incoherence.

A similar puzzle can be raised against denials of the second principle. Imagine now a third character, Charlie, with \(n - 1\) cents. The proof that there are sharp cut-off points now appeals to the following kinds of instances of the principle:

(2) If Bob is rich if Alice is, and Charlie is rich if Bob is, then Charlie is rich if Alice is.

Once again, I cannot fathom how (2) could be denied, or how one could take the phenomenon of vagueness to motivate the denial of this claim. If Bob’s rich if Alice is, and Charlie is rich if Bob is, then how on earth could Charlie fail to be rich if Alice is rich?

Unsurprisingly, these principles are not derivable in the logics that are standardly appealed to in the context of the sorites paradox (this includes the 3-valued Kleene and Łukasiewicz logics, infinite valued Łukasiewicz logic, and various logics that Hartry Field has appealed to to solve the paradoxes of vagueness).8 The above shows that this is not really an accident, since these principles feature essentially in derivations establishing the existence of sharp cut-off points.

Let me mention there are certain paraconsistent logics that do accept CS and PMP: the paraconsistent logic \(LP\) based on the dual of the three valued Kleene logic, for example.9 Since this logic accepts these principles and the background logic required to derive the existence of sharp cut-offs, these logicians must accept the existence of a last rich person in a sorites for richness. Thus this kind of paraconsistent logician is no better off than the classical logician with respect to the existence of sharp cut-offs, and they are probably worse off, for this logic does not have the rule of modus ponens.10 Unlike the classical logician,

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6The logical assumptions I make are modest in the sense that they are shared by all the non-classical logics for dealing with vagueness on the market: Łukasiewicz logic, Kleene logic, and various logics that Hartry Field has applied to the paradoxes of vagueness.

7Note that in denying instances of these principles these theorists are not necessarily committed to asserting the negation of those instances. Such theorists typically take denying to be a \textit{sui generis} speech act not reducible to assertion.

8See for example Field [45]. Interestingly the first principle, CS, does not appear to be as straightforwardly problematic when combined with the semantic paradoxes (see Bacon [4]).

9See Priest [95]. The precise details of this logic need not concern us here.

10Modus ponens is the rule: from \(A\) and \(A \rightarrow B\) infer \(B\). This is distinct from the principle PMP, which is a single sentence and not a rule. One cannot derive the rule of modus ponens from PMP without applying the rule of modus ponens itself. (Conversely, one cannot derive PMP from modus ponens without applying conditional proof – conditional proof is therefore missing in the logics we have been considering, which contain the rule of modus ponens but not PMP.)
these theorists are also committed to the claim that there are no sharp cut-offs in a sorites for richness, but it is unclear how this extra belief helps make their commitment to sharp cut-offs any more palatable.\footnote{One might think that the paraconsistent logician can say something about their commitment to sharp cutoffs to make it sound better: unlike the epistemicist, they will typically be able to say that there are lots of sharp cutoffs, and so in some sense the feeling of arbitrariness is curtailed. But again, this is also a belief they have in addition to one they share with the epistemicist – for just like the epistemicist, this kind of paraconsistent logician thinks that there is exactly one cutoff point. This seems to me to be the very thing we find so puzzling about epistemicism.}

It should be acknowledged at this juncture that there are non-classical logicians who have made their peace with failures of principles such as PMP, and are quite upfront about it. Hartry Field is probably the most prominent example.\footnote{See Field \cite{Field}, for example.} One interesting thing to note about this is that these theorists are often engaged in a more ambitious project: that of producing an all purpose non-classical logic that not only deals with the sorites paradoxes but also accommodates the liar paradox. Field, for example, draws a number of parallels between both paradoxes in \cite{Field}, for example. Given this background project some comfort can be derived from the fact that there are independent reasons be suspicious of the principle PMP – it is susceptible to a version of the liar paradox known as the ‘Curry paradox’.

Be this as it may, it is worth noting that there is no similar independent argument against CS. There are several consistent approaches to the liar paradox that can recover a significant amount of reasoning about truth without relinquishing CS.\footnote{The failures of PMP and CS do not exhaust the problems. Related principles are also problematic: contraction – the inference from \(A \rightarrow (A \rightarrow B)\) to \(A \rightarrow B\) – and conditional proof – the meta-inference from \(\Gamma, A \vdash B\) to \(\Gamma \vdash A \rightarrow B\) – must also be dropped to avoid the existence of sharp cutoff points.}

My main point, which I think most can take on board, is that the costs of the classical/non-classical debate are frequently mischaracterised as a debate about the status of the law of excluded middle. I am not dogmatic about LEM, and I can conceive of situations where it would be reasonable to revise it. However the real cost of such accounts, in my opinion, are that they typically give up principles like pseudo modus ponens and conjunctive syllogism.\footnote{See Bacon \cite{Bacon}, for example. Brady also develops some weakenings of relevant logic that deal with the liar paradox but but contain CS in \cite{Brady}.}

A final issue worth highlighting, is that even if a non-classical logic can give us a satisfactory account of the sorites paradox, there are other puzzles relating to vagueness that need to be addressed before we have a fully adequate theory. One of these further puzzles is the ‘problem of the many’ (see \cite{Field2}) – a problem not evidently directly related to the sorites. There are no doubt a number of different issues that need to be addressed here, but here is something that seems initially puzzling about the non-classical theories we have been considering so far. Firstly it seems as though the property of being at least 29,000ft is a precise property. Although the classical and non-classical logician disagree about much, one might have thought that they could surely agree about the subject matter of their investigations: which properties are vague and precise. Thus one might have thought that the non-classical logician can agree with the classical logician about the preciseness of this property. Since precise properties satisfy the law of excluded middle we have:

\[
\text{Everywhere is either at least 29,000ft tall or not at least 29,000ft tall.}
\]

Now, all of the logics we have considered so far allow us to infer from a universal generalisation, ‘everything is \(F\)’, a specific instance, ‘\(a\) is \(F\)’ (indeed many endorse a stronger axiom form of this inference: that if everything is \(F\) that \(a\) is \(F\)). Thus in all of these logics we can infer from the above sentence:

\[
\text{Either Mt. Everest is at least 29,000ft tall or Mt. Everest is not at least 29,000ft tall.}
\]

Where \(F\) is being understood as the disjunctive property of either being at least 29,000ft or not at 29,000ft, and \(a\) as Mt. Everest. Now evidently the non-classical logician,
because it is borderline, should not accept this conclusion. It follows that the non-classical
logician must either make some revisions to the ordinary conception of precision, or make
some revisions to the quantified logic beyond those that have to be made to accommodate
the sorites paradox (which tend to be modifications to the propositional logic).

1.1.2 Nihilism

If we are granting classical logic, then the only other way to avoid the existence of sharp
cut-offs is to either deny the first premise – that anyone with 100,000,000 cents is rich
– or accept the conclusion – that someone with 1 cent is rich. Call these views nihilism
and universalism respectively. For whatever reason, philosophers who make these kinds of
responses usually make the nihilist response and so I shall focus on that response. The
two views are, however, very closely related and for most of the things I say about nihilism
there is an analogous thing that can be said about universalism.

Note that the nihilistic response, if it is to serve as a general response to the sorites
paradox, must generalise in a few ways. It must firstly extend to people who have more
than 100,000,000 cents – thus no-one, whatever their wealth, is rich. Secondly, if the
response is to be general it must extend to other properties that can be the subject of a
sorites sequence; thus a nihilist thinks that nobody has ever been to the moon, learnt to
swim, kissed, flipped a pancake, and so on, for clearly each of these properties is susceptible
to a sorites sequence. Note that this is more radical than accepting a mere conspiracy
theory, for the nihilist not only thinks that nobody has been to the moon, they also think
that nobody has even appeared to have gone to the moon, no-one has seemed to have
flipped a pancake, and so on – straightforward observational facts about how things seem
to us also go out of the window if nihilism is to be accepted. Symmetrical things must be
said about universalism.

How, then, do nihilists explain these wacky sounding claims? Typically they do so by
invoking some peculiar feature of the language that is used to express the strange sounding
claims – according to nihilism vague language is defective in some way. Perhaps sentences
involving vague predicates simply always fail to express a proposition, or perhaps they
always express a false proposition – presumably the inconsistent proposition, to avoid ar-
bitrariness. Thus in response to the argument we opened with the nihilist will say things
like the following:

Nihilism:

– The predicate ‘rich’ does not apply to anybody.
– Sentences of the form ‘S is rich’ are never true.
– Belief tokens corresponding to ‘S is rich’ are never true.

They will of course say similar things about other vague predicates.

At this point it will be useful to distinguish two different ways one could be a nihilist.
The first way, which I take it is the most common way, is to accept the semantic claim
about the word ‘rich’ but to nonetheless retain one’s common-sense beliefs about who is
rich and who isn’t. Let’s call this the ‘standard nihilist’. The other kind of nihilist – the
‘radical nihilist’ – not only comes to think that all of their ordinary beliefs are untrue, they
come to reject those beliefs as well.

Let me start with the standard nihilist. The thinking behind it goes as follows: the
belief that Warren Buffett is rich, while strictly speaking semantically defective and thus
untrue, is nonetheless an incredibly useful belief to have and thus we should retain the
belief that Warren Buffett is rich despite this semantic defect. Without beliefs like these

\footnote{Peter Unger \cite{Unger}} is probably the most famous instance of this response. For more recent defences see
Braun and Sider \cite{BraunSider} and Beall \cite{Beall}.
it would be impossible to go about our ordinary business. I don’t know exactly how much money I have, but I have a rough idea. Since my beliefs about how much money I have aren’t precise, they’re untrue – but I shouldn’t drop them altogether, otherwise I wouldn’t be able to make economic decisions: I wouldn’t know what I can and can’t afford.

Parallel things can be said about assertion on this view. Although this type of nihilist will maintain that uttering the sentence ‘Warren Buffett is rich’ involves uttering an untruth, they will still retain the practice of uttering this sentence provided they believe that Warren Buffett is rich. The thought is that in uttering strictly untrue sentences one communicates beliefs that, while also strictly speaking false, are useful.

Note that we have to be careful about what we mean by the word ‘true’ in these contexts. The nihilist I’ve described believes both that Warren Buffett is rich and also that their belief that Warren Buffett is rich is untrue. Let us say that a belief is ‘Ramsey-true’ if it is a belief that \( P \) and, in fact, \( P \).\(^{16}\) Thus any moderately reflective person who believes that Warren Buffett is rich believes that their belief that Warren Buffett is rich is Ramsey-true. They believe that Warren Buffett is rich, and since they are moderately reflective they realize that this belief is a belief that Warren Buffett is rich and therefore are in a position to conclude that this belief is Ramsey-true. Thus the nihilist I am describing is not talking about Ramsey-truth when they talk about their vague beliefs being untrue. These theorists rather think that there is another property, more substantial than Ramsey-truth, that plays an important role in semantics and they moreover believe that their vague beliefs do not have this property.

For this kind of nihilist it is important to distinguish sharply between the following two questions:

1. Are there any rich people?
2. Does the word ‘rich’ apply to anyone in English?

Whether or not you are a nihilist, these are quite clearly not the same question. If you are not a nihilist they both have the same answer, but they might not have done. To see this imagine a world in which English speakers use the word ‘rich’ in such a way that it applies only to round squares – let’s say, they use the word ‘rich’ in roughly the same way we actually use the phrase ‘round square’. Let’s also suppose that, much like the actual world, this world suffers from widespread wealth inequality. In this world the answer to whether there are rich people is clearly ‘yes’ while the answer to second question is clearly ‘no’ – the word ‘rich’ applies to nothing.

This example is actually not that far fetched: the nihilist we have been considering thinks the actual world is like the world described above in the relevant respects. Due to the way we actually use the word ‘rich’, and other vague predicates, it ends up expressing an inconsistent property (or perhaps no property at all). However, these nihilists maintain that despite the fact that the word ‘rich’ does not apply to anyone in English, this is not a sufficient reason to abandon the common-sense belief that there are rich people. Thus according to the beliefs of this nihilist the actual world is a bit like the world described above: there are rich people, but the word ‘rich’ doesn’t apply to anyone.

This highlights the most important feature of this kind of nihilist: they have a completely standard conception of what it is to be rich – they retain the common sense belief that billionaires, like Warren Buffett, are rich and that people with only a few cents to their name are not. It is their understanding of ‘truth in English’ and ‘applies in English’ that is non-standard: they will agree with everyone about who is rich and who isn’t, and only disagree about who the word ‘rich’ applies to in English. In particular, they think that Warren Buffett being rich does not suffice for the truth of the sentence ‘Warren Buffett is rich’, since they believe that Warren Buffett is rich, but that the sentence ‘Warren Buffett

\(^{16}\)This notion is due to Ramsey – see [102].

8
is rich’ is not true in English.\(^{17}\)

These nihilists are guilty of the very mistake I raised at the beginning of this chapter. Consider the following argument, that is somewhat analogous to the argument we opened with:

1. The word ‘rich’ applies to anyone with 100,000,000 cents (a millionaire).
2. For any \(n\) if ‘rich’ applies to a person with \(n + 1\) cents, it applies to a person with \(n\) cents.
3. Therefore ‘rich’ applies to anyone with 1 cent.

In response to this sorites it is completely clear which premise this nihilist denies: since the word ‘rich’ does not apply to anything, premise 1 is denied.

However, we took care to distinguish the above paradox from the paradox we opened with, which did not assume that the word ‘rich’ applies to millionaires, it simply assumes that millionaires are rich. And we have noted already that the nihilist I have been describing has a completely standard conception of what makes someone rich. Thus the nihilist grants the first premise of our original argument, and we are back to where we started. If the nihilist retains the common sense beliefs about richness, then they accept the first premise and reject the conclusion. So given classical logic, they must accept that there is a last rich person in a sorites sequence for richness. Of course, there is no last person to which the word ‘rich’ applies, but this is of little comfort if you already have to accept the existence of a sharp cut-off point for richness.

This type of nihilism is one instance of the general tendency in the philosophy of vagueness of changing the subject from puzzles about richness, baldness, and so on, to questions about language. In doing so we often leave the original puzzles unanswered.

These problems stemmed from the fact that the nihilist kept her common-sense false beliefs. There is a much more radical way of being a nihilist – not the kind that is typically endorsed – that seems be in a better position to resist the original sorites paradox we opened this chapter with. This is the kind of nihilist who not only declares common-sense beliefs to be untrue, but also abandons those beliefs. The radical nihilist therefore not only thinks that the word ‘rich’ does not apply to anybody, she also thinks that nobody is rich and is therefore well placed to answer our original paradox by denying the first premise. (Strictly speaking, the nihilist could abandon the belief that millionaires are rich by being agnostic, but then she would have to be agnostic about the existence of cut-off points; I shall therefore focus on the view that there are no rich people, etc.)

While it is consistent for a radical nihilists to continue the practice of assertively uttering vague sentences whatever the reason she gives for that practice, it presumably isn’t about passing along useful beliefs. Presumably the only beliefs the radical nihilist has are the kinds of beliefs you can state in logico-mathematical language, and perhaps the language of fundamental physics, since vagueness is pervasive in other realms of discourse. Thus in uttering ‘Warren Buffett is rich’ the only contingent belief I could be passing on is a belief about fundamental physics, and it seems relatively clear that I do not learn anything about fundamental physics upon hearing utterances like these.

However, a little further reflection reveals that the radical nihilist doesn’t really need to explain why English speakers go about uttering vague sentences because, according to the

\(^{17}\)In fact, their account of truth in English, and their overall meta-semantics, will be much more radical than this. They maintain that the practice of making assertive utterances of the sentence ‘Warren Buffett is rich’ is perfectly legitimate amongst people who believe that Warren Buffett is rich. They will therefore agree that this sentence is used to communicate the belief that Warren Buffett is rich. Many people simply take such patterns of usage to indicate that ‘Warren Buffett is rich’ means that Warren Buffett is rich. The nihilist will not – they will not take these patterns of usage to be sufficient for that sentence to mean that Warren Buffett is rich in English, and consequently, will not take the richness of Warren Buffett (which is something they accept) to suffice for the truth of this sentence in English.
radical nihilist, they don’t. In fact, radical nihilists believe that there aren’t any English speakers and there is therefore no need to explain their practices.

Unfortunately this type of reasoning highlights why it’s almost impossible to be a radical nihilist – for if you are a radical nihilist it is unclear why you need to do anything. A radical nihilist should feel that she has no need to do the groceries because she believes that there are no grocery stores, and moreover no need to eat since not only is eating impossible, but there is no food to eat and no-one ever dies of starvation anyway. If a radical nihilist does decide to go grocery shopping, surely it is not her logico-mathematical beliefs that explain this action, nor her beliefs about fundamental physics – and if it is none of these it is unlikely to be a precise belief, for very little is precise outside of those realms.

One response you could make, at this juncture, is that there is another attitude, not belief, which our hypothetical radical nihilist holds towards the vague. Maybe this attitude is something like ‘pretending that $P$ for such-and-such purposes’, or ‘believing that according to the fiction of grocery stores, $P$’ or maybe it is just a *sui generis* attitude that plays a particular causal functional role. Whatever this attitude is, perhaps it is this attitude and not belief that explains your actions: when the agent goes to the grocery store this is because she has this attitude towards the proposition that there is food in the grocery store whilst desiring to have food. And perhaps it is the passing on of this attitude that communication achieves.

It is clear that this attitude, whatever it is, pretty much plays the role that belief plays according to a common-sense view. Once we have introduced an attitude that plays the belief role, then I think we have just introduced belief under another name; radical nihilism collapses into the more moderate nihilism we rejected earlier. In the important respects this kind of nihilist agrees with me, for the attitude that rationalises action is one which is held towards the proposition that millionaires are rich, and the proposition that people with nothing are not rich. Moreover, assuming that this attitude is belief-like enough to be subject to the norms of classical reasoning, the attitude must also be held to the proposition that there is a last rich person in a sorites for richness. In the sense in which this nihilist can be said to be committed to anything at all, this nihilist is committed to sharp cut-off points.

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18Here I am appealing to the facts that eating, being food and dying of starvation are all soritesesable.
Chapter 2

Vagueness and Language

As is often the case in philosophy, competing philosophical theories, purportedly about the same subject matter, can be couched in very different vocabulary. In such cases it is often quite hard to state, in a neutral way, exactly what the disagreement is – choice of ideology can sometimes contain assumptions of its own. Sometimes it is not even clear whether two theories are in competition or whether they are just talking about two different things.

As a case in point, consider the two distinctions that will be the focus of this chapter:

**Sentential Borderline ness:** Some sentences exhibit features that are correlated, in some way or other, to the presence of vagueness: call this feature ‘sentential borderline ness’. Examples of English sentences that are sententially borderline include the sentences ‘if a glass is two thirds full it is pretty full’, ‘France is hexagonal’ and ‘Harry is rich’. Examples that are not sententially borderline include the sentences ‘there are at most three bald people on the planet Earth’ and ‘the smallest tall person is tall’.

**Propositional Borderline ness:** Some propositions exhibit features that are correlated, in some way or other, to the presence of vagueness: call this feature ‘propositional borderline ness’. Examples of propositions that are propositionally borderline include the the proposition that if a glass is two thirds full it is pretty full, the proposition that France is hexagonal and the proposition that Harry is rich. Examples that are not propositionally borderline include the the proposition that there are at most three bald people on the planet Earth and the proposition that the smallest tall person is tall.

Related to these distinctions are parallel distinctions between vague propositions and sentences; the sentence ‘there are at most three bald people on Earth’ is sententially vague, even though it is not borderline, because it is potentially borderline.

I take it that it is pretty much universally agreed that the first of these distinctions exists. There is, for instance, a distinctive reason not assert the sentence ‘Harry is rich’ when you know that Harry falls within the borderline region, and it clearly has something to do with vagueness. The existence of the second distinction, however, is a bit more contentious. On the face it it just seems obvious that there’s a principled distinction to be drawn between, say, the proposition that Harry is rich, and the proposition that Harry has at least $200,000. For example the truth of the former proposition seems not to be amenable to investigation, whereas the truth of the latter is. However some philosophers maintain that this apparent difference is an illusion and that these seemingly distinct propositions might even be identical, for all we know. By analogy, one might note that there is no fundamental difference between the last small number, whichever it might be, and the number 5 – they

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1 Harry, recall, is a particular person who is borderline rich.
are both precise entities, although there is vagueness concerning the relation between these precise entities and the description ‘the last small number’. Perhaps the entities denoted by ‘the proposition that Harry is bald’ and by ‘the proposition that Harry has at least $200,000 dollars’ are just as precise as one another, although the relation between the former expression and the proposition it picks out is more complex than it seems. Insofar as propositions are the objects of attitudes, however, one will have to explain why it appears as though we do not know that Harry is rich even though we do know that he has at least $200,000.

In this chapter I shall argue that the best way to make sense of our ignorance is to accept both distinctions; both the distinction between sentences and the distinction between propositions, or the objects of thought whatever they may. For someone who accepts the existence of both distinctions, however, it is natural to wonder what the dispute between the linguistic and propositional approaches to vagueness amount to. Are both sides not just giving an account of different things?

The answer, I take it, is that those espousing one or the other of these approaches typically take themselves to be doing more than just exploring a side effect of vagueness, whatever it may be; they take their project to be giving an explications of what vagueness is, and providing the means to address the fundamental problems of vagueness. Thus I take it that the issue at stake concerns the following two hypotheses:

1. Hypothesis one: it is sentential borderlineness (or, perhaps, a related property of some other linguistic category) which is at the root of, and will feature in the explanations of all the fundamental puzzles associated with vagueness.

2. Hypothesis two: it is propositional vagueness that is at the root of, and will feature in the explanations of all the fundamental puzzles associated with vagueness.

The fundamental puzzles of vagueness, I take it, include at least the sorites paradox, and problem of explaining the puzzling kind of ignorance that comes along with vagueness. Given the first hypothesis our theoretical investigations ought be focussed around a metalinguistic relation to languages: ‘S is vague in language L’, or something similar. Propositional vagueness should be reduced to and explained in terms of sentential vagueness. According to the former hypothesis it is rather a notion of propositional vagueness that plays a more basic role in explaining the relevant puzzles and we should instead theorise about this monadic property of propositions, which we might express using a predicate taking a that clause, or more commonly, an operator taking a sentence. Sentential vagueness and the phenomena associated with it should be explained in terms of propositional vagueness.

The linguistic approach to vagueness, characterised by its preoccupation with the aforementioned relation between sentences, utterances or more generally linguistic items, is pervasive within the philosophy of vagueness. The following prominent theories, for example, all appear to subscribe to something like the first hypothesis:

- Semantic indecision (Lewis [82], McGee & McLaughlin [90], Keefe [71], Dorr [31], Rayo [103])
- Semantic plasticity (Williamson [131])
- Inconsistent meaning constitutive principles (Eklund [36])
- Contextualism (Graff Fara [59], Raffman [100], Shapiro [114])
- Ambiguity (Sider and Braun [18])

Horwich perhaps has some affinities with the second hypothesis, however since propositions for Horwich are, for most intents and purposes, just sentences, it’s clear that he belongs on the linguistic side of the fence.
• Use theories of meaning (Horwich [64])
• Nihilism about vague language (Unger [122])

Although not complete, I take it that this list is fairly representative of the state of the literature on vagueness at present.³

An analogy with a related debate might be illuminating here. Historically, linguistic accounts of modality were quite pervasive. According to that view the fundamental issues surrounding contingency and necessity were to be best understood by looking at the way we use language, and it was common to theorize about modality by drawing a distinction between certain sentences.⁴ Although that view is hardly held by anybody today, it is also surely true that nobody denies the difference between the sentence ‘Hesperus is Phosphorus’ and ‘Hesperus is bright’. The modern view is not that it was a mistake to draw this distinction between sentences, but that it is a mistake to think that an account of this distinction exhausts all there is to be said about modality. By contrast, once we have an adequate account of propositional necessity, either a theory of a propositional necessity predicate, or (as is more common) of a necessity operator, the distinction between sentences can be explained in terms of the distinction between propositions: in the above example the former sentence expresses a necessary proposition whereas the latter expresses a contingent proposition.

Critical to the failure of the linguistic approach was the fact that the converse reduction does not seem to be possible: one cannot explain propositional necessity in terms of sentential necessity. Contingency concerning the brightness of Venus itself is contingency Venus would have had even if there hadn’t been any language users, and it looks on the face of it as though no linguistic account of necessity could account for this.

The situation here is similar in the case of vagueness. I of course acknowledge the existence of sentential borderlineness, but think that it is not a theoretically central notion: sententially borderline sentences merely express borderline propositions, and it is in elucidating the latter notion that the action lies. Accounts that theorize purely in terms of sentential borderlineness, I will argue, do not have the resources to explain propositional borderliness and are therefore at best incomplete.

Indeed, the analogies between the two debates is much closer than I have suggested above. Much like the linguistic account of modality, linguistic accounts of borderlineness are subject to a number of technical difficulties that need to be worked out before we can compare them. The purpose of this preliminary section is to describe the best version of each kind of formalism, so that we will have a more precise target in the later sections. For those wishing to skip the technicalities, one can leave out sections 2.1.1-2.1.6 and move straight onto section 2.1.7.

2.1 Differences between Linguistic and Non-linguistic explanations of vagueness

2.1.1 Grammar

In the last chapter we drew a distinction between people that depended on how much money they had. Not the distinction between being rich and not rich, but a slightly more demanding one – that of being clearly rich as opposed to not clearly rich. The most natural way to express this distinction in English is to prefix, as I just have done, an adverb such

³There are some notable exceptions of course. For example Field [44], and Barnett [8] do not appear to ascribe to a linguistic theory. More controversially, Fine [51], might best be understood as a non-linguistic theorist, although he certainly relates his theory to language.
⁴See for example Ayer [2], Carnap [23] and Malcom [88]. Of course, matters were slightly muddled by the fact that these authors often conflated analyticity, necessity and knowability a priori. Presumably they were right about analyticity.
as ‘clearly’ or ‘definitely’ to a verb or verb-phrase, as in ‘clearly rich’, ‘clearly richer than’, and so on. These expressions fall under the same grammatical class as many other words of interest to logicians such as ‘not’ and ‘possibly’; compare ‘Harry is clearly bald’ to ‘Harry is not bald’ and ‘Harry is possibly bald’. These adverbial expressions also play an important role in the way that many philosophers informally talk about vagueness.

It is common among logicians to regiment adverbial modification in formal contexts using operators – things which result in a sentence when prefixed to a sentence – so that ‘Harry is not bald’ becomes the slightly more awkward ‘it’s not the case that Harry is bald’ and ‘Harry is possibly bald’ becomes ‘it’s possible that Harry is bald’. This regimentation is innocuous in my view and I shall frequently adopt it in what follows. The operator locutions can further be divided according to whether they take the complementizer ‘that’ (‘it’s possible that’) or ‘whether’ (‘it’s contingent whether’). The most common ways of drawing the distinction I have just alluded to, I claim, all use expressions that fall into one of these classes.

ADVERBS: Harry is definitely/clearly/determinately/as a matter of fact/borderline tall.

‘THAT’ OPERATORS: It’s clear/determinate/definite/a fact that Harry is tall.

‘WH’ OPERATORS: It’s borderline/it’s indeterminate/it’s vague/there’s no fact of the matter/it’s unclear [whether/when/how Harry became tall] / [who/where/what the size of/which the oldest tall person is].

A common way of representing these operators in logical notation is to use $\Delta p$ for the operators expressed in the second class (e.g. ‘it’s determinate that $p$’) and $\nabla p$ for the operators in the third class (e.g. ‘it’s indeterminate whether $p$’). It is typically assumed that the latter can be defined in terms of the former by writing $\sim \Delta p \land \sim \Delta \sim p$, and the form from the latter by $p \land \sim \nabla p$ – in other words it is indeterminate whether $p$ iff neither $p$ nor its negation is determinate, and it is determinate that $p$ iff $p$ is true and isn’t indeterminate. I shall adopt this notation and the two assumptions throughout the book.

One notable thing about expressions in these classes that distinguish them from verbs, say, is that adverbs and operators can iterate. One can combine different adverbs with other adverbs, such as with ‘clearly not bald’ and ‘not clearly bald’, as well as iterating them, such as in ‘not not bald’ and ‘clearly clearly bald’. Analogous remarks apply to operators. This way of speaking does not obviously relate to the metalinguistic attributions of vagueness the linguistic theorist is prone to make. If I say that it is not the case that Harry is bald, for example, it is not at all obvious I have said anything about the English sentence ‘Harry is bald’. More generally, attaching an operator to a sentence does not involve saying something about the sentence that it precedes; it would be analogous to suggesting that appending ‘runs’ to ‘Alice’ results in a sentence that says something about the name ‘Alice’ – the resulting sentence is not about names at all, it is about the person Alice.5 This suggests, in particular, that when I say ‘it is clear that Harry is bald’ I have not said anything about the English sentence ‘Harry is bald’, I have said something about Harry and the status of his hairline; this way of speaking seems, therefore, to be fundamentally unsuited to a linguistic theory of vagueness.

If we want to talk about the sentence rather than whatever that sentence is about it is useful to have some canonical way of referring to that sentence; the use of quotation marks serves this purpose well. A linguistic theorist must reject the thought that adverbial and operator expressions listed above play a central role in the theory of vagueness; she might therefore prefer to theorise with a predicate – something which attaches to a name to form a sentence such as ‘S is definitely true’ (see McGee’s [89]; many other linguistic theorists

5As Prior put it in the context of tense operators ‘when a sentence is formed out of another sentence or other sentences by means of an adverb or conjunction, it is not about that sentence, but about whatever those sentences are themselves about’ [97].
use this locution as well). She may then combine this with the relevant quotation names of sentences to draw distinctions that at least seem to correspond to the distinction we introduced earlier, thus saying, for example, ‘the sentence ‘Gandhi was bald’ is definitely true’ instead of ‘Gandhi was definitely bald’.

One thing to note about these particular locutions is that they do not completely do away with the adverbial notion of clarity. In the sentence ‘S is clearly true’, ‘clearly’ appears as an adverb and the verb it is modifying is a linguistic truth predicate. We can regiment this in terms of operators with the sentence ‘it’s clear that S is true’; as we mentioned already, this does not appear to be about the sentence ‘S is true’ but what that sentence itself is about (which in this case happens to be yet another sentence, S.) Thus, while, the distinction between clear and unclear truth is perfectly acceptable to the adverbialist, it is therefore not obvious whether linguistic theorists ought to be using the locutions ‘definite truth’ and ‘clear truth’ to state their theories; if anything is linguistic in this example it is the truth predicate and not the adverb ‘clearly’. Something more would need to be said about the non-linguistic notion of definiteness or clearness that modifies the truth predicate if it were to play an important role. Better, then, that the linguistic theorist adopt some primitive simple predicate, ‘is vague’ or ‘is definite’, to do the work that she wants; it will be this linguistic notion of vagueness that this theorist will be attempting to analyse.

Another structural difference between linguistic theorists preferred terminology and the adverbialist comes out with the point about iteration. When one applies an operator to a sentence one gets a sentence back, so that an operator can be grammatically applied again and again. This is not so with predicates – one cannot say ‘Harry is tall tall’, so we must find some other way to paraphrase apparently coherent talk involving iterations of the above adverbs and operators. While one can say, for example, that the sentence S is vaguely vague, the word ‘vaguely’ here is an adverb which acts as a modifier on, but is not about, the sentence ‘S is vague’.6 The linguistic theorist might capture something like the original thought by instead saying ‘S is vague’ is vague, which is to ascribe vagueness to one sentence, ‘S is vague’, which in turn ascribes vagueness to another, S.

2.1.2 Parameters

However the linguistic theorist cashes out this linguistic notion, it is not obvious that an adequate theory can be stated using a simple monadic predicate such as ‘S is borderline’ or ‘S is vague’. For one thing there is no sense in which the sentence ‘Harry is tall’, on its own, is borderline without further qualification. There are a range heights such that if Harry is within that range then, in ordinary contexts, it is not permissible to assert the sentence ‘Harry is tall’ or its negation; relative to these contexts the sentence is intuitively borderline. In a context where we are talking about basket ball players, however, it may be perfectly acceptable to outright assert the sentence ‘Harry is not tall’. Presumably the sentence ‘Harry is not tall’ is not borderline in this context, even though it is in the former context: whether a sentence is borderline can depend on the context of utterance. Thus a linguistic theorist cannot take as her basic theoretical term a simple monadic predicate, she must take a relation which holds between a sentence and some parameters that includes, at the very least, the context of utterance.

What must these parameters include other than the context of utterance? It is natural to think that we must also relativise to disambiguations of a given sentence. The sentence ‘some banks are full of money’ might be borderline if ‘bank’ means a financial institution but definitely false if it means the bank of a stream or river. (Unfortunately this kind of relativisation would be disastrous for a linguistic theorist that thought that vague sentences typically have lots of disambiguations, and that each of these disambiguations is completely

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6The word ‘borderline’ seems to be able to function as both a verb and an adverb, thus one can say ‘S is borderline borderline’. However, just as with ‘S vaguely vague’, we have an uneliminated adverbial use of the word ‘borderline’.
Presumably we must also specify a language; if ‘Harry is tall’ means that 1=1 in some language \( L \) at context \( c \) then this sentence is borderline only in English at \( c \) but not in \( L \) at \( c \). Similarly, sentences within a single language change their meaning over time – some terms start off vague and become more precise, and vice versa.\(^7\) For similar reasons one ought to relativise to dialects. Last, but not least, we must relativise to a world – the sentence ‘Harry is bald’ could have been used in exactly the same way as the sentence ‘5 is prime’ is actually used. Even though there is an intuitive sense in which the sentence ‘Harry is bald’ would not be borderline at such a world, it will become clear later that we will need a way of ascribing vagueness to a sentence that does not depend on how that sentence is used in the world of evaluation. In this sense we will say that at the world \( x \), ‘Harry is bald’ is borderline as used at \( x \) and parameters \( \ldots \) even if this sentence is used in a precise way at \( x \) (what this means is that the sentence ‘Harry is bald’ counts as borderline at \( x \) according to the way that sentence is used at \( w \).

2.1.3 Quantifying in

Another technical issue that must be surmounted by a linguistic theorist is related to what Quine calls the third grade of modal involvement: how should a linguistic theorist say what we would normally say by quantifying into the scope of an operator or adverb. Formally speaking, quantifying in is straightforward for the adverbialist, and is semantically well understood from modal logic. The naïve way of translating a formula like \( \exists x \Delta Fx \) into vocabulary that only attributes determinacy to sentences would result in the following piece of nonsense:

\[ \exists x \text{Def}^*Fx \]

supposing here that ‘Def’ represents our linguistic predicate ascribing definiteness to sentences, then this paraphrase suffers from two problems. Firstly ‘Fx’ is not a sentence, it is an open formula and our primitive was introduced so as to apply truthfully only to sentences. Secondly, even if a sensible notion of definiteness could be introduced for open formulae, the above quantification is vacuous, since \( x \) does not appear free in ‘Def^*Fx’, it only appears free the sentence that is mentioned by that sentence.

Before one dismisses this as a technical side issue, note that one of the crucial concepts in the study of vagueness is the notion of a predicate being vague. An adverbialist would introduce a parallel notion for properties: a property, \( F \), is vague if there is something which is borderline \( F \) (or perhaps, if it’s possible that there is something which is borderline \( F \)). This involves exactly the kind of quantification into the scope of an operator that is problematic for the linguistic theorist to emulate.

A natural way to make sense of the above formula is to invoke a substitutional understanding of the quantifiers. For instance, ‘something is definitely \( F \)’, becomes: ‘for some name \( a \), ‘Fa’ is a definite sentence’. Provided we restrict ourselves to proper (and hence rigid) names this strategy is not a complete disaster when applied to linguistic accounts of modality. By contrast, in the case of vagueness the results are disastrous. This paraphrase would, for instance, count the predicate ‘is 29,000 ft’ as having borderline cases. Since the sentence ‘Mt Everest is 29,000 ft’ is a borderline sentence, the substitutional understanding of the quantified claim is ‘for some name, \( a \), ‘\( a \) is 29,000 ft’ is borderline’ and this is true when \( a \) is substituted for ‘Mt. Everest’.

Indeed, skeptics of modal logic, such as Kneale and Quine, took similar points to suggest that properties cannot ‘be said to belong to individuals necessarily or contingently, as the case may be, without regard to the ways in which the individuals are selected for attention.’ ( Kneale [72].) The analogous move here would be to say that Harry is only borderline tall.

\(^7\)Of course, the context provides the time of utterance but this does not tell you what version of English you are speaking. One might still be able to speak ancient versions of English by uttering sentences today.
relative to the name ‘Harry’, but is determinately tall relative to the description ‘the shortest

tall person’ (assuming, for the sake of argument, that he is the shortest tall person.) Or,

that Mt. Everest is a borderline case of being over 29,000ft relative to the ‘Mt. Everest’

way of selecting it for attention, but not relative a precise name for the fusion of rock and

soil that constitutes Everest.

A much less radical move would be to introduce further ideology. Rather than theorising

with a sentential notion of vagueness, one could start instead with a notion that relates an

object to a predicate when it is a borderline case of that predicate relative to the relevant

parameters: ‘x is a borderline case of predicate F at parameters p’. Note that this move
does not allow us to simply dispense with the old sentential notion of vagueness. We must
have that too, or we would not be able to draw the sentential distinction anymore (at least,
we cannot straightforwardly state that ‘Mt. Everest is 29,000ft’ is vague in terms of the
borderline cases of the predicate ‘is 29,000ft’, for example.) If we are to go this route, then,
we will need separate accounts of both predicate and sentential vagueness.\footnote{Or perhaps we

could get by with accounts of predicate vagueness and referential vagueness for names.}

Not only that, the above primitive is only good for monadic predicates. We will need a

four place relation to make sense of cases where it’s borderline whether two objects stand

in a relation: ‘x and y, in that order, are borderline cases of the relation R relative to

parameters p’. And we will need a five place relation to ascribe borderline cases to ternary

relations, and so on. (Of course, rather than multiply primitives, one could talk about

monadic predicates of tuples.)

The difficulties do not stop there. Most of the issues raised above arise for plural

predicates and plural quantification. An adverbialist can say things like the following: it’s

borderline whether Tom, Dick and Harry have enough hair between them to make a hairball,

so there are some people such that it’s borderline whether they have enough hair between

them to make a hairball. A similarly complex theory would have to be constructed for the

linguistic theorist to make sense of this sentence.

While it would not be at all surprising that such a theory could be carried out rigorously,
it is natural to wonder if the complexity is a symptom of an incorrect underlying picture.

Note, by contrast, that from a purely formal perspective the devices that allow one to
quantify into the scope of a determinacy operator are no more complicated than quantifying
into the scope of the negation operator. It is rather the metaphysical commitments of
this approach that are problematic. If we accept existential generalisation, it follows from
the fact that Mt. Everest is determinately identical to Mt. Everest, that something is
determinately identical to Everest. Moreover, it at least appears as though no fusion of
soil, rock and snow is determinately identical to Mt. Everest since there are particles that
are neither determinately a part of Mt. Everest nor determinately not a part. Assuming
Leibniz’s law (cf. Evans [38]) it follows that Mt. Everest is determinately distinct from any
fusion of particles. This is puzzling to say the least.

In this respect there is an exact parallel with the case of modality – Quine, for example,

was suspicious of the adverbial view of modality because of the metaphysical implications.
As we know, in this debate the adverbialist approach to modality now dominates, meta-
physical puzzles and all.

One puzzle for the adverbialist account of modality, that is strikingly similar to the
above one, is the problem of the statue and the lump of clay. The lump of clay out of which
the statue is constituted could have been deformed without being destroyed, whereas the

statue couldn’t; thus the statue, it is concluded, is not identical to the clay for they have
different modal properties. If this conclusion is correct then it is not at all surprising that
Mt. Everest is not identical (even indeterminately) to a lump of rock, soil and snow – this

is a conclusion that is motivated by an already established response to the corresponding
modal puzzle. Indeed there are many solutions to this puzzle in the modal case that can
be applied fairly straightforwardly in the present case. Although there are many analogies between the two puzzles, some strategies that are sometimes applied to the vagueness case are less appealing in the modal case. For example, the standard supervaluationist line is to treat names like ‘Mt. Everest’ as non-rigid, in the sense that it picks out different objects relative to different precisifications. The result allows one to reject the principle of existential generalisation: without existential generalisation you cannot infer that there is an entity over and above the fusions of particles that is determinately identical to Everest, and you can even say this whilst maintaining that Everest is determinately identical to Everest. Yet another strategy, one that is much less attractive in the modal case, but perhaps defensible in the case of vagueness, is to reject Leibniz’s law. It would then be possible to say that Mt. Everest is identical to exactly one fusion of particles, although it is indeterminate which.

Although I do not know exactly what to say here, the elements to a solution to these puzzles are out there in the literature. I see no particularly pressing objection here to an adverbialist conception of vagueness that is not equally an objection to an adverbialist conception of modality.

### 2.1.4 Montague’s paradox

Despite the pervasiveness of the linguistic approach to vagueness, it is somewhat surprising to see that the logical theory surrounding the study of vagueness focuses almost exclusively on the operator formalism. In that setting two principles are almost universally taken for granted. These are that the determinacy operator is factive, which means

$$\text{If it’s determinate that } p \text{ then } p$$

and a rule of proof call necessitation, which guarantees the following

$$\text{If ‘} p \text{’ is provable in classical logic with factivity, then ‘it’s determinate that } p \text{’ is a theorem.}$$

The rule of necessitation is actually strictly stronger because it can be applied repeatedly, however the above consequence suffices for my discussion. These two principles, and many more, are naturally modelled by the same formal tools used to study modal logic: the kind of framework involving indices and accessibility relations. A framework, incidentally, whose invention coincided roughly with the rise of non-linguistic theories of modality, and which was crucial to the success of those theories (see Kripke [76]).

As was noted by Richard Montague at that time (see [92]), results concerning the above logic and the corresponding model theory cannot be straightforwardly transferred to linguistic theories of modality. Similar things must also be said about the linguistic approach to vagueness. For example, one might naively think that a linguistic theorist could develop a theory completely parallel to the operator theory by adopting the following two analogous principles:

$$\text{If ‘} p \text{’ is definite then } p$$

$$\text{If ‘} p \text{’ is provable in classical logic with factivity, then ‘} p \text{’ is definite’ is a theorem.}$$

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9One strategy I find particularly promising is the idea that there are many coincident fusions of the particles that fuse Mt. Everest with different modal profiles (see, for example, Hawthorne [62]. See also Cotnoir and Bacon [26] for a way of spelling out the mereological picture). A similar view could be adopted to the case of vagueness (see the discussion of vague parthood in Korman [73]).

10Unfortunately the standard supervaluationist view has some fairly surprising consequences: one might have naively thought that there is certain mountain which is widely known to be the tallest mountain in the world, that was first climbed in 1953, and so on. Of course it is widely known that Mt. Everest has these properties, but without existential generalisation you cannot infer that there is something that is widely known to have these properties. Moreover, since nothing determinately has these properties according to the view in question, we have positive reason to doubt that there is something which is known to have those properties since one cannot know something unless it is determinate.
However, perhaps surprisingly, the above two principles are inconsistent, unlike their operator variants. The reason is exactly parallel to the problems Montague raised against linguistic accounts of necessity. Montague proves it formally within a background theory of syntax represented in arithmetic (in which there is no explicit self-reference), but we can give the informal jist of the argument using self referential sentences. Let $D$ be the sentence ‘$D$ is not definite’, then by Leibniz’s law we can infer that if $D$ is definite, then ‘$D$ is not definite’ is definite, and by the first principle, that ‘if $D$ is not definite’ is definite, then $D$ is not definite. Putting these together we get that if $D$ is definite, it isn’t definite. Thus $D$ isn’t definite, and we have proved this classical logic with the analogue of factivity. So by the second principle we may conclude that ‘$D$ isn’t definite’ is definite. But this is just the conclusion that $D$ is definite, which contradicts our earlier conclusion that $D$ isn’t definite.

No parallel argument can be levelled at the operator formalism, or even the variant formalism utilising a predicate of propositions, without making substantial assumptions about propositions. For example, if one represented propositions as simply sets of indices of some kind, it is natural to simply deny the existence of a proposition, $p$, identical to the proposition that $p$ is not true, just as we are forced to deny the existence of a proposition, $p$, identical to the proposition that it’s not the case that $p$ in this setting, for no set is identical to its set theoretic complement.\(^\text{11}\)

It is thus not generally safe to assume that a linguistic account of vagueness can simply piggy-back off the success of formalisms formulated using operators. Few linguistic theorists are careful about this, and simply theorise using an operator assuming that it can safely be reinterpreted within their preferred account vagueness. The most notable exception to this exclusive focus on operators among linguistic theorists is McGee [89], who has done more than anyone to spell out the consequences of using a linguistic definiteness predicate. McGee’s theory relaxes the factivity requirement of definiteness and keeps necessitation.

However the costs of McGee’s approach are more than just the failure of factivity. One might think that it’s never the case that a sentence and its negation are both definite at the same time. However, even this principle must be relaxed in McGee’s theory. Indeed, one of McGee’s own limitative results suggests that some concession beyond factivity will have to be made. In the operator formalism it is standard to assume that a determinate conditional with a determinate antecedent has a determinate consequent. Also important, in a first order theory, is the Barcan principle which says that if it’s determinate that everything is $F$ then everything is determinately $F$. This principle, among other things, helps rule out the possibility of indeterminate existence. However if we were to adopt the linguistic analogues of these principles,\(^\text{12}\) along with the rule of necessitation and the principle that no sentence and its negation are both definite (and the background theory of syntax), the theory would be ‘$\omega$-inconsistent’: while no contradiction could be derived from it in a finite number of steps, contradictions could be derived if one could perform infinite inferences.

While I by no means think that these kinds of costs are decisive, it does highlight the fact that linguistic theories are usually logically highly complex and cannot be modelled by the simple model theory that has proved so fruitful in the case of the operator approach.

\(^{11}\)A common, albeit flawed argument, that there must be propositional paradoxes is that it seems as though it should be possible for Martha, say, to have the following uniquely favourite proposition: the proposition that Martha’s favourite proposition isn’t true. From this we can derive a paradox assuming a propositional variant of the T-schema. But this argument seems in many ways parallel to the following flawed argument. Surely it is possible for Martha to uniquely have the following favourite number: the successor of Martha’s favourite number. Again one gets a contradiction, but this time from purely numerical facts and no propositional T-schema. I think in this case it is clear, even if Martha goes about declaring ‘my favourite number is the successor of Martha’s favourite number’, that she hasn’t succeeding in making this her favourite number.

\(^{12}\)One would have to be careful about how one formulates the Barcan principle, given the points made in the last section. See McGee [89], for one way of making this precise if you are in the language of arithmetic.
2.1.5 Can one explain propositional borderlineness in terms of sentential borderlineness?

I have so far been characterising the contrast between linguistic and non-linguistic theories of vagueness as being concerned with which terms will play a more basic role in the explanations of the vagueness related phenomena: an adverbial or operator locution such as ‘it’s vague whether’ or the metalinguistic predicate ‘S is borderline in L relative to parameters ...’. According to the non-linguistic view when I say that something is definitely the case I am no more talking about sentences or linguistic items than I would be if I were talking about what will be the case, what could be the case, what is not the case, and so on.

As with many philosophical disputes in which different sides adopt different basic ideology, it is often desirable to be able to explain the vocabulary of one theory in terms of the other. I have suggested that the notion of a sentence being borderline in language L relative to parameters $\bar{p}$ can be explained in terms of propositional borderlineness: it is just for the sentence in question to express a borderline proposition in L relative to parameters $\bar{p}$. As I mentioned earlier, some linguistic theorists accept both the distinction between sentences and the distinction between propositions. It is crucial, then, that they be able to offer some kind of explanation of this notion in their preferred vocabulary.

It might tempting, given all the technicalities mentioned above, for the linguistic theorist to adopt the adverbialist way of talking for the sake of convenience, but to insist that when it gets down to it this way of speaking is really an innocuous shorthand for something else. Indeed, my suspicion is that this attitude is actually quite common since the formalism of determinacy operators is ubiquitous in the philosophy of vagueness, yet most of those who employ the formalism informally endorse a linguistic theory of some sort.

The practice of using the operator formalism as shorthand for something linguistic, however, is not innocuous. We have encountered two reasons to think this already. Firstly, quantification into the scope of an operator is a simple affair, whereas the counterpart of this with a metalinguistic predicate is potentially problematic. Secondly the standard logic assumed in discussions of determinacy operators includes the principle of factivity and the rule of necessitization described in section 2.1.4, yet the counterparts of these two principles for a metalinguistic predicate are jointly inconsistent. The practice is a bit like using the phrase ‘it’s ambiguous whether $p$’ to say something about the ambiguity of a particular sentence: strictly speaking this locution makes no sense and needs to be spelt out explicitly in some other way. In the present case I think there is no way to spell it out satisfactorily, even if we set aside the issues raised in the last two sections. In short, the practice of using adverbial and operator locutions in place of linguistic predicates is misleading and should be abandoned altogether.

Consider a typical example of something we would state using an operator:

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It’s borderline whether Harry is bald. (2.1)
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As a first stab: it is natural to paraphrase (2.1), by picking a language, L (along with other parameters), and sentence, S, such that the claim that S is borderline in L (at parameters $\bar{p}$) is a reasonable paraphrase of the claim expressed by (2.1); one of the things this paraphrase must do is play the same role that (2.1) does within a theory of vagueness. An example of the paraphrase strategy, therefore, would be to choose English as our language, and the sentence “Harry is bald” as our sentence. Our paraphrase of (2.1) is thus that the sentence “Harry is bald” is used by English speakers in whatever way is required to make it a vague sentence of English.\footnote{One can think of it as simply composing the quotation subnective (of type te) with the borderlineness predicate (of type et) to produce an operator (of type tt).}

This is effectively the strategy Quine adopts in [99] for paraphrasing modal sentences in which only closed formulae appear in the scope of intensional operators. In the context of vagueness the relevant paraphrase is rarely made explicit. When it is made explicit, such
as in Dorr [32] for example, this is the kind of strategy adopted, so it seems like a natural place to start.

An obvious objection is that it is not equivalent to (2.1): English speakers could have used “Harry is bald” in a precise way, to mean that $1+1=2$ say, yet it would still have been vague whether Harry is bald provided he has the same number of hairs he in fact has. The equally obvious reply, which we pre-empted in an earlier section, is that we should instead paraphrase (2.1) with the claim that “Harry is bald” is a vague sentence of English as it is actually used.

But even this seems inadequate: a non-English speaker may not know anything about how English is actually used, but still know that the number of hairs Harry has falls within the borderline region for baldness. It seems, in this case, that one can know whether Harry is borderline bald without knowing whether the sentence “Harry is bald” is vague as it is actually used in English. Conversely, someone who doesn’t speak English may have it on good authority that the sentence “Harry is bald” is vague in English as it is actually used, but have no idea whether Harry has 0 hairs, 1,000,000 or a borderline number, since she might not know what “Harry is bald” actually means in English. Although these are only hyperintensional differences they are important: it is natural to think that the notion of propositional borderliness is the notion that regulates our epistemic and doxastic attitudes for both English and non-English speakers. It is irrational, for example, to believe that Harry is bald but not determinately so; it is not irrational, if you were rationally mistaken about how English is used, to believe that Harry is bald but that ‘Harry is bald’ is not definite as it is actually used by English speakers.

A more pedestrian worry is the dependence of the paraphrase on the choice of sentence and language: the claim that “Harry is bald” is borderline in English as it is actually used, and the claim that “Harry ist kahl” is borderline in German as it is actually used are two completely different paraphrases of (2.1). One could, for all we know, be true while the other false – they state completely different facts about different sentences in different languages. Without smuggling in any suspect imperialist assumptions, neither paraphrase is superior to the other – if two incompatible paraphrases are equally good, neither are perfect paraphrases. On the other hand, what is asserted by (2.1) can be said in a number of languages without mentioning English or German. The operator formulation does not seem to be about sentences, the English language or the German language, it seems to be about Harry and the status of his head.

Perhaps we could do better with language independent paraphrases:

E. There is some sentence, $S$, in some language, $L$, whose linguistic community actually uses $S$ (i) in such a way that it says that Harry is bald and (ii) in whatever way it takes to make $S$ vague in $L$.

U. Every sentence, $S$, in any language, $L$, whose linguistic community actually (i) uses $S$ in such a way that it says that Harry is bald (ii) uses $S$ in whatever way it takes to make $S$ vague in $L$.

These paraphrases still mention languages and sentences, unlike (2.1), but perhaps they do better with regard to our first problem of language dependence.

Unfortunately, according to some linguistic theories (i) and (ii) are incompatible: the vagueness of a sentence relative to its use in a linguistic community precludes that use determining any proposition as being uniquely expressed by $S$ in $L$.

However, I think the most problematic feature of this strategy is that it’s either inadequate or it assumes the distinction between propositions we are trying to dispense with. Consider the following two possibilities.

The proposition that electrons are positively charged is expressed in some language by a vague sentence.

The proposition that Harry is bald is expressed in some language by a precise sentence.
If the first claim was metaphysically possible then E. would be inadequate and if the second statement was metaphysically possible U. would be inadequate.

It should be noted straight off the bat that coarse grained theories of propositions, such as the view that propositions are sets of worlds, are already committed to both these possibilities. For example, for some \( N \), the proposition that Harry is bald is necessarily equivalent, and thus identical to the proposition that Harry has \( N \) hairs. It follows that the precise sentence ‘Harry has less than \( N \) hairs’ expresses the proposition that Harry is bald, demonstrating the second possibility. An analogous argument can be made for the first possibility. The paraphrases E. and U. are therefore simply not adequate on a coarse grained theory; such a theorist would have to resort to a language dependent paraphrase.

Fine grained theories (theories that distinguish the proposition that Harry is bald from the proposition that Harry has \( n \) hairs, for each \( n \)) do not have this problem. It follows that if the paraphrases are adequate neither statement is possible: if E is adequate the proposition that electrons are positively charged couldn’t possibly be expressed by a vague sentence in some language, and if U is adequate the proposition that Harry is bald couldn’t possibly be expressed by a precise sentence.

But facts like this call out for explanation! What is so special about the proposition that Harry is bald that means it can’t even possibly be expressed by a precise sentence? Yet it seems hard to see how the difference between propositions which could be expressed by a precise sentence and those which couldn’t could be explained without appealing to the propositional notion of vagueness that we are attempting to eliminate.

To put it another way, while we know that the proposition that Harry is bald is expressed in English by the vague sentence “Harry is bald” and in German by the vague sentence “Harry ist kahl”, what is it about the proposition that Harry is bald, that prevents it being expressed by a precise sentence? Couldn’t there be a language with a completely precise predicate, such that there is no unclarity about when it applies in that language and when it doesn’t apply, which is such that it applies in \( L \) only when the object is bald, and doesn’t apply otherwise. How are we supposed to distinguish the proposition that Harry is bald from a proposition that can be expressed by a precise sentence, like the proposition that Harry has less than 2,000 hairs, if not by employing the distinction between vague and precise propositions we are trying to eliminate? It seems like we need something similar to the operator way of talking to distinguish vague from precise propositions. The upshot of the possibility of the proposition that Harry is bald being expressed by a precise sentence is that (2.1) may come apart from its paraphrase.

2.1.6 More vague propositions than sentences

Here another analogy between the debate about linguistic accounts of modality might be helpful. David Lewis once objected to linguistic theories of modality that identified possible worlds with certain kinds of sets of sentences on the grounds that there are simply more possible worlds than sets of sentences. A related point applies here. It certainly seems like there are more vague propositions than vague sentences. At the very least, there appear to be vague propositions which are not expressed by any sentence of English.

To see this imagine that we encounter an alien race who have evolved in a stellar system orbiting a white dwarf. Consequently the range of light that stimulates their visual system is in the ultraviolet spectrum. Like us they have different names for different segments of that region depending on how things appear to them. For example, just as we’re unsure whether to apply the term ‘red’ on the region of the visual spectrum that is borderline red, there are corresponding regions of the ultraviolet spectrum where the aliens are unsure whether to apply their terms. It seems very natural to say that there is something the aliens can say which we can’t. These things aren’t synonymous with the precise claims we can make about the ultraviolet spectrum in our language, and therefore encode pieces of information not expressible in English. If the vague propositions exist independently of us and the way
we happen to speak, just as precise propositions do, the vague propositions we described would exist even if there weren’t any aliens speaking this way. It seems particularly hard to imagine how the unknowability of these vague propositions could be explained by any sentence in English being borderline.

We can massage this point into a much more general worry about the prospects of giving a reductive account of propositional borderlineness in terms of sentential borderlineness. The problem is that there are simply not enough sentences to account for all of the vague propositions. There are, for example, at least uncountably many different vague propositions. Take the colour spectrum and divide it into three adjacent connected region such that (i) the two outermost regions are each roughly the same width as the range of colours that we are unable to know are red, and (ii) the inner region is roughly the same width as the range of colours we are able to identify as red. There are uncountably many such divisions, but it is natural to think that each of these could be the range of unknowability and knowability of some other vague property much like the property of being red except shifted. There are simply not enough sentences of English to account for all such properties.\textsuperscript{14}

2.1.7 Vagueness in the objects of thought and vagueness in the world

It is very natural to associate the view I am going to defend, according to which the adverbial and operator ways of speaking about vageness are basic, with the view that it is language independent entities – i.e. propositions – that are the primary bearers of vagueness and precision.

I do in fact endorse this further claim, but it is important be clear about how this strengthens the initial claim. By analogy, it ought to be obvious that the operator and adverbial ways of speaking about negation are conceptually basic compared to the linguistic analogue of negation – the falsity predicate.\textsuperscript{15} Yet it’s clear that a nominalist about propositions can perfectly consistently resist the further claim that propositions can be negated.\textsuperscript{16} Nominalists could in principle avoid commitment to propositions, but paraphrase many of the things I say in this book by engaging in a way of talking in which one can quantify into the position that a sentence occupies. This way of talking does not commit you to propositions, or any kind of singular entity, in the same way that quantification into the position of a plural term does not commit you to special kinds of set-like singular entities. I suspect that one could accomodate most of what I say here using this way of speaking, however it would be cumbersome and quantification into sentence position is not clearly a part of English proper.

I will generally simply help myself to these entities and leave it to the nominalist to do whatever they need to in order to make sense of what I say. Once we have helped ourselves to these entities we must get clear on the kinds of things they are supposed to do. In my view there are a number of different roles that philosophers want propositions to play. For some philosophers these entities are, like ‘facts’ or ‘states of affairs’, supposed to do heavy duty metaphysics. For others propositions are the semantic values of sentences and provide their truth conditions. Yet another important characterisation, the one I will adopt, identifies propositions with the denotations of that-clauses and the objects of propositional attitudes. It may turn out that no single kind of entity can play all these roles, in which case a dispute can arise about which entities are the real propositions. These disputes are verbal – if entities filling each of the roles in question exist then, so long as we are careful about

\textsuperscript{14}For each real number, $\gamma$, we have a name for, there is the property of being a colour $\gamma$ hertz above red. But there are only countably many reals nameable in English.

\textsuperscript{15}To see this, try to explain what it would mean for a sentence to be false in a language at a context without using the negation operator.

\textsuperscript{16}It is not quite as awkward to speak of propositions as bearers of vagueness as it is awkward to say that propositions are the bearers of negation.
which things occupy which roles, no issue of substance will turn on which of these entities we choose to call ‘propositions’. For my purposes a proposition will be just whatever the denotation of a that-clause is. Since propositional attitudes are grammatically relations taking names and that-clauses as arguments this view might naturally be expressed by the slogan that propositions are the objects of thought – the objects of propositional attitudes such as believing, knowing, desiring and so on. Given the operator approach it’s natural to think that propositions, qua denotations of that-clauses, are also the bearers of vagueness, and indeed negation, necessity and so on. However as this book continues it will be clear that it is the role that propositions play as the objects of thought that define them, so I shall keep to the slogan that propositions are the objects of thought.

This way of talking about propositions suggests that they are abundant. Whenever there is a sentence, for example ‘Harry is bald’, there is also a singular term ‘that Harry is bald’, or more perspicuously, ‘the proposition that Harry is bald’, which denotes a singular entity, a proposition, that is true if and only if Harry is bald.17

It might seem that the theory that merely says that propositions are the denotations of that-clauses tells us very little about their nature. On the contrary, we can deduce quite a lot about them. Indeed one can assume that they satisfy what I’ll call ‘the proposition role’ described below. Each of these principles simply fall out of the stipulation that propositions are the things expressed by that-clauses.

The Proposition Role:

– One believes, (knows, desires, asserted, said etc) that $P$ if and only if one believes (knows, desires, asserted said etc) the proposition that $P$.
– It’s necessary that $P$ if and only if the proposition that $P$ is necessary.
– It’s true that $P$ if and only if the proposition that $P$ is true.
– A sentence means that $P$ if and only if it means the proposition that $P$.
– It’s determinate that $P$ if and only if the proposition that $P$ is determinate.

From these facts we can deduce many things, including that propositions can be said and asserted, are the objects of our attitudes, are necessary and contingent, are the meanings of our sentences, and the objects of truth and falsity, and perhaps also determinate truth and falsity.18

In choosing to use the word ‘proposition’ in this way I have made at least one substantive commitment – that the proposition role is consistent, and that there can be entities that occupy the role. However with that assumption,

Of course, I don’t mean to suggest that this is a complete theory: more concrete theories of propositions tell us whether propositions are structured entities, whether they are set theoretic constuctions, whether they have a Boolean structure, and so on. For all I’ve said the things playing the proposition role are just linguistic entities – sentences of a particular language, or equivalence classes of sentences from different languages. However these theories must all agree that these entities satisfy the proposition role if they are engaging in the project of describing the denotations of that-clauses, and it is only facts like those described in the proposition role that I shall need in what follows; it is unnecessary to be more specific than this.

17The thesis that the proposition that $\phi$ is true if and only if $\phi$ is importantly different from disquotational schemata which are well known to be problematic (‘the sentence ‘$\phi$’ is true in English at $c$ if and only if $\phi$’ is a disquotational schema; our thesis does not mention sentences or involve quotation marks at all.) Unlike the disquotational variants, the propositional T-schema I have just presented is known to be consistent.

18It should be stressed that there is more to the story here than I am letting on. For example, it seems as though one can fear that $p$ without fearing the proposition that $p$, and one can hope that $p$, but it is not even grammatical to ‘hope the proposition that $p$’. These complications can mostly be avoided by substituting ‘the proposition that $p$’ for ‘that $p$’ in the following discussion, although for the sake of readability I shall not do this. It is also worth noting that most of the theses in this book can be reformulated by quantifying directly into sentence position and eliminating talk of propositions altogether.
Note that a consequence of this view, if you accept the distinctions drawn by the ad-
verbialist, is that there are vague propositions: if it’s not determinate that Harry is bald
then *that Harry is bald* (a proposition by our lights) is not determinate. If it is furthermore
not determinate that Harry is not bald, then we may also infer that *that Harry is bald* is a
borderline, or vague proposition.

One issue I want to address is whether a non-linguistic view – one that accepts the
operator and adverbial ways of speaking about vagueness as basic – has to be a view in
which vagueness is, in some sense, ‘in the world’ (or, alternatively, whether I am committed
to ‘metaphysical vagueness’. ) In effect my answer to this question will depend on how the
question is posed. On a very deflationary use of the word ‘fact’ there will be vague facts
according to this view. For if a fact is just a true proposition, and furthermore, if to say
that the proposition that *p* is true is equivalent to simply saying that *p*, then it follows that
there are vague facts. Suppose that the proposition that Harry is bald is borderline (and
thus, it follows, that its negation is also borderline.) If Harry is bald, then the proposition
that Harry is bald is true, and is thus by definition a fact. So in this case we have a vague
fact. If Harry is not bald then the proposition that Harry is not bald is true and borderline,
so again we have a vague fact. Either way (assuming the law of excluded middle) there is
a vague fact.

This is just one thing that ‘metaphysical vagueness’ might mean; it might mean other
things, and these too might require clarification. At any rate, I suspect that once it is clear
what I mean by ‘fact’ in the above argument many philosophers will find the sense in which
I am committed to there being ‘vagueness in the world’ an uninteresting one. The real
issue at stake, in my view, is whether the entities which serve as the objects of thought are
vague in a way that cannot be reduced to the relations they stand in to public (or private)
language sentences.

2.2 Vagueness and Ignorance

According to a widely held intuition there is an important, systematic connection between
borderline and knowledge. One instance of this general thought can be described as
follows: there are certain heights such that when a person is that height it is impossible
to know whether they’re tall (even if we know their height.) Moreover, the fact that we
are ignorant in cases such as these has *something* to do with vagueness, and a good theory
of vagueness ought to explain this ignorance. For the linguistic theorist, however, this
connection seems at least initially mysterious. What is special about the people with these
particular heights, according to that theorist, is that certain words in certain languages
bear a special relation to them – how could this special relation to words explain why we
can’t know whether these people are tall?

Note that this connection to ignorance sheds light on the question, raised in section
2.1, of whether all propositions are precise or whether some propositions are vague. For
if you think that there are some propositions (such as the proposition that Harry is bald)
which in certain circumstances enjoy a kind of incurable ignorance, and other propositions
(such as the proposition that Harry has less than 1000 hairs) which do not, and you can
reliably distinguish between the cases where the incurable ignorance is due to vagueness,
then you have done everything except verbally accept the distinction between propositions
that is being claimed to exist. You might think that the distinction is a product of a more
basic phenomenon associated with vague language, but you must at least accept that the
distinction between propositions exists and is non-trivial. To go beyond a verbal rejection
of the distinction, I suggest, you must rather maintain that the propositions being grouped
together as ‘vague’, such as the proposition that Harry is bald, do not really exhibit these
epistemic features; that every vague proposition is identical to some precise proposition
that does not exhibit the epistemic phenomenon.
I should mention straight away that while the connection between vagueness and ignorance is certainly widely accepted, some philosophers have recently attempted to resist it. According to this view, it is consistent that all propositions are precise (even the proposition that Harry is bald!). I consider these philosophers in section 2.4. I will, however, begin by treating the majority of philosophers who do accept the basic intuition behind the ignorance idea gestured at above. The fact that there are certain heights such that we are unable to know whether people with those heights are tall calls out for explanation. The phenomenon responsible for this fact is vagueness, so a linguistic theorist had better be able to accommodate this.

In our preliminary characterisation of the phenomenon, we said that there are certain heights such that it is impossible to know that a person is tall when they are that height, a certain number of hairs such that it is impossible to know that a person is bald when they have that number of hairs, and so on. One issue that needs to be addressed is how to characterise these heights, hair numbers and so on. A non-partisan way of doing this would be to talk about the heights of people who are borderline tall, or the number of hairs that people who are borderline bald have and so on. This way of characterising the cases, of course, uses the adverbial way of talking about vagueness and carves out a distinction between people. Some linguistic theorist would want only to draw a distinction between sentences, leaving the distinction between people derivative at best, and so will say something different in its place. As we discussed in section on quantifying in there are several different ways to do this, and perhaps other ways I haven’t considered.

My argument against linguistic theories will revolve around the following example (adapted from Dorr [31]):

Before us is a glass of water that is filled so that it is exactly $70\%$ full. There is a large international team of people ready to inspect the glass, armed with many different measuring devices for calculating every possible dimension of the glass and the water in it. Some of these people speak multiple languages, some of them only speak one, and perhaps some of them, if you can imagine it, do not speak any languages at all. I have, moreover, given each of them the task of determining every truth they can about the glass. After I have allowed them to do whatever measuring they need, the ones who speak a language have been instructed to communicate to me, using some previously determined signal, whether the glass is pretty full.

It should be obvious to everyone that an unqualified positive or negative answer to this question would be inappropriate, even among those who happened to have measured all the relevant precise facts about the exact volume and shape of the glass, the exact volume of water in it, and so on. Modulo a small number of dissenters, mentioned earlier, most philosophers think that the explanation for this fact is that they simply do not know that the glass is pretty full, despite the fact that they know numerous exact facts, such as the proportion of the glass that is filled with water.

Once we have conceded that, for example, the English speakers do not know whether the glass is pretty full, it becomes pretty hard to imagine that the Chinese speakers are in a better position to know than the English. Indeed it becomes hard to imagine that anybody is better placed to discover whether Harry is bald – even the people who do not speak any languages at all. Thus I think we have good reason to accept the following:

**Ignorance:** Nobody knows whether the glass is pretty full.

We can also make it explicit that this is not because they are ignorant about how full the glass is:

**Knowledge:** For each $0 \leq n \leq 100$, somebody knows whether the glass is at least $n\%$ full.

This should also be obvious assuming, as I have been, that among the people measuring the glass are people who have measured the exact percentage of the glass that is filled. We
could extend Knowledge to include knowledge of other precise facts without changing the case.

Ignorance, I take it, is somehow or other a result of vagueness, and a very puzzling result at that. Each person in our team of measuring experts measures the glass, but without fail comes away ignorant about whether the glass is pretty full. This isn’t just an accident, it is a general fact that calls out for an explanation, and a theory of vagueness ought surely provide one, or ought at least be capable of providing one within the resources it invokes.

We must be careful to distinguish Ignorance from Linguistic Ignorance, which I expect is also true

Linguistic Ignorance: Nobody knows whether the sentence “the glass is 70% full and pretty full” is true in English in 2014, at context c, world w (and ...)

Without a doubt, not one of the measurers knows this fact either. However it is important to bear in mind the difference. Among our international team, we may suppose, are monolingual Chinese speakers who are ignorant of the second fact for reasons that have nothing to do with vagueness. In general, if you do not know what the sentence “the glass is 70% full” means in English (at context c and ...) then you may not know whether it is true. Conversely, you might know that “the glass is 70% full” is true in English (at c and ...) without knowing whether the glass in question is 70% full. A competent English speaker knowledgeable of the glass might have told the monolingual Chinese speaker that the sentence in question is true, without telling her that the glass is 70% full.

There are therefore two distinct things we are ignorant about. In fact, there are more than two things: there are countless other languages with sentences like the English one mentioned above whose truth statuses we do not know. But at any rate, the point is that they are all different things to be ignorant about: we are ignorant of the semantic status of a number of different sentences in different languages, and then we are ignorant about whether a particular glass is pretty full.

It is this latter fact, the ignorance about whether the glass is pretty full, that I think is hard for the linguistic theorist to explain and it is this fact that I shall concentrate on in what follows. If the linguistic theorist is right about the nature of vagueness it would be fairly easy to come up with explanations for Linguistic Ignorance. However doing so does not exempt her from the burden of addressing one of the central issues in the philosophy of vagueness: explaining Ignorance.

It’s worth mentioning that other facts about propositional attitudes need explaining, and would have served equally well as the basis of my criticism in the following. For example it is natural to think that both following are true

Bouletic If you know exactly how much water there is in the glass (and any other precise things that you care about), you should not further care whether it is pretty full or not.

Doxastic If you are rationally certain that it’s borderline whether the glass is pretty full you cannot be rationally certain that it’s pretty full.

These principles will be defended later in the book; for now I will focus on the more familiar principle Ignorance. Like Ignorance, there are linguistic versions of these principles that we should take care to distinguish.

What follows from the conjunction of Ignorance and Knowledge? An important consequence of these two assumptions is that, given a natural supervenience thesis, propositions are more fine-grained than sets of worlds. Moreover, this increase in the fineness of grain is a result of vagueness. One way to gloss this result would be to say that there are ‘vague propositions’ in addition to precise propositions. This way of putting things, however, is more contentious than it needs to be. Really all we are saying is that there
are some propositions, like the proposition that the glass is 70% full, whose truth the team of measurers have no problem discovering, and other propositions, such as the proposition that the glass is pretty full, whose truth the measurers have difficulty discovering, and moreover the propositions from the former class are not identical to propositions in the latter class. It is tempting to call the former proposition precise because I referred to it using a description consisting of mostly precise words, and tempting to call the latter proposition vague for symmetrical reasons. However, this takes sides on the terminological question about the bearers of precision and vagueness – all that the conjunction of ignorance and knowledge ensures is that these propositions are distinct in virtue of the different attitudes that people hold towards them.

To see why ignorance and knowledge require this level of fine-grainedness, take any set of worlds that is putatively identical to the proposition that the glass is pretty full – let’s say, the set of worlds where the glass is at least n% full. According to knowledge, somebody knows whether the glass is at least n% full, yet by ignorance nobody knows whether the glass is pretty full. Thus, applying the proposition role specified earlier and Leibniz’s law, the proposition that the glass is at least n% full (i.e., the set of worlds at which the glass is at least n% full) is not identical to the proposition that the glass is pretty full. Here is the argument explicitly (assume, without loss of generality, that the glass is in fact at least n% full):

1. Somebody knows that the glass is at least n% full. (By knowledge.)
2. Nobody knows that the glass is pretty full. (By ignorance.)
3. Thus nobody knows the proposition that the glass is pretty full, and somebody knows the proposition that the glass is at least n% full. (Applying the proposition role.)
4. Therefore the proposition that the glass is pretty full is not the same as the proposition that the glass is at least n% full. (Leibniz’s law.)

Let me clear up two possible misunderstandings about this argument. The first concerns philosophers who reserve the word ‘proposition’ for sets of worlds, states of affairs or for some other coarse-grained entity. Such philosophers can verbally reject any conclusions one might draw from this argument involving the word ‘proposition’ by divorcing proposition talk from that-clause talk (for example, they might reject the locution ‘that snow is white’ as a term for denoting the proposition the sentence ‘snow is white’ expresses – they will end up talking in convoluted ways, but there is nothing that in principle stops them from making this move.)

Such philosophers are therefore denying that propositions play what I called the ‘proposition role’. But of course, something has to play the proposition role – at least the way we use that-clauses in English suggests that something does – and so this is just a disagreement about which entities we should grant the honorific title ‘proposition’ to. No conclusion that matters to us here can only be stated using the word ‘proposition’ – the important upshot of the above argument concerns the objects of thought, the denotations of that-clauses – i.e., the things I have been calling ‘propositions’.

The second misunderstanding concerns a certain approach to the semantics of attitude reports. A common response to Frege puzzles involving Leibniz’s law, such as the one above, is to maintain that knowledge and belief, despite their surface form, are fundamentally three-place relations between a person, a proposition and a ‘mode of presentation’ which represents the way in which you come to believe that proposition (see Crimmins and Perry [27], and Richard [106].) According to this view no-one ever simply stands in this relation to a proposition – they stand in this relation to a proposition relative to a way of entertaining that proposition – a mode of presentation. One might object that in making this argument

\[19\] This choice is natural if we make the simplifying assumption that whether the glass is pretty full supervenes on the percentage of the glass that is full.
I have begged the question against these theorists by not being explicit about the mode of presentation. I think that whatever that view says about the fundamental psychological structure of propositional attitudes, it still has to account for ordinary language belief reports which have a binary structure and make no mention of modes of presentation. The most natural way to do this is to suppose that a natural language belief report which on the surface appears to be a binary relation in fact states a ternary connection between a person and a proposition (supplied by the referents of the subject and the that-clause respectively) and a contextually supplied mode of presentation which does not appear grammatically as a third term on the surface.20

The point to stress here is that my argument was not stated using the fundamental ternary relation, it was stated using the ordinary language binary relation that is expressed in the present context by the verb ‘knows’; which binary relation this verb expresses is context sensitive on this view, but that is not to say that I didn’t express a particular binary relation when I stated IGNORANCE and KNOWLEDGE. The correct response to this argument for a contextualist is not to reject the validity of the argument – it was literally an application of Leibniz’s law, along with a stipulation about what I meant by ‘proposition’ – but to reject one of the premises. Suppose that the proposition that the glass is pretty full is, in fact, identical to the proposition that the glass is at least n% full – call this proposition p. If the contextually salient mode of presentation is the vague one then, presumably, nobody stands in the three place knowing relation to p relative to that mode of presentation. In this context KNOWLEDGE expresses a falsehood. On the other hand if the contextually salient mode of presentation is the precise one then presumably the people who measured the percentage of the glass that is full do stand in the three place knowing relation to p relative to the precise mode of presentation. In these context IGNORANCE expresses a falsehood. Either way the argument is valid, and it is one of the premises that fails.21

Thus, I contend, anyone who accepts both IGNORANCE and KNOWLEDGE in the same breath, without changing the context, must acknowledge the thesis that there are vague propositions.22

I will treat those who deny the conjunction of IGNORANCE and KNOWLEDGE in section 2.4. The simplest way to deny the conjunction is to deny IGNORANCE flat out. However the contextualist variant of the no ignorance type view, briefly described above, allows certain attitude reports to depend on a contextually salient mode of presentation. There will be some contexts where they behave like the straightforward kind of no-ignorance theorist by denying IGNORANCE (and asserting KNOWLEDGE,) However, there will be many other contexts in which they can assert IGNORANCE (albeit, in these contexts they must deny

20 There are those who accept the ternary analysis of belief but are not contextualists (see Salmon [109].) For Salmon an ordinary belief report merely states that one stands in the ternary relation to a proposition relative to some mode of presentation. This type of view simply rejects the principle IGNORANCE altogether; the validity of above argument is therefore not in question on Salmon’s view, it is the truth of the premises. I shall therefore set Salmon aside along with others who reject IGNORANCE – I will come back to these theorists in section 2.4.

21 A different objection one might have is that according to some views Leibniz’s law has to be rejected, at least when stated as a schema involving proper names like ‘Hesperus’ and ‘Phosphorus’. According to these views, however, one cannot existentially generalise on names appearing within the scope of attitude reports. That said, I think that a good case can be made that Leibniz’s law is valid when restricted to proposition terms (‘that p’ or ‘the proposition that p’) since we need to be able to existentially generalise on these terms in order to do the kind of theoretical work we need to put them to (e.g. as prescribed by the proposition role). If there are entities fine-grained enough to be distinct whenever the psychological facts require it, and if by ‘proposition’ we just mean whatever satisfies the proposition role, then we may always quantify out on propositions terms and apply Leibniz’s law to them. This strategy is not available for concrete entities like Venus.

22 Note that for all I’ve said vague propositions are just ordered pairs of sets of worlds and vague modes of presentation. Although this view appears only to deviate minimally from the contextualist mode of presentation view, it is different in the respects that matter, namely, that IGNORANCE and KNOWLEDGE are jointly true in the same context and so propositions are fine-grained enough to be the objects of vagueness and precision.
Knowledge) thus allowing themselves more flexibility than the simplest no-ignorance view. Of course, at no context will both ignorance and Knowledge express a truth. In the next section, however, I shall simply assume that Ignorance and Knowledge are being granted, and that propositions are therefore somewhat fine-grained.

Before we move on, let me stress that the question we are dividing our discussion around is over whether one accepts the conjunction of Ignorance and Knowledge, and as a result, over how coarse or fine-grained we treat the objects of attitudes. However, as I’m sure many have noticed, deciding this question leaves a number of different possibilities regarding the relation between sentences and propositions open. The two most salient options to choose between are the views that vague sentences (i) express exactly one proposition and (ii) that they express several.23

A coarse grained account of propositions could be combined with (i) by either maintaining that the sentence ‘Harry is bald’ expresses exactly one precise proposition, whilst maintaining that we don’t know enough about linguistic practices to work out which proposition that is, or by maintaining that it expresses exactly one coarse-grained precise proposition but it is indeterminate which.24 On the other hand, if we were to combine (i) with a more fine grained account one could simply say that the sentence ‘Harry is bald’ determinately expresses a unique proposition, and that we furthermore know which that proposition is: the proposition that Harry is bald. We could also combine the coarse and fine-grained views in various ways with option (ii). The most natural view is a coarse grained view in which a vague sentence expresses a bunch of precise propositions, but one could also maintain that vague sentences express a collection containing both precise and vague propositions, or even exclusively vague propositions. My purpose, in bringing this up, is to stress that the subsequent discussion relies only on the stance we have taken towards the possibility of ignorance in the vague, and on how coarsely or finely we individuate the objects of ignorance; the status of the relation between sentences and propositions will, for most part, be absent from this discussion, and nothing I say turns on how we ultimately settle that question.

2.3 Explaining Ignorance about the Vague

Ignorance is an instance of one of a tightly knit cluster of phenomena that we associate closely with borderlineness. There are systematic connections between facts like Ignorance and borderline cases – facts like this just call out for some kind of general explanation. In our toy scenario, for example, not one of our international team of glass measurers was able to determine whether the glass was pretty full or not; this kind of thing is puzzling and seems to be just the kind of phenomenon a theory of vagueness is supposed to explain.

According to the theory I ultimately defend, that explanation takes the form of a general principle, which in turn falls out of a general theory of indeterminate propositions:

\[ \text{Epistemic} \] Necessarily, if it’s borderline whether \( p \), then it’s not rationally known whether \( p \).

A linguistic theorist who has no paraphrase for Epistemic in her own ideology cannot accept it as an explanation. If she is to offer a general explanation of Ignorance which

\[ \text{A variant of (ii) would be that sentences don’t express proposition simpliciter, but only relative to a precisification, much like a context sensitive sentence only expresses a proposition relative to a context. I haven’t considered the more radical view that borderline sentences don’t express propositions at all, since this view is subject to straightforward problems. For example the sentence ‘Either Harry is bald and electrons have charge or Harry is not bald and electrons have charge’ is a precise sentence equivalent to ‘electrons have charge’, and should therefore express a proposition. However it is hard to see how to compute that proposition compositionally according to this account, given that neither disjunct expresses a proposition.} \]

\[ \text{Note that although some self-described linguistic theorists adopt the latter idea, it is stated in the adverbialist vocabulary so it is unclear whether the latter theory is open to a linguistic theorist.} \]
still has something to do with vagueness she must instead seek to explain IGNORANCE in
terms of her favoured vocabulary: \( S \) is borderline in \( L \) relative to context \( c \) and other
parameters. But this raises a number of questions

The fact that some sentence, \( S \), is borderline as used in language \( L \), by community
\( C \), at context \( c \), at time \( t \) and world \( w \) purportedly explains why no-one in the
international team of glass measurers knows whether the glass is pretty full (i.e. explains IGNORANCE.) We must ask

(a) Which language \( L \) and linguistic community?
(b) Which sentence of this language is such that its borderlineness explains IGNORANCE?
(c) At what time must the sentence be borderline?
(d) What context must the sentence be borderline in?
(e) At what world must the sentence be borderline?

Before we move on let me stress that the demand is not just to explain why the English or
Spanish or Chinese speakers among our team don’t know whether the glass is pretty full.
The demand is to explain why nobody knows this, no matter how much they have inspected
the glass, no matter what language they speak or whether they even speak a language, no
matter what the architecture of their brains and so on.

So let us begin with the first question, (a). Suppose, without loss of generality, that the
sentence is simply the English sentence ‘this glass is pretty full’, uttered in a context where
‘this’ refers to the 70% full glass in question. Could the fact that this particular sentence is
borderline in English (relative to the relevant context and other parameters) explain why
nobody knows that the glass in question is pretty full? Could it, for example, explain why
none of the monolingual Chinese speakers who inspected the glass knows that the glass is
pretty full? The answer to this question seems to be obviously ‘no’ – the linguistic practices
of people in England can do nothing to prevent monolingual Chinese speakers from knowing
whether the glass is pretty full.

The best we can do is explain my ignorance by appealing to the borderliness of an English
sentence, a monolingual Spanish speakers ignorance by appealing to the borderliness of a
particular Spanish sentence, and so on. Although it is unclear to me whether even these
explanations are possible, it is natural to object that even if they were, they would be
incomplete – they say nothing of intelligent creatures that do not speak a public language.
Would it be easier for me to find out whether the glass is pretty full if I didn’t speak a
language? And even if I do speak a language, why should my co-speakers linguistic habits
bear on whether I can find out whether the glass is pretty full?

More importantly, it is not clear that we have an explanation of fact that nobody who
tried to determine whether the glass was pretty full succeeded. What we have here is
just a bunch of distinct and very local explanations of one-off facts. Mary doesn’t know
whether the glass is pretty full because the English use the sentence ‘this glass is pretty full’
in a certain way, whereas Pablo doesn’t know whether the glass is pretty full because the
Spanish use the sentence ‘este vidrio es bastante completo’ [CHECK SPANISH] in a certain
way. But, one might ask, what is the general reason that neither Mary and Pablo know, or
could come to know, whether the glass is pretty full. Also, if one didn’t speak Spanish (or
English) one might be puzzled by the explanation of Pablo’s (or Mary’s) ignorance – why
is it the way the Spanish use ‘este vidrio es bastante completo’ that prevent Pablo from
knowing, and not some other sentence? For the explanation to be explanatory it must also
include a description of what these sentences mean; but what holds the sentences used in
these explanations together cannot be that they all express a borderline proposition, for
that is to concede these explanations to the adverbialist.

Perhaps the general fact is that both Mary and Pablo speak a language containing a
sentence which both expresses the proposition that the glass is pretty full, and is moreover
used in whatever way suffices to make a sentence borderline in that language. This expla-
nation is adequate only on the assumption that every language under consideration has a
sentence which expresses the proposition that the glass is pretty full, and which is used in
that special way that makes the sentence borderline. But some of the languages in question
might fail to have a sentence which expresses that proposition; maybe there is no perfect
translation of ‘this glass is pretty full’ into German, for example, even if there might be
something pretty close. But this should not make it any easier for a German to find out
what we cannot in this situation.

These remarks cast some doubt on the possibility of answering our second question.
Even if we were just concentrating on explaining Mary’s ignorance we’d still need to supply
a borderline sentence. If I’m not around to point at the glass I would have to describe the
glass in some way or other, and so there will be lots of different sentences to choose from.
However we can’t explain Mary’s ignorance in terms of the borderliness of the sentence
‘John in tall’ – presumably it must be a sentence which expresses the proposition that
the glass is pretty full in L. As mentioned already, there might be languages in which no
sentence expresses the proposition that the glass is pretty full. This would not make it
easier for Mary to find out whether the glass was pretty full. Furthermore, what if, as some
theories claim, vagueness prevents a sentences expressing a unique proposition? Must the
proposition that glass is pretty full be merely among the propositions S expresses?

Finally, it seems we must specify a time, world and context at which the sentence is
borderline. Words often become more precise as language evolves. If the selected sentence S
(‘this glass is pretty full’ say) had once been precise it would presumably still be impossible
to know whether the glass is pretty full. Conversely, if ‘electrons have positive charge’ had
once been vague that shouldn’t prevent me from finding out that electrons have positive
charge. Maybe S has to be vague at the time I’m trying to find out whether the glass is
pretty full. If we have to pick a different sentence each time we want to explain why someone
can’t figure out whether the glass is pretty full then the explanation loses its generality.

Furthermore, had we used S in a precise way we wouldn’t be in a better epistemic
situation regarding Harry’s head. We may therefore be able to explain why no-one in fact
knows whether the glass is pretty full by appealing to a selected sentences borderliness, but
in order to explain why we couldn’t have known whether the glass is pretty full, even if S
had been used in a precise way, we need also to relativise to worlds. This opens up the
possibility that we can’t know that the glass is pretty full because the sentence ‘John is
tall’ is used in a vague way at another world to mean that the glass is pretty full.

Let me end by mentioning one more point which, although not a fully fleshed out
objection, is something I find worrisome. In many ways ways explaining IGNORANCE is one
of the easier jobs for a linguistic theorist. Other facts are harder to explain. Suppose that
Harry is as before a borderline case of baldness, and that we rationally believe this. Then
it seems that

AGNOTICISM: It would be irrational to believe (given what we know) that the glass
is pretty full, and it would be irrational to believe that the glass isn’t pretty full.

This fact is prima facie quite puzzling. After all you know that either Harry is bald or he
isn’t, so at least one of the two beliefs above is true. Furthermore you have all the evidence
you could possibly have available. It feels like there should be some general fact about
vagueness related phonemona that explains this.

Since there appear to be plenty of things we do not know which are rational to believe,
it seems, therefore, that there is a separate and harder problem of explaining what feature
of linguistically vague sentences prevents us from rationally believing propositions. This
seems a lot harder to do. For example, in a discussion of Williamson’s epistemicism, Horwich
writes: “the ignorance due to vagueness is attributed to a special form of unreliability – an
external failure – and not, as it should be, to the internal difficulty in making a judgement.”
[64]. But surely any explanation of our ignorance that appeals to the use of a term in a public
language is going to be an external one.\textsuperscript{25} Even if we bracket the problems surrounding the explanation of \textit{Ignorance}, I am much less confident that an explanation of \textit{Agnosticism} in terms of language use can be given.

### 2.3.1 Explaining Ignorance via Metalinguistic Safety Principles

Let me now turn to a specific attempt to explain \textit{Ignorance} within a linguistic theory. This attempt arises in the context of the epistemicist account of vagueness defended by Williamson [131]. Williamson’s view as I am characterising it is linguistic – vague sentences are \textit{semantically plastic}: slight variations in the use of language will result in that sentence expressing a slightly different proposition. A sentence is borderline relative to a linguistic community $C$ and parameters on this view if, there are close worlds, with respects to how that language is used, where that sentence says something false and close worlds where it says something true. It earns the name ‘epistemicism’ as it allegedly \textit{entails} the ignorance thesis. If this is true it is a significant benefit of the view over other linguistic theories. That said, even if one rejects Williamson’s analysis of vagueness in terms of semantic plasticity, one might still think that Williamson’s explanation of ignorance in terms of semantic plasticity is fundamentally sound by maintaining that vagueness and semantic plasticity, although not identical, come hand in hand. Thus Williamson’s explanation of vagueness related ignorance has interest that is independent of the success of his brand of epistemism.

The crux of Williamson’s explanation is a controversial principle that has become to be known as the ‘metalinguistic safety principle’. In order to understand the principle we firstly need to introduce some definitions. Say that $A$’s belief that $P$ is \textit{safe} iff it couldn’t easily have been the case that (i) $A$ believes that $P$ and (ii) it is not the case that $P$. $A$’s belief that $P$ is \textit{metalinguistically safe} iff it couldn’t easily have been the case that (i) $A$ produces the belief token that actually resulted in a belief that $P$ and (ii) that belief token expresses a false proposition (let us say that a belief token ‘expresses’ a proposition $P$ if it constitutes a belief that $P$). The expression ‘it couldn’t easily have been the case that $P$’ is a term of art, and means something roughly like $P$ isn’t true in any nearby world, where nearby is determined by some measure of similarity that is understood to have some epistemic significance. It is unclear whether one can get an understanding of the relevant notion of similarity without already having a grip on the concept of knowledge, but however this question turns out, the notion a safe belief is still of interest and can be used to put some important structural constraints on knowledge that would be hard to motivate without invoking the notion.

Although it is by no means uncontroversial, a sizeable number of philosophers take the notion of a safe belief to have some epistemic force. In particular, these philosophers subscribe to something like the following safety principle:\textsuperscript{26}

\textbf{Safet\'y:} One knows that $P$ only if one does so via a belief that is safe.

Roughly, the thought is if you could easily have falsely believed that $P$ then even if you were in fact correct about $P$ you were lucky to be correct, and so your belief could not constitute knowledge. It is important to contrast this principle with its metalinguistic variant:

\textbf{Metalinguistic Safety:} One knows that $P$ only if one does so via a belief that is metalinguistically safe.\textsuperscript{27}

\textsuperscript{25}Horwich’s own view may be an exception: Horwich, unlike most linguistic theorists, takes the language in question to be an internal language of thought instead of a public language. This exception aside, the problem of explaining \textit{Agnosticism} seems harder for most linguistic theories.

\textsuperscript{26}The principle as stated below will probably need some refinement. For example, it is common to additionally stipulate that a belief that $P$ is only safe if it couldn’t easily been the case that the agent falsely believed that $P$ by the same \textit{method} she actually came to believe that $P$.

\textsuperscript{27}Let me mention a variant metalinguistic safety principle that I find more plausible, although I won’t
The safety principle says that if you know that \( P \) then you couldn’t easily have been wrong about \( P \). The metalinguistic safety principle entails no such thing: being easily wrong about \( P \) is neither necessary nor sufficient for having a metalinguistically unsafe belief – all you need to do to be metalinguistically unsafe is a have a false belief in some proposition in a nearby world, although not necessarily a false belief that \( P \), it needs only belong to a belief type tokens of which are beliefs that \( P \) in the actual world.

It should be clear that the ordinary safety principle cannot explain Ignorance. For example, in the present example, given my knowledge, the glass is 66% full in all worlds that count as nearby for me, and since whether the glass is pretty full supervenes on how full it is it is either pretty full in all nearby worlds or not pretty full in all nearby worlds. Thus for all the safety principle says, a belief that the glass was pretty full could constitute knowledge provided it was in fact a true belief. Metalinguistic safety does better in this regard. One could make a reasonable case that the semantic properties of belief tokens are correlated to the semantic properties of corresponding public language sentence tokens. On that hypothesis we can attribute a similar degree of semantic plasticity to vague beliefs corresponding to vague public language sentences, and we can also attribute to them the feature Williamson identifies with borderlineness: in some nearby worlds they express a true belief and in others they express a false belief. If I form a belief that the glass is pretty full by forming a belief that is borderline in this sense, then this belief is not metalinguistically safe: although I couldn’t easily have been wrong about whether the glass is pretty full, I could easily have been wrong about another proposition – one that, in that same nearby world, would have been expressed by the same belief token.

Let me firstly note, along with many others, my reservations about the metalinguistic safety principle. Suppose that, unbeknownst to me, my fellow English speakers had decided to start using the numeral ‘1’ to mean 100. Grant also the assumption, needed in the explanation above, that the semantic properties of beliefs and their corresponding public language sentences are correlated. Since the actual world certainly counts as nearby, it appears as though there are nearby worlds where a belief token corresponding to ‘1+1=2’ expresses the false proposition that 100+100=2; in other words my belief that 1+1=2 is metalinguistically unsafe. Yet, I hope, it should be clear that this form of unsafety does not in any way undermine my knowledge that 1+1 = 2, a fact which I can verify by performing a simple calculation.

Although the metalinguistic safety principle seems somewhat suspect, notice that the ordinary safety principle, which is on much firmer footing, does allow us to explain a related fact: that we cannot know whether the sentence ‘this glass is pretty full’ is true or not in English relative to the context and other parameters described. If that sentence is borderline in that context then it expresses a false proposition in a nearby world, and so, we are to suppose, the belief corresponding to this sentence is not metalinguistically safe.

Recall, however, that I took pains to distinguish Ignorance from Linguistic Ignorance – the latter claim only states that it is impossible to know whether the sentence ‘this glass is pretty full’ is true or not in English relative to the relevant parameters. The fact that there are close worlds where we use ‘this glass is pretty full’ differently may explain, by ordinary safety principles, why we cannot know that this sentence is true in English. But whether the glass is pretty full or not is not unstable in the same way: it is just as full as it actually is at all the relevantly close worlds. So there is also the further fact that, given the

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28 For more criticisms of the metalinguistic safety principle see Magidor and Kearnes [70]. See also Hawthorne [61], Sennet [113], Caie [22] and Mahtani [87] for related discussion.
status of Harry’s head, we cannot know whether he is bald, and there is no obvious way to infer ignorance of the latter fact from ignorance of the former linguistic fact.

One way to bridge this gap would be to invoke knowledge of the disquotational schema. Suppose I know that if this glass is pretty full then the sentence ‘this glass is pretty full’ is true in English, at the present context, etc. However, since we have just shown that I do not know the consequent on the basis of an orthodox safety principle, one can infer that I do not know that the glass is pretty full, assuming a small amount of closure. A parallel argument could be made to show that I do not know that the glass is not pretty full either. Of course, this argument still suffers from the defect that it is not completely general. To explain why a monolingual Spanish speaker does not know whether the glass is pretty full we’d appeal knowledge of a principle that does not take the form of a disquotational principle when it is stated in English: if Pablo knows that this glass is pretty full then he knows that ‘este vidrio es bastante completo’ is a true sentence of Spanish in this context. The fact that neither I nor Pablo know whether the glass is full seems to call out for a general explanation—something holding both the cases together—which the explanation I have just given does not seem to provide.

Generality aside, there are some important limits to when knowledge of disquotational reasoning can be appealed to, even among native English speakers. When words change their meanings, or we are no longer sure of a words meaning, we are generally not in a position to know instances of disquotational principles involving those words. To demonstrate the general point, let us suppose that Alice is looking at Madagascar for the first time, and correctly concludes that it is an island. However Alice is living during the time in which the word ‘Madagascar’ was being used to refer to a portion of mainland Somalia. So although Alice does in fact have a correct belief that Madagascar is an island, and in fact she knows it is an island, she also knows that the sentence ‘Madagascar is an island’ is not a true sentence of English. Similarly cases where a words meaning is unknown gives rise to cases where the relevant disquotational principles are unknown. If Alice is observing a particular cow chewing grass, then it’s plausible that she knows that the cow is masticating, although if she doesn’t know that ‘masticating’ means chewing, she might not know that the sentence ‘the cow is masticating’ is true in English in her context.

The critical point here is that if the argument from semantic plasticity is to work, then there are nearby worlds in which the borderline sentence does not mean what it actually means. Assuming the ordinary safety principle, it follows that unless our beliefs about meanings are extraordinarily sensitive to very slight differences in meaning due to slight differences in use, we simply do not know what borderline sentences mean. Thus it follows that the above argument for IGNORANCE appeals to exactly the kind of knowledge of disquotational principles that we have seen to be suspect.29 Indeed we can effectively prove that this instance of the disquotational principle is unknown from SAFETY. That is, we can argue that although the sentence “Harry is bald’ is true if and only if Harry is bald’ expresses a truth at all nearby worlds (even if it is a different truth at each world), the regular safety principle predicts that, were there nearby worlds where ‘Harry is bald’ is true even though Harry isn’t bald (for example), we would not count as knowing that ‘Harry is bald’ is true iff Harry is bald. Let us suppose for the sake of argument that Harry is not bald (a symmetrical argument can be made if he is) and that he has the same number of hairs at every nearby world (lets suppose I know how many hairs he has.) So there are no nearby worlds at which Harry is bald, since baldness supervenes on hair number. Yet by hypothesis the sentence is borderline, so there are nearby worlds where ‘Harry is bald’ expresses a truth and nearby worlds where it expresses a falsehood. So there are nearby

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29Hawthorne [61], notes that there is a case to be made that the T-schema “p’ is true if and only if p’ is true in all nearby worlds, due to penumbral connections between the word ‘true’ and the vague words appearing in ‘p’. However this is still not sufficient to guarantee that one knows that ‘p’ is true if and only if p, unless one has knowledge of the more complicated instance of the T-schema: “p’ is true if and only if p’ is true if and only if ‘p’ is true if and only if p. But knowledge of this is suspect for the same reason that knowledge of the initial instance is suspect.
worlds where ‘Harry is bald’ is true even though Harry is not bald, so by the safety principle it follows that I do not know that ‘Harry is bald’ is true if and only if Harry is bald.

The epistemicist might insist at this juncture that although there are nearby worlds where Harry is bald, even though ‘Harry is bald’ isn’t true in English (because it means something else), our beliefs are surprisingly sensitive to the differences in meaning of ‘Harry is bald’ between these different worlds. Sensitive enough that we are able to have the belief that Harry is bald iff ‘Harry is bald’ is true in English only at the worlds where ‘Harry is bald’ does indeed mean something that’s true iff Harry is bald. On its face this suggestion looks absurd: according to the picture described the meaning of ‘Harry is bald’ in English depends on very particular features of usage, and can change on the basis of slight differences of use that are clearly far beyond the knowledge of ordinary humans. On appearances this sounds like the type of view where we cannot know what vague sentences mean – the best we can hope for is to know the range of meanings a word could have.

The epistemicist might try defend the position by noting that at the nearby worlds where ‘Harry is bald’ means something else, so does the corresponding disquotational principle: ‘Harry is bald’ means that Harry is bald in English’. Indeed, whatever ‘Harry is bald’ means at this world – $p$, lets say – the instance of the disquotational principle will mean something true, namely that ‘Harry is bald’ means that $p$, and not the false claim that ‘Harry is bald’ means that Harry is bald. The thought, then, is that we get to know what ‘Harry is bald’ means simply by tokening some mentalese equivalent of the sentence “Harry is bald’ means that Harry is bald’, which is guaranteed to express a truth whatever it means.

It is hard not to think there is something extremely fishy about this way of acquiring knowledge of something. Consider again Alice, who does not know what ‘masticate’ means, but can see, and therefore knows, that a particular cow is masticating, even though she has no idea whether the sentence ‘the cow is masticating’ is true in English. I take it that there is something obviously wrong about inferring from her non-linguistic knowledge about the cows eating behaviour, that a particular English sentence, whose meaning she is completely unsure about, is true. To conclude that ‘the cow is masticating’ is true in English in Alice’s position seems just as obviously wrong even once we have pointed out that if she were to form a belief of the form ‘if the cow is masticating then ‘the cow is masticating’ is true in English’ her belief would most likely express a truth, even if she does not know which truth it is.

2.4 Denying Ignorance about the Vague

If vagueness is a special source of ignorance, I argued, then a linguistic account of vagueness will have a hard time explaining this ignorance. But our discussion there rested on two suppositions: firstly that vagueness is a source of ignorance, and secondly that ignorance due to vagueness cannot be understood without adopting a fairly fine-grained account of the objects of propositional attitudes.

These two assumptions were related in a number of ways. Firstly, it was shown that if you have a coarse grained theory of the objects of attitudes – one that identifies the proposition that the glass is pretty full with the proposition that it is at least $n\%$ full (for some $n$) – then you should deny the conjunction of ignorance and knowledge (at least, when evaluated in a single context). The converse of this claim, however, while not forced on us, is extremely natural. If you deny ignorance and knowledge and you reject the adverbial way of speaking about borderlineness, then there you have very little reason to postulate any fineness of grain due to vagueness.

A particularly simple theory of propositions that identifies the proposition that the glass is pretty full with the proposition that it is at least $n\%$ full, for some suitable choice of $n$, is the view that propositions are just sets of possible worlds. No-ignorance views are therefore consistent with the interpretation of propositions being sets of possible worlds, and for convenience I shall often speak as though this is the background theory of propositions.
Little I say should turn essentially on this assumption. Perhaps there are examples having nothing to do with vagueness that require one to posit propositional structure (for instance); if so then every I say should straightforwardly extend to structured theories of propositions.

2.4.1 The Straightforward No-Ignorance View

Failures of principles like Ignorance should strike us extremely surprising. To deny such principles would be to take seriously the idea that, for example, not only is there a nanosecond at which I stopped being a child, but that we in fact typically can, and often do, know which nanosecond this is! The view seems to be open to a simple refutation – as a matter of sociological fact, no-one, not even those who reject the ignorance thesis, will ever go so far as to try answer a question like ‘what is the length of my childhood in nanoseconds?’

While claims like Ignorance are widely acknowledged, their acceptance isn’t universal. Both David Barnett [9] and Cian Dorr [31] argue that when people are knowledgeable about the relevant precise facts people often do have knowledge in borderline cases. Since Barnett is not a linguistic theorist, and since I treat his version of this view in chapter 5, I shall focus here on Dorr’s view.

Why is it, then, that we don’t say ‘yes’ or ‘no’ to borderline questions whose answers we know? Let’s consider a specific example: Suppose that Harry is a borderline case of the predicate ‘bald’. By the law of excluded middle, an assumption Dorr accepts, Harry is either bald or he isn’t. Let us suppose for the sake of argument that Harry is bald; so by the no-ignorance view I know that Harry is bald, assuming I have the relevant precise facts to hand (hair number, distribution and so on.) Why is it, then, that I don’t simply say ‘yes’ when someone asks me the question ‘Is Harry bald?’?

Dorr’s explanation is that, in virtue of the audiences knowledge of the conventions of the language, assertively uttering this sentence would increase their confidence in a number of false propositions. It is natural to pair the view with a metasemantic account of vagueness according to which producing assertive utterances of the sentence ‘Harry is bald’ (or utterances of ‘yes’ in answer to the above question) results in one asserting a great number of similar propositions; perhaps including, whenever \( n \) is in an admissible range, the proposition that Harry has less than \( n \) hairs. According to this view when Harry is in the borderline region then some of the propositions you assert will be false, and for this reason one shouldn’t assert such sentences.

Note also that in the absence of ignorance, the view is compatible with a coarse grained theory of propositions in which the proposition that Harry is bald just is the proposition that Harry has less than \( n \) hairs, for some suitable choice of \( n \) (although the sentence ‘the proposition that Harry is bald is identical to the proposition that Harry has less than \( n \) hairs’ will be unassertable for the kinds of reasons ‘Harry is bald’ is unassertable, since the descriptions flanking each side of the identity do not determinately refer to a single proposition.) If so then knowing that Harry is bald just is knowing, for some \( n \), that Harry has less than \( n \) hairs.

It should be evident that disquotational principles for dissent and assent cannot be a part of this view. Take for example:

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30. Crispen Wright [137] also denies principles like Ignorance, although his theory seems to require than one accept an intuitionistic logic. Jeremy Goodman, [58], has also questioned whether vagueness in general precludes knowledge, although his arguments do not cast doubt on the majority of instances of the general principle, including Ignorance.

31. Indeed, while Dorr wasn’t specific about this in [31], this is his present view.
Dissent: Don’t dissent to ‘A’ if you know that A

By ‘dissent’ I mean to include a range of linguistic responses that include flat out denial as well as a principled and persistent refusal to assent. According to Dorr one should dissent from ‘Harry is bald’ in some cases, even when one knows that Harry is bald.

Now of course the scope of Dissent needs to be limited somewhat if it is remain at all plausible. It doesn’t apply if verbally dissenting to ‘A’ will have other bad consequences – you might, for example, wake the baby, or perhaps you’re in a case where lying is justified all things considered, or ‘A’ has a presupposition that you want to avoid. However it seems like a prima facie cost if one cannot accept some suitably limited principle of this form – such principles seem to be integral to the way that we learn what people believe on the basis of their linguistic behaviour: if I look all puzzled and refuse to say ‘yes’ or ‘no’ when you ask me a question like ‘is Harry bald?’ English speakers typically conclude, mistakenly according to Dorr, that I don’t know whether Harry is bald.

Here is another worry. Supposing that I know the number of hairs on Harry’s head, then even though I know that Harry is bald I won’t assert the sentence ‘Harry is bald’ for that would be to assert (or at least, raise my audiences confidence in) a number of similar but false propositions. Surely, one might think, if I know that Harry is bald it should be simple for me to introspect on that fact, and come to know that I know that Harry is bald. Why then couldn’t I just assert the sentence ‘I know that Harry is bald’ and allow my audience to conclude that Harry is bald from that? According to Dorr it is for exactly the same reasons I cannot assert the sentence ‘Harry is bald’ – the knowledge ascription is also borderline. Generally, when S is borderline ‘I know that S’ is also borderline provided I’m in possession of the relevant precise facts.

Note that there are two aspects of our linguistic behaviour that need explaining. One is our refusal to accept a sentence or its negation when it is a simple borderline sentence that does not involve attitude reports, such as ‘Harry is bald’. Dorr’s explanation seems to account for this adequately. It is not clear, however, whether Dorr can accommodate our linguistic behaviour regarding borderline sentences involving knowledge and belief ascriptions like ‘Alice knows that Harry is bald’. Unlike the simple borderline sentence ‘Harry is bald’, where we are not inclined to assert it or its negation, we are inclined to outright deny things like ‘Alice knows that Harry is bald’ and to even assert its negation. Yet according to Dorr, this sentence is also borderline, and we should not be asserting either it or its negation.

Not only are utterances of ‘Alice knows that Harry is bald’ utterances of borderline sentences, according to Dorr, but one would expect at around a half of these utterances to be untrue in cases where the knower is in possession of the relevant precise facts. It seems, then, that this theory must endorse some kind of error theory regarding attitude reports: speakers frequently assent to the false sentence ‘Alice doesn’t believe/know that Harry is bald’.

Things get worse. The sentence ‘Alice believes that Harry is bald’, let’s suppose, is borderline. Now according to Dorr, if I’m in possession of the relevant precise facts (perhaps the fact that Alice believes that Harry has exactly n hairs) then I know whether Alice believes that Harry is bald. Thus I am in an even worse position than someone who goes about assertively uttering false sentences: I am going about asserting ‘Alice does not believe that Harry is bald’ when I in fact know that Alice believes that Harry is bald.

2.4.2 Non-linguistic Behaviour

Setting aside the troubles with attitude reports, an adequate theory ought to do more than just account for linguistic behaviour that is associated with vagueness. It seems to be extremely hard to eradicate vagueness related uncertainty from explanations of our non-linguistic behaviour as well. It is therefore hard to see, even in principle, how a view in which vagueness is a public language phenomenon could accommodate this behaviour.
For example: I may know exactly how much cheese I have, but still be unsure about whether I have enough to make a cheese sandwich that tastes reasonably good – that, for example, doesn’t have to high a ratio of bread to cheese. It is natural to attribute this uncertainty to the fact that I have a borderline amount of cheese. There is no precise fact that I am unsure of, but yet I still hesitate: it would be a waste of bread if I don’t have enough cheese, but it would satisfy my need for food if I do. How is this to be explained if all ignorance is ignorance about the precise? You might try to attribute the hesitation to uncertainty about some precise fact about what the resulting sandwich would taste like. Let me clarify then: I know exactly what it would taste like, I just don’t know whether that taste is tasty – with that cheese to bread ratio, the result would be borderline. The hesitation is not like the refusal to assert or deny a sentence – I needn’t even speak a language to be in this situation. The problem is completely internal to me: I don’t know whether the sandwich would be tasty, and so my actions will depend on how confident I am about its tastiness. Phenomenal sorites show that these kinds of scenarios are ubiquitous – the states you are in when something looks red to you, when you feel cold or feel hungry, and so on, are all soritesable, and these states all seem to play an important role in guiding our non-linguistic behaviour in ways that couldn’t explained if we were always certain about whether we were cold or hungry.

Another example in which vagueness makes its way into our thought, although plausibly not via a public language, is when we acquire evidence through imperfect perceptual faculties. If I see a tree in the distance I learn some things about its height. It seems implausible that my total evidence, after seeing the tree, is that the tree is between \( x \) and \( y \) centimeters tall, or any precise proposition of this type – my credences will presumably fit some kind of smooth curve over the possible precise heights, but no credence that is gotten by conditioning on a precise proposition would have this smooth shape. It is natural to think that in this case my total evidence is vague; yet the visual experience and my resulting epistemic state had nothing to do with my ability to speak a language. Of course, there is much more to be said about this argument, and we’ll return to that in chapter 4, but on the face of it vagueness is pervasive in our non-linguistic mental lives.

2.4.3 The Contextualist No-Ignorance View

One of the main problems with the no-ignorance account described above is its treatment of attitude reports. Indeed, one can see this as an instance of a more general problem of trying to account for propositional attitude ascriptions in a theory where the objects of attitudes are reasonably coarse grained. If the proposition that Harry is bald just is the proposition that Harry has less than \( n \) hairs, for some \( n \), then the fact that we are disinclined to say that someone believes that Harry is bald even if we are inclined to say that they believe that Harry has less than \( n \) hairs might be subsumed by a more general theory for dealing with Frege puzzles.

Of course, one solution is to adopt a fine-grained account of the things appearing in the complements of attitude ascriptions (i.e. ‘propositions’, in my terminology.) This would allow you to accept the conjunction of IGNORANCE and KNOWLEDGE, and falls under the fine grained views we have already considered in earlier sections. If we want to maintain a coarse grained account of propositions it seems as though we’d either have to bite the bullet and accept the problematic attitude reports, or adopt a contextualist view in which the word ‘knows’ expresses a different relation between people and propositions in different contexts (see [27] and [106] for views of this kind in the context of attitude reports involving proper names.31) On this view it is consistent to say that the expressions ‘that Harry has

\[32\]Whether it’s rational to care intrinsically about whether the sandwich is tasty, as opposed to the particular tastes it could have, is another question; but at any rate, some people do care, and this fact about them explains what they do.

\[33\]I do not know of a fully worked out defence of this view in the context of vagueness in print, however both John Hawthorne and Tim Williamson have pressed me to respond to this kind of view, and it is
less than $n$ hairs' and 'that Harry is bald' denote the same proposition, $p$, but still maintain
that utterances of 'Jane knows that Harry has less than $n$ hairs' and 'Jane knows that Harry
is bald' can have different truth values provided that they are made in different contexts.
In one context 'knows' must express a relation that Jane bears to $p$, whilst in the other
context it must express a relation which she doesn’t bear to $p$.

Let me suggest one way to cash this idea out a bit further. When a person has a belief-
like relation to some proposition $p$ they typically do so by being in a particular kind of
mental state. Naturally lots of different mental states can result in the same proposition
being believed: I might mentally token something like the sentence ‘Hesperus is Hesperus’ in
believing the necessary proposition, or I might mentally token something like the sentence
‘Hesperus is Phosphorus’. Presumably it would be fairly easy to come to know the necessary
the proposition by believing it the first way, and not so easy to come to know it by believing
it the second way. For convenience, call these different ways of believing a proposition
‘modes of presentation’. Presumably before the discovery that Hesperus was Phosphorus,
nobody knew the necessary proposition via second mode of presentation, although everyone
knew it via the first mode of presentation.

So far I have been using locutions like ‘$S$ knows/believes $p$ via $m$’. Let us grant the
empirical assumption that this is really what is going on psychologically when we believe
things – that this expression picks out a natural three place relation between propositions
people and modes of presentation. How does this technical ternary relation relate to the
binary ordinary language expressions ‘knows’ and ‘believes’ and so on? These are the terms
that we have been theorising with, and are the terms relevant to the puzzles we have been
developing. According to contextualism there is no one relation that these expressions pick
out: in contexts where a mode of presentation $m$ is salient, ‘knows’ picks out the relation of
knowing via $m$, and in other contexts knowing via $m'$. A context, then, can be thought of
as providing a way of matching up coarse grained propositions with modes of presentation.

It’s natural to think that the that-clause we use when making an attitude ascription
brings to salience certain modes of presentations and not others. Thus when I say ‘Alice
doesn’t know whether Harry is bald’ I bring to salience a vague mode of presentation
corresponding to the vague sentence ‘Harry is bald’, and when I say ‘Alice does know
whether Harry has less than $n$ hairs’ I bring to salience a precise mode of presentation,
even if, in both cases, I am just ascribing some relation between Alice and one and the
same proposition. Note that the view in question is importantly different from the view
which treats proposition, the denotations of that-clauses, to be ordered pairs of sets of worlds
(or some other coarse grained entity) and modes of presentations. There are affinities, but
this view is a version of the fine graining strategy: this view can straightforwardly make
sense of the conjunction of IGNORANCE and KNOWLEDGE, and is therefore covered by the
criticisms in the previous section.

The contextualist cannot accept the conjunction of IGNORANCE and KNOWLEDGE – at
least, not unless the context changes mid sentence.\footnote{One could just insist that the contextually salient mode of presentation changes to match the embedded sentence with which attitude ascription is made, but this view seems hard to distinguish from the fine graining view where one takes propositions to be ordered pairs of sets of worlds and modes of presentations. One could try to pin the difference on some important line between pragmatics and semantics, but it seems unlikely to me that this would draw a substantive difference between the two views.} For whatever mode of presentation is
salient, since we are relating the agent to one and the same propositions, it cannot be both
known and not known relative to that mode of presentation.

At this point it is worth drawing the analogy between this type of view and a similar
view in the philosophy of modality. For example, when railing against the third grade of
modal involvement, Quine writes “being necessarily or possibly thus and so is in general
not a trait of the object concerned, but depends on the manner of referring to the object.”
[99]. According to Quine an object is only necessarily $F$ relative to some linguistic ‘mode
of presentation’ of that object: the number nine is necessarily composite relative to the
\begin{footnotesize}
\footnote{One could just insist that the contextually salient mode of presentation changes to match the embedded sentence with which attitude ascription is made, but this view seems hard to distinguish from the fine graining view where one takes propositions to be ordered pairs of sets of worlds and modes of presentations. One could try to pin the difference on some important line between pragmatics and semantics, but it seems unlikely to me that this would draw a substantive difference between the two views.}
\end{footnotesize}
mode of presentation ‘the square of three’ but not ‘the number of planets’. It’s natural for the contextualist to make an analogous move here concerning determinacy operators. Rather than taking the relation ‘S is borderline in L relative to parameters \( p \)' as primitive (as suggested in section 2.1.2), this theorist might theorise instead with a slightly more complex expression ‘It’s borderline whether \( p \) relative to the mode of presentation \( S \) in \( L \) and parameters \( \bar{p} \).

The view does not completely solve all the problems. It predicts, for example, the existence of a puzzling context in which it is OK to assert ‘Alice knows whether Harry is bald’. Moreover, there will be no context in which it is ok to assert ‘John knows that Jane’s height is less than \( x \) m but does not know whether she’s tall’: whatever mapping from propositions to modes of presentation the context provides, if the proposition that Jane is less than \( x \) m is identical to the proposition that she’s tall, it will be assigned the same mode of presentation by the context and John must either know the proposition or not know it relative to that mode of presentation.

### 2.4.4 More on Non-linguistic Behaviour

The puzzles raised about the role of vague beliefs in guiding our non-linguistic behaviour in section 2.4.2 seem to be just as pertinent here. If it is modes of presentations that are vague, and these are vague in virtue of their relation to public language sentences (perhaps they are in some sense synonymous with vague public language sentences), it is hard to see how they play a role in behaviour that doesn’t seem to involve language.

Moreover, most of our vague beliefs aren’t articulable, either in a public language or even in a private language of thought. If I’ve been blindfolded and rolled down a hill, I have lots of beliefs about the rough direct up is, which I acquire through some form of proprioception. I certainly have these beliefs, since they affect the actions that I make, but I could not articulate these beliefs in English, or even mentally. They are just beliefs that I have, they do not appear to be beliefs about any precise fact, and they do not appear to be linguistic. Such facts are puzzling for a view on which it is only linguistic items that are vague.

### The Problem from Decision Theory

Alice ran because she believed that Jack the Ripper was following her – not because she believed that Friendly Fred was following her. In order to explain why Alice acted in the way that she did we must appeal to a potentially hyperintensional distinction in her attitudes. The way we behave depends on what we believe and desire and any decent theory of propositional attitudes ought to be able to accommodate explanations like this one. It is crucial to note that hyperintensional differences in a persons beliefs do not just generate differences in linguistic behaviour – the contextualist would not be hard pressed to accommodate differences in linguistic behaviour within her theory of modes of presentation – they also generate differences in non-linguistic behaviour as well. Alice’s behavioural profile could be instantiated by someone who didn’t speak any languages whatsoever.

Decision theory is an extremely general and powerful framework for representing a rational agents decisions, beliefs and desires. A now very standard way to formulate decision theory, due to Richard Jeffrey [67], takes as a starting point a set of indices, a probability function over the algebra of sets of indices and a utility function on these indices and a probability function over the algebra of sets of indices. The expected utility, or ‘news value’, of a proposition is then given by summing (or integrating if necessary), over each index, the result of multiplying the utility of that index with the probability probability of that index conditional on that proposition. According to the orthodox interpretation of this formalism the indices correspond to epistemic possibilities.

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35I should note here that many people feel that Jeffrey’s specific theory delivers the wrong verdict on Newcomb’s paradox. However even these dissenters accept the features I have listed here – my discussion extends mutatis mutandis to these theories.
sets of them to propositions, and the probability and utility function correspond to graded attitudes equivalent in nature to the ungraded attitudes of belief and desire. The intuition behind this is completely straightforward: to find out how good things are conditional on $A$ first of all assume $A$, and calculate a weighted average of how good the remaining epistemic possibilities are, with each possibility weighted by its probability given $A$.

Decision theory is well established. There should, I think, be a default obligation on those who wish to abandon it to provide some reasonable alternative. Now, a crucial feature of this framework is that it is a theory of rational action that operates directly on the contents of the agent’s beliefs and desires. The different ways of having beliefs with those contents, the modes of presentation, play no role whatsoever in its formulation – perhaps such things are needed in the correct semantics of belief reports, but for orthodox decision theory they remain completely idle. Conclusions that can be drawn about fineness of grain in this framework, then, cannot be explained away by standard contextualist manoeuvres invoking modes of presentation.

According to orthodox decision theory, then, whether an agent’s action is rational or not supervenes on the contents of her beliefs and desires (construed broadly to include assignments of credence and utilities): 

**Content to Action:** If Alice and Bob’s beliefs and desires have the same contents, then they will act the same way if they are rational and have the same actions available to them.

**Content to Action** allows us to draw conclusions about how fine grained contents are in a way that are impervious to the standard contextualist responses. For example, it entails that if two rational people can behave differently whilst having only beliefs and desires with necessarily equivalent contents then contents are more fine grained than sets of worlds.

Let us demonstrate this strategy with an utterly trivial example. If we were willing to relativise all non-truth functional operators to guises, in the way the contextualist does to attitudinal operators, then it is a live option that there are only two propositions: the true and the false. However, even if we can account for attitude reports and other non-truth functional operators using the contextualist apparatus there is a distinct problem deriving from decision theory. Given **Content to Action** there are only sixteen types of people, most of which are probabilistically incoherent, differing only in the combinations of beliefs and desires they hold towards the two propositions. It is clear, however, that there are more than sixteen different rational ways of behaving.

This is, of course, a very boring version of the argument; however you might think a similar argument can also be applied to show that propositions are more fine-grained than sets of worlds too. The basic point here is that the argument from attitude reports and the argument from decision theory are two distinct problems that both need addressing.

One can see how these considerations might also be relevant to the case of vagueness: almost all of our beliefs and desires, involving ‘medium sized dry goods’, are vague. Although I concede that one could in principle have a perfectly precise belief or desire, perhaps about some aspect of fundamental physics or mathematics, we would not be equipped to deal with ordinary day to day life if all our beliefs and desires were like this. I will not pursue the application of these ideas to vagueness here, however, as I will resume discussion of these issues in chapter 6 which is devoted to this topic.

Returning to our initial line of thought, it should be clear, I think, that a contextualist possible worlds theorist can at least in principle accommodate our intuitions about the two sentences beginning with ‘Alice ran because ...’ by postulating a change in context. The problem is not that the contextualist cannot explain particular utterances that purport to

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36 Modes of presentation might contribute to individuating the indices themselves. This might occur, say, on a view in which propositions are identified with ordered pairs of sets of worlds and modes of presentation – but it is important to remember that this is a view in which propositions are not sets of worlds and is thus importantly different from the contextualist we are considering.
explain actions in terms of beliefs and desires. The problem is the rather more theoretical one of integrating particular explanations of this sort into a general theory of rational action. To my knowledge, no theorist of this stripe has ever developed anything looking like a half decent decision theory involving modes of presentations that avoids CONTENT TO ACTION.\textsuperscript{37} To do this one would have to replace orthodox decision theory with a theory where the rationality of an action depends not only on the contents of your beliefs and desires, but on how you have them, or in other words, that depends on the modes of presentation under which you assign credences and utilities.

I think the technical obstacles to this project would be immense. Here a just few problems I can foresee, although I doubt these will exhaust the difficulties. Firstly, in order for credence to depend on modes of presentation one needs to relativise credences to modes of presentations. I can see what it might mean to have a credence of 0.6 in the necessary proposition via a mode of presentation corresponding to ‘Hesperus is Phosphorus’, but then I presumably won’t have a credence of any defined value in the proposition that snow is white relative to this mode of presentation. This makes it unclear how to even formulate probabilistic axioms for mode-of-presentation-relative-credences so understood. Secondly recall that the basic thesis of decision theory is that the expected value of a proposition \( p \) is the sum, weighted according to probabilities conditional on \( p \), of the utilities of a partition of epistemic possibilities. On the present view people only assign probabilities and utilities to epistemic possibilities relative to modes of presentation, and there is no canonical way of pairing epistemic possibilities with modes of presentations of the possibilities. There is therefore no reason to expect that one can assign a unique expected utility to \( p \) relative to a mode of presentation for \( p \) since one has a choice about how to pair the epistemic possibilities with modes of presentations. Even worse, I think, is that there is no guarantee that the probabilities defined relative to some pairing will sum to one, leaving it open whether the resulting notion of expected utility will conform to the ordinary constraints on rational preferences.

Until basic problems like these are solved it’s totally unclear how to go about formulating a decision theory that takes into account modes of presentations. The simplest way to avoid these troubles would be to treat propositions as ordered pairs of sets of worlds and modes of presentation, and to treat negation, disjunction and so on as operations on both coordinates of these things simultaneously. This is, of course, just a version of the fine graining strategy I have been arguing for – a view in which the objects of our attitudes are more fine grained than sets of worlds.

\textsuperscript{37}Note that you can treat the objects of thought as coarse grained proposition/modes of presentation pairs. This kind of theory has the ability to accept Ignorance and Knowledge simultaneously and is the kind of view discussed in the first half of this chapter. This section is aimed at theories that identify propositions (objects of thought/denotations of that-clauses) with the coarse grained propositions.)
Chapter 3

Vagueness and Modality

The notion of a precisification (or, sometimes, ‘sharpening’) of a language has been appealed to in many areas of philosophy\(^1\), however this formalism is most saliently associated with a certain semantical apparatus for dealing with vague languages: supervaluationism. Supervaluationist semantics is an extremely influential way to make precise the idea that vagueness consists in semantic indecision. A precisification of a vague language, according to this framework, is a way of making each word of that language completely precise. A precisification not only tells us what the extension of the predicate ‘bald’ is, and where the cut-off point is in a given sorites sequence, it also tells us what the extension would have been if those people had had different amounts of hair. They are things which, when given a possible world as input, tells us how to assign cut-off points (i.e. complete extensions) to each predicate (and words of other categories) at that world.

Of course, it is formally possible to assign the word ‘bald’ any extension we like at each world – there are therefore ways to make the word ‘bald’ precise that will include people who are clearly not bald. A precisification is admissible if it is furthermore compatible with the practices of those using the language.\(^2\) If the language is vague there is typically more than one admissible precisification – the determinate truths will be those that come out true relative to all admissible precisifications, whereas the borderline cases will be true on some but not all admissible precisifications. The laws of classical logic are retained because they are true relative to any admissible way of making the language completely precise.

Since the rise of supervaluational semantics, however, it has become clear that it is not only theorists who identify vagueness with semantic indecision that can make use of the formalism of precisifications. Epistemicists will appeal to the notion of an interpretation of a language which is not knowable (for distinctive reasons) by the speakers to be incorrect (see [131].) Inconsistency theorists will talk of ‘acceptable assignments of semantic values’: precise interpretations of the language that come ‘maximally close in satisfying the meaning-constitutive principles for the expressions involved’ (Eklund [36].) The formalism, in one guise or another, is completely ubiquitous amongst classical approaches to vagueness. Given an appropriate interpretation of ‘admissible’ and ‘precisification’, these theorists can all accept the structural claim that a proposition or sentence is borderline iff it is true relative to some but not all admissible precisifications. Many of the things I say about supervaluationism generalise to other approaches that accept this formalism.

Supervaluational semantics is not exclusive to linguistic theorists either. A non-linguistic theorist must adopt a relatively fine-grained theory of propositions: it must include vague as well as precise propositions. If they also accepted the ideology of possible worlds they could think of a precisification as a way of precisifying vague properties and propositions

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\(^1\)In the philosophy of science (Mehlberg [91]), empty names (van Fraassen [124]), the semantics of conditionals (Stalnaker [118]) and the liar paradox (Kripke [75]) to name a few.

\(^2\)The notion of a precisification being compatible with certain linguistic practices needs to be spelt out in much more detail, however specifics of the view need not concern us here.
in just the same way that a linguistic theorist would think of one as precisifying predicates and sentences. In short, a precisification on a non-linguistic conception would be a function telling us what the truth value of each proposition is at each possible world, telling us what the extension of each vague property is at each world, and so on. Thus general arguments against linguistic conceptions of vagueness, such as those presented in chapter 2, do not necessarily pose problems for the supervaluational way of modelling vagueness.

However we understand it, the formalism of supervaluationalism presupposes a clean division of facts into those which are grounded, in some sense, by the world, and those which are not: a precisification only tells you where the cutoff points are once you’ve told it what the world is like. In this chapter I shall raise a number of problems for this assumption, and I shall develop a different account, in which the ideology of possible worlds and precisifications is absent altogether. I shall argue that if we want to introduce entities playing something roughly analogous to the theoretical role that worlds play for the supervaluationist, there will be vagueness concerning which entities they are and what their structure is. This fact, I think, rules out any metaphysically inflationary way of understanding of what it means for a truth to be grounded by ‘the world’. Similar problems will arise for the notion of a precisification.

3.1 Supervaluationism

The supervaluationist formalism appears to assume a neat separation between the kind of factors that cause variation in truth value that depend on facts to be found in the world, and variation of truth value that is purely a matter of linguistic convention – differences in truth value that are merely a result of how we precisify our language. We postulate a theoretical entity, possible worlds, to represent the different ways the world could be – these settle things like heights and hair number. The other theoretical entities, precisifications, take a world as input and then settles what the extension of words like ‘bald’ and ‘tall’ would have been if the heights and hair numbers had been this way. If the world always settled who was bald there would be no need for precisifications.

Call this the ‘two ingredient’ picture – every truth gets determined by two independent ingredients: the way the world tells us things really are, and a way of settling the extensions of vague predicates. The second ingredient is to be thought of as being, in some sense, more arbitrary than the first.

What exactly does it mean to say that the truth of some propositions are settled by the world while others are not? On some conceptions of propositions this way of talking is misleading. For if what one means by a ‘proposition’ is a state of affairs, a physical chunk of reality, set of possible worlds, or something like that then strictly speaking, all propositions are the first-rate ‘worldly’ kind of proposition.3

However, even those who posit the existence of entities such as states of affairs which are metaphysically incapable of being vague have need of another kind of theoretical entity that is not so coarse-grained. We need entities fine grained enough to play the role of the objects of thought while accounting for vagueness related ignorance and other vagueness related attitudes (recall the discussion in section 2.2 of chapter 2.) Indeed it is natural to think there is a single entity that plays the role of the objects of attitudes, the meanings of sentences and the things that are necessary, true and determinate. Moreover, if sets of worlds, states of affairs (or what have you) cannot play this role then it is the former entities that are the more theoretically interesting and more deserving of the name ‘proposition’. To this end I shall use the word ‘proposition’ to denote whatever kind of entity plays the ‘proposition role’, as given by the principles listed below and others like them:

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3These theorists might retreat to the hypothesis that is is merely representations – sentences and the like – that are settled or unsettled, where all this really means is that they somehow fail to latch on to a determinate states of affairs.
The Proposition Role:

- One believes, (knows, asserts, desires, etc) that $P$ if and only if one believes (knows, asserts, desires, etc) the proposition that $P$.
- It’s necessary that $P$ if and only if the proposition that $P$ is necessary.
- It’s true that $P$ if and only if the proposition that $P$ is true.
- A sentence means that $P$ if and only if it means the proposition that $P$.
- It’s determinate that $P$ if and only if the proposition that $P$ is determinate.

It should be clear that coarse-grained entities like sets of worlds cannot play the proposition role, even when we limit ourselves to the relatively narrow considerations that arise in relation to vagueness. For example, assuming that whether you are bald supervenes on how many hairs you have it follows that for some $N$ the proposition that Harry is bald is necessarily equivalent to the proposition that Harry has at most $N$ hairs. But these propositions cannot be identical, since it is determinate whether Harry has at most $N$ hairs while it is not determinate whether he’s bald; applying the proposition role it follows that the proposition that Harry is bald is distinct from the proposition that Harry has at most $N$ hairs for they have different properties. Similar conclusions can be obtained without invoking the ideology of a determinacy operator by drawing on the issues surrounding vagueness related ignorance (see chapter 2 section 2.2.)

Once this point is realised it remains to say what kind of entities do play the proposition role. A natural thought would identify propositions with sets of ordered pairs consisting of worlds and admissible precisifications. This won’t quite do: because of higher order vagueness the determinacy operators do not iterate in a straightforward manner. This effectively corresponds to the idea that it’s vague which things are admissible precisifications. The result of this proposal would then be that it is vague which proposition exist. This cost on its own may be acceptable, however the proposal also entails that propositions are individuated by necessary determinate equivalence\textsuperscript{4}, and this is not fine grained enough to satisfy the proposition role. Possibilities that are necessarily determinately false, but not necessarily determinately determinately false can still play an important role in ones beliefs, desires and so forth. Thus, at a second parse, it’s natural to think the supervaluationist should endorse the following:

**Supervaluationist Propositions:** The entities playing the proposition role are (isomorphic to) sets of world precisification pairs.\textsuperscript{5}

Sets of world precisification pairs have just enough fine grainedness to play the determinacy aspect of the proposition role, and their relation to this role ensures that the supervaluationist formalism is not just an idle wheel.

3.1.1 The Relation between World Propositions and Precise Propositions

The basic insight behind supervaluationism is the idea that borderline cases lack a certain semantic status: they are neither supertrue nor superfalse. Something is supertrue if it is true relative to every admissible way of making the language precise, and superfalse if it is false relative to every admissible way of making the language precise. Despite the existence of these apparent semantic gaps, the laws of classical logic are upheld because every way of making a language precise removes the gaps by assigning every sentence a classical truth value, even if this is in some cases achieved arbitrarily. Thus a disjunction

\textsuperscript{4}Which, assuming the supervaluational semantics, is the same as determinate necessary equivalence.

\textsuperscript{5}We can think of this principle as partly giving us a handle on what precisification is, by its relation to the proposition role.
might be supertrue even if neither disjunct is – every precisification might make at least one disjunct true, but it might be a different one for different precisifications.

To demonstrate the semantics let us consider its application to a relatively simple language: the language of the propositional calculus with two unary connectives for expressing necessity, written \( \Box A \), and supertruth (or simply ‘determinacy’), written \( \Diamond A \). In this semantics sentences will be evaluated relative to a pair of objects, a world, \( (w, x, y, z) \) an a precisification (which I’ll refer to with \( u, v, \)). The set of worlds, we denote \( W \), and are related to one another by a reflexive accessibility relation, \( Rxy \), meaning that everything necessary at \( x \) is true at \( y \).\(^6\) Many philosophers employing this formalism assume that \( R \) is simply the universal relation that holds between every pair of worlds. Thus for most purposes this relation can be ignored.

In addition to \( W \) there is also a set of precisifications, \( V \), with a reflexive relation of relative admissibility, \( Suv \). Relative admissibility requires a little more explanation. A precisification is admissible, roughly, if it is an interpretation compatible with the way English is used at the actual world.\(^7\) Note that the concept of being admissible, as informally described above, is itself vague – there is vagueness, for example, concerning how English is used. Thus one can precisely the notion in various different ways. Perhaps according to one precification of ‘admissible’ \( v \) counts as admissible, and relative to another it doesn’t. This is the relative notion of admissibility – of one precisification being admissible according to another – and it is a bit like an accessibility relation in modal logic. For short we shall write \( Suv \) to mean that \( v \) is admissible relative to \( u \).

Sentences of our language are then evaluated at pairs of worlds and precisifications. The truth functional connectives are computed as one would expect: a disjunction is true at a pair iff one of the disjuncts is, a conjunction iff both conjuncts are, and so on. The crucial clauses are those for the necessity and determinacy operators:

\[
\begin{align*}
(\Box) & \quad x, v \vDash \Box A \text{ if and only if } y, v \vDash A \text{ for every } y \text{ such that } Rxy \\
(\Delta) & \quad x, v \vDash \Delta A \text{ if and only if } x, u \vDash A \text{ for every } u \text{ such that } Svu
\end{align*}
\]

\( \vDash \) here represents the relation of a sentence being true at a world precisification pair. Note that for necessity we keep the precisification fixed and check for variation with respect to accessible possible worlds. For determinacy we keep the world fixed and look for variation with respect to relative admissible precisifications.

Now that we have this formalism on the table, there is a question of how the new ideology of worlds and precisifications relates to the things we were trying to model – namely, vagueness and precision. The obvious, and most flatfooted interpretation is that the function of the entities occupying the first co-ordinate of an ordered pair is simply to determine the truth values of all and only the precise propositions, and the second co-ordinates function is to determine the truth values of the remaining vague propositions. When we think of the first and second co-ordinates as possible worlds and precisifications respectively we can spell this out as follows:

**The Naïve Interpretation:** Each possible world represents a (metaphysically possible) complete totality of precise facts, and each metaphysically possible complete totality of precise facts is represented by some possible world. Precisifications settle the remaining facts that would have been borderline had those precise propositions obtained.

\(^6\)There is a problem with this informal gloss which I think is sometimes not appreciated by supervaluationists: it is not clear what it means for something to be true at world simpliciter. The framework only seems to make sense of things being true relative to both a world and a precisification. One might similarly gloss the notion of relative admissibility (introduced below) as everything determinate at \( v \) is true at \( u \) – this suffers a similar problem.

\(^7\)It is important to note that this guarantees that it is not contingent which precisifications are admissible. The results would be disastrous if, for example, we counted a precisification as admissible at a world of evaluation \( w \) if it was compatible with the way English is used at \( w \).
By the totality of facts of a certain sort I mean the collection of all the facts of that sort, thus less informally a possible world is a collection of propositions which could have been the set of all the precise truths. Thus the naïve interpretation guarantees that, necessarily, there is a possible world representing the conjunction of all the precise truths, and for every possible world it’s possible that the conjunction of all the precise truths is represented by that world. The naïve interpretation is compatible with the idea that possible worlds could be eliminated altogether from ones vocabulary in favour of just talking about maximally strong precise propositions that are metaphysically possible. Analogous things can be said about precisifications.

The naïve interpretation seems to be implicit in the informal explanations supervaluationists give of the formalism. Unfortunately, as I shall now show, the naïve interpretation is simply not compatible with assumptions held by most supervaluationists: there are precise propositions whose truth values are not settled by the world proposition. Moreover, I shall here argue that if you accept the supervaluationist framework it is not possible to eliminate the ideology of possible worlds from your theorizing in favour of talk about propositions (as given by the proposition role) and the notion of precision and vagueness as it applies to these entities, either in the way suggested by the naïve interpretation, or any other way.

Let us start by expanding on the role the possible worlds do play in this formalism. The first thing to note is that any formalism that treats propositions as sets of ordered pairs will impose a partition within the space of propositions. In our case, each element of this partition corresponds intuitively to a possible world, and can be identified with the proposition associated with the set of ordered pairs that have that world as the first coordinate. Call these propositions the ‘world’ propositions. If a proposition is a disjunction of world propositions call it a ‘worldly’ proposition. Although this is a purely formal distinction within our model, worldly propositions are often interpreted by supervaluationists to be stating facts that in some sense, correspond directly to reality – that they are some sense metaphysically first-rate.

A completely analogous distinction can be drawn between propositions corresponding to the set of pairs that share the same precisification coordinate. One can call these precisification propositions, and arbitrary disjunctions of them can be called precisificational (note that these propositions will never be precise if there’s more than one precisification).

To take a very simple example, suppose that there are only four possible worlds. Then we can think of the space of propositions as divided into four quarters, as depicted by the four squares in either of the two diagrams in figure [REF], with each quarter representing the pairs that have a particular world as its first coordinate (ignore, for the time being, the grey lines and the circles.)

Anyone who takes the supervaluationist formalism at face value can introduce another operator, ‘it’s settled by the world that’, written $S A$, into our formalism which corresponds to being true throughout your local square. The basic idea is that $A$ is settled by the world at a world precisification pair iff $A$ is true there and $A$’s truth depends only on the world coordinate – i.e. it is true in a way that does not depend on the precisification. Formally we would add this to our language as follows:

($S$) $x, v \vDash S A$ if and only if $x, u \vDash A$ for every precisification $u$.

If we already have the ideology of ‘world’ propositions we can simply introduce $S A$ as stating that the true world proposition entails $A$.

Note the distinction between the $S$ operator and the determinacy operator: the latter only quantifies over accessible precisifications, whereas the former quantifies over all precisifications. This distinction is no accident; we need to restrict attention to accessible precisifications when we model the determinacy for otherwise we would not be able to account for higher order vagueness. Given $S$ you can say that $A$ is a worldly proposition internally in the object language with the sentence $\Box(SA \vee S \neg A)$; this formula is only satisfied at a pair if the set of pairs at which $A$ is true is a worldly proposition. In accordance
with the driving intuition, this account delivers the idea that borderline propositions are not settled by the world. The converse, however, is not true: borderline borderline propositions are also not settled by the world, yet a borderline borderline proposition might in fact be determinate (although whether it’s determinate or borderline would be unknowable to us).

It is natural to wonder whether someone can count as being a ‘supervaluationist’ without accepting the distinction between worldly and non-worldly propositions introduced above, or the coherence of the $S$ operator defined in terms of it. Someone could, in principle, adopt the supervaluationist model theory but reject these distinctions as being merely artefacts of the model. According to this view, the choice to evaluate formulae at pairs of entities, rather than at single unstructured entities, is incidental. It seems clear to me that anyone who takes this attitude towards the supervaluationist model theory has rejected the core insight of supervaluationism, which stresses the theoretical and heuristic importance of what I earlier called the ‘two ingredient picture’.

In addition to the worldly/non-worldly distinction between propositions, the supervaluationist also admits a distinction between propositions which are precise and propositions that are vague. For the supervaluationist this distinction is not taken as primitive, and must be defined in terms of the notions they do take as primitive, such as borderlineness and determinacy: the standard definition is that a proposition is precise if it is not possibly borderline. This can be represented in our object language with the formula $\Box(\Delta A \lor \Delta \neg A)$. (The notion of precision is related to the determinacy operator in the following way: a proposition is determinate if and only if it is entailed by a true precise proposition. This distinguishes precision from the notion of being worldly, $\Box(SA \lor S\neg A)$, for example, which doesn’t stand in this relation to the determinacy operator.)

What does the distinction between the precise and vague propositions look like in the present picture? Interestingly, unlike the worldly/non-worldly distinction, this is a distinction that depends on which precisification you are at. In the two diagrams in figure [REF] we have represented two equivalence classes of ordered pairs under the modal accessibility relation, represented by the grey lines connecting them together. At each pair in each equivalence class the worlds accessible to that pair via the determinacy accessibility relation is represented by the circle enclosing it. Note that the formalism guarantees that each circle must be completely included within a world proposition ($S$ accessible pairs always have the

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*I shall return to the adequacy of this definition in section [REF].
same world co-ordinate), and that exactly one circle in a given equivalence class under the modal accessibility relation will be contained in each world proposition.

As with the worldly propositions we can get a fix on the precise propositions by describing the maximally strong precise propositions. Let \( \langle w, v \rangle \) be one of the four pairs depicted in the first diagram in figure [REF]. Then a proposition is a maximally strong precise proposition, relative to \( \langle w, v \rangle \), if and only if it is either (a) one of the circles or (b) it is the singleton of any ordered pair not in one of the circles. An analogous characterisation can be made of the second diagram. A proposition is then precise, relative to a pair, if it's a disjunction of maximally strong precise propositions relative to that pair. It should be clear, then, that the precise propositions and the worldly propositions have very different structures, and we cannot simply understand worlds as corresponding to maximally strong precise propositions as the naïve interpretation would suggest.

Three things bear emphasising about these two distinctions. Firstly, as mentioned already, the distinction between the precise and vague propositions is itself a vague distinction, for as you can see the partition of logical space into maximally strong precise propositions depends on the precisification. Indeed it is quite simple to conjure up models at which \( \nabla^2 (\Delta A \lor \Delta \neg A) \) is satisfiable – a formalisation of the claim ‘it is borderline whether \( A \) is precise’. On the other hand the way we partition the space into world propositions in this model theory is always precise. In both our two diagrams the world partition is the same, and formally this is substantiated by the fact that \( \nabla^2 (SA \lor S \neg A) \) is not satisfiable in any model of the kind described – the claim that it is borderline whether \( A \) is worldly is false in all models.

The second point is that the two-fold distinction between worldly and precise seems otiose and extravagant. Clearly the philosophy of vagueness cannot dispense with the distinction between the precise and vague, but what work is the other distinction doing? One might try to do without both distinctions by trying to reduce one to the other. The most natural way to do this is to try to reduce the notion of a worldly proposition to a proposition that is precise\(^*$, where a proposition is precise\(^*$ if it is (i) precise, (ii) the proposition that it is precise is precise, (iii) the proposition that the proposition that it’s precise is precise is precise, and so on. In our picture, the precisely precise propositions – those that satisfy both (i) and (ii) – have a similar structure to the precise propositions, except the four circles will be a bit larger (although still within the confines of their local square): there will still be vagueness concerning which propositions are precisely precise, and the precisely precise propositions will not correspond to the worldly propositions. As you increase the number of ‘precisely’s the circles get bigger, but unless every precisification can be reached from every other precisification in a finite number of \( S \) transitions, we have no guarantee that the precise\(^*$ propositions will match up with the worldly propositions. (Indeed, in chapter 8 I argue that there are strong reasons to think that \( S \) is not a symmetric relation which is one way for this to fail to happen. More on this later.)

Someone less sympathetic to the metaphysical notion of a proposition being ‘settled by the world’, but generally impressed by the mathematical framework might be tempted to brush aside the notion of a world propositions as a formal artefact of the model. But as I mentioned already, this is tantamount to denying the importance of the distinction between world and precisification: without that distinction one might as well represent propositions as sets unstructured entities, rather than sets of ordered pairs, and not call oneself a ‘supervaluationist’ at all.

The last point worth highlighting is that the precise propositions have a very strange structure according to this picture. The maximally strong consistent precise propositions are just the circles and the remaining singletons, so that a proposition is precise at a given pair in the models pictured provided each of the four circles is either completely within or completely excluded by that proposition. This means, among other things, that the degenerate maximally strong precise propositions – the singletons – will settle lots of seemingly precise questions, such as the exact wealth at which people stop being rich.
and similar things. Indeed this strange structure is shared by any theory which takes the notion of being borderline as primitive, instead of the notion of a proposition being precise (see section [REF].) At any rate, since the supervaluationist formalism does not have the means to explicate the notion of a precise proposition, except by defining it in terms of borderlineness and necessity, it seems as though this structure is going to be essential to the supervaluationist picture.

3.2 Against Supervaluationism

Everyone theorizing about vagueness must be able to make sense of the distinction between vagueness and precision. My primary thesis in what follows is that this distinction is sufficient: the worldly/non-worldly distinction is not needed in addition to it. Naïvely, one could picture the resulting view by simply deleting the horizontal and vertical that divides the two squares in in figure [REF] into quarters. However I think there are also positive reasons to prefer the picture I am describing: the worldly/non-worldly distinction, in the supervaluationist incarnation, gives rise to a number of puzzles that can be avoided simply by rejecting it. I shall turn to these now.

3.2.1 What does it mean to be settled by the world?

The formalism we have just outlined commits us to a distinction between to two types of proposition which we have suggestively labeled with the word ‘worldly’. However, this is a purely formal distinction and is compatible with a wide variety of interpretations of that formalism. That said, the formalism appears to sit extremely naturally with the thesis that some sentences communicate thoughts or propositions that are, in some sense, metaphysically first-rate – grounds for whose truth can be found in the world (as represented by the world coordinate in a world precisification pair) – while others do not. Amongst the former first-rate truths are truths expressed by precise sentences such as facts about hair number, whereas amongst the latter are things expressed by vague sentences, such as truths concerning the baldness of borderline bald people. This interpretation of the formalism is especially prominent among supervaluationist interpretations of the formalism. For example, Keefe ([71] p153), says of vague predicates:

[...] there is nothing in our language, its use, or the world that determines particular locations for such hidden boundaries.

By contrast, one must assume, the location of the boundary of a precise predicate is straightforwardly determined by the world and our use of language. Thus while truths about hair number might correspond to concrete facts about the way physical objects are arranged, truths about the locations of vague boundaries do not. This way of talking is ubiquitous in the philosophy of vagueness, although some people prefer different expressions. Sometimes people talk about \( p \) corresponding to reality or to the world. Being factual, metaphysically settled, an objective fact, describing a state of affairs, being a thick proposition, saying something about the way things are, I take it, are all different ways of elucidating about the same distinction. (Sometimes people also use the word ‘true’ to make the distinction, and go on to claim that borderline propositions are neither true nor false. In what follows I’ll continue to use the word ‘true’ in a deflationary way – for example I will consider the proposition that Harry is bald to be true if Harry’s bald and false otherwise. I’ll reserve the technical word ‘supertrue’ for the non-deflationary use.)

The distinction is supposed to be a metaphysical one. By contrast, one might try to elucidate a difference between factual and non-factual propositions by looking at their role in thought – how the different kinds of propositions behave as the objects of belief, desire and knowledge. A theory that elucidates the difference purely in terms of how we think

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9See chapters 4-6, Field [44] and Schiffer [110].
with vague propositions would, I think, be at odds with the dominant understanding of the distinction. For example Barnett [8] writes that ‘on the dominant view of vagueness, if it is vague whether Harry is bald, then it is unsettled, not merely epistemically, but metaphysically, whether Harry is bald’ – it does not seem as though this metaphysical unsettledness could be explained purely in terms of how we think about things.

Although the world/precisification formalism is theoretically separable from this interpretation, they are mutually reinforcing. Anyone who accepts worldly/non-worldly ideology can reconstruct possible worlds as maximally strong consistent worldly propositions (and precisifications as maximally strong non-contingent propositions.) Conversely the pure formalism gives rise to a distinction between propositions that correspond to sets of worlds in the way described in the last section.

However, despite the pervasive use of this distinction among supervaluationists, it is actually quite contentious whether such a metaphysical distinction can be articulated in a satisfactory way (see, for example, Field [49], and Dreier [33].) A skeptic of the distinction might wonder, for example, why the proposition that Harry is bald doesn’t correspond to the world, even in cases where Harry is borderline bald. After all, this proposition is saying that Harry is bald, and since it is being granted that either Harry is bald or he isn’t, either this proposition corresponds to the way things are (if he’s bald) or it doesn’t (if he’s not). To deny this would be to accept that either Harry is bald, but not bald-according-to-the-world, or he’s not bald but not not-bald-according-to-the-world. This renders talk about ‘the world’ utterly mysterious for it allows things to be some way, without them being that way ‘according to the world’.

There are also analogous puzzles with existential quantification. For according to the semantics it can be a truth according to the world that there are bald people, even if there is nothing that is, according to the world, bald. Naively one would have thought that if the world grounds an existential truth it does so by grounding a truth about some object in the world. It should be clear, then, that whatever intuitive, pretheoretic grip we thought we had on the notion of objective ‘truth in the world’ it is not the one that is being employed by these theorists. Thus these theorists owe us an explanation of this notion.

A strategy I have occasionally seen employed for defending the distinction is to describe the distinction under a certain hypothesis about the nature of possible worlds in which the distinction seems to be particularly clear. Roughly, the thought goes, you must pretend that worlds are Lewisian concrete maximal fusions of spatio-temporally connected objects. Under this assumption, the story goes, the distinction seems to be absolutely clear: surely a Lewisian world settles some questions (for example, how many space-time points there are) but does not settle others (such as where the boundary between red and green lies.) The notion of truth according to the world, according to the thought, is exactly like truth-at-a-Lewis-world, except without the Lewis-world.

One might quite reasonably question the last step, in which one throws away the Lewisian metaphor. That is indeed a serious worry – however, even granting that step it is not clear to me that the Lewisian metaphysics makes the distinction any clearer in the first place. The thought, I take it, is that Lewis-worlds don’t merely represent Harry as being one way or the other, they literally are Harry and his surroundings, and they contain him and his properties as parts. Whereas it is quite simple to construct an ersatz world which represents Harry as being bald, whilst simultaneously representing him as lying in the borderline region for baldness, Lewisian worlds are not created so easily – it takes more than a stipulation to have a Lewisian world.

As attractive as this thought is, it is not clear that Lewisian worlds really enjoy this benefit. Note firstly that the distinction we are after, if it applies at all, applies to the relatively fine grained entities that satisfy the proposition role: things that are about as

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Although see Fine [53], Russell [107], Schroeder [111].

\[11\]

I have heard this defence of the distinction given in conversation.
fine-grained as sets of world-precisification pairs.

However it is far from clear what it means for a fine grained proposition of this type to be true at a Lewisian world. In addition to propositions about how many hairs Harry has, there is also the proposition that Harry is bald. Now clearly there is some difference between the former proposition and the latter – the truth of the former can simply be ‘read off’ the Lewisian world. All one has to do to determine its truth is inspect the part of this large hunk of concreta that constitutes Harry’s head. Whether Harry is bald, however, is not something we can simply check by inspection.

Unfortunately, while this distinction is relatively intuitive, it is quite clearly not the distinction we are looking for. The distinction between the truths that can simply be read off the the Lewisian world by looking is an epistemic one. The fact that we can’t tell whether Harry is bald just by inspection doesn’t mean the truth isn’t there in the world – after all, we are conceding that either Harry is bald or he isn’t.

One could instead proceed by introducing a primitive binary relation between Lewis worlds and fine-grained propositions – the relation of being ‘true-at’ – and theorize directly in terms of that. It is thus abundantly clear that the relation of being true-at-a-world, as applied to fine grained propositions, now assumes a heavy explanatory burden. Since the facts about what is true at a Lewisian world are not being simply read off the distribution of properties over space-time points, questions about which questions are settled by a Lewisian world are far from straightforward, and crucially, requires helping ourselves to the very distinction the Lewis picture was supposed to elucidate – the difference between propositions whose truth is settled by a world and those which aren’t. The idea that on the assumption of Lewis’s metaphysics there is a straightforward distinction is an illusion.

\subsection{Higher order vagueness}

The formalism of worlds and precisifications, and the metaphysical picture thinking that comes with it, seems to commit us to problems connected to the puzzles of higher order vagueness. The problem can arise in a couple of different ways. The first problem arises purely from the formal properties of the distinction between worldly and non-wordly propositions. According to the formalism a worldly proposition is one such that, for any two world-precisification pairs with the same world coordinate, the proposition either contains both pairs or neither. Due to the way determinacy is defined, propositions like this are always determinate relative to every pair in the model, so there can be no indeterminacy about what follows from the true world proposition. This leads to certain kinds of sharp cutoffs concerning what follows from the true world proposition. The second version of the problem relies on a metaphysical underpinning of the notion of a ‘world proposition’ in conjunction with the idea that there is no metaphysical vagueness. The loose idea is that if we are thinking of ‘truth according to the world’ as carving out a metaphysical joint, then borderline cases of the distinction would involve some form of vagueness in the world.

Let us start by just assuming the division between worldly and non-worldly propositions guaranteed by the supervaluationist semantics. For now we shall not assume that this division corresponds to a metaphysically substantive distinction. However, I shall assume that the theorist accepts some kind of distinction between propositions whose members agree about the world coordinate, and propositions that don’t (according to the intended model). Perhaps this isn’t a deep metaphysical distinction, but it is not merely an artefact of the model either. (If the world/precisification division were merely an artefact of the model, then one could dispense with ordered pairs altogether and just work with unstructured indices instead; little would be left of the supervaluationist insight.)

The distinction between worldly and non-worldly propositions can be introduced into the object language with an operator, $S_A$, which holds at a world precisification pair if $A$ is true at any pair that agrees with the first pair about the world coordinate. One might be tempted to read this operator as saying ‘it is settled by the world that $A$’, although the
formalism alone does not force this reading. Both $\mathcal{S}A \rightarrow \Delta \mathcal{S}A$ and $\neg \mathcal{S}A \rightarrow \Delta \neg \mathcal{S}A$ will be true at any world precisification pair in any model of the kind described in section [REF]. In other words it is never borderline whether or not this operator applies or not.

The problem from higher order vagueness is this. If you accept the world precisification formalism you are committed to admitting that this operator is coherent. Whether something is settled by the world coordinate, as defined above, is always a sharp matter at that world relative to any precisification. But the problem is that facts about what the world settles seem to be just as soritesable as any other facts. For example, consider a sorites sequence, $a_1, a_2, a_3, \ldots$ for baldness starting with people with very little hair and ending with people with lots of hair. A sorites paradox can then be run as follows:

1. It’s settled that $a_1$ is bald.
2. If it’s settled that $a_n$ is bald, it’s settled that $a_{n+1}$ is bald. (For $1 \leq n < 1,000,000$.)
3. So, it’s settled that $a_{1,000,000}$ is bald.

As with the ordinary sorites we can either reject the first premise, the second premise-schema, or accept the conclusion. The conclusion, of course, entails that $a_{1,000,000}$ is bald, which is a version of universalism already considered and rejected in chapter 1, so I take it our responses are limited to rejecting 1 and 2.

If we reject 2 then it follows that not only will there be a last person in a sorites such that the world settles that they are bald, as required by classical logic, but it will be a completely precise matter which person that is. This is because of the observation, noted above, that it is never borderline what is settled. This is therefore much worse than the analogous move of rejecting the corresponding premise of the ordinary sorites paradox for baldness. There one accepted the existence of a last bald person, but could maintain that it was borderline, and thus unknowable, which person that was. The conclusion that it is not borderline which the last settled-to-be bald person is is surely paradoxical, for without vagueness what barrier is there to finding out where that boundary is? (I take it for granted that we do not know where this boundary lies. A challenge for anyone who thought otherwise would be say where the boundary is in a concrete sorites sequence.) The assumption that one can delineate propositions into those that are settled by the world and those that are not leads straightforwardly to this paradox.

The remaining option, then, is to reject 1. This could be achieved by allowing very little to remain fixed throughout each world proposition. Perhaps at one world precisification pair it can be true that Harry has no hairs and is bald, but by varying the precisification coordinate while keeping the world fixed, one can eventually get to a pair at which Harry is not bald.

This response, of course, quickly raises the question of which kinds of facts do remain fixed throughout a world proposition. Thinking about our example, one might expect facts about hair number to remain fixed while the facts about baldness vary. However this thought doesn’t stand up to scrutiny: the property of being a hair is just as soritesable as the property of being bald, so hair number should also vary through out each world proposition. Perhaps one could maintain that the facts that remain constant throughout a world proposition are just those facts corresponding to the basic and fundamental aspects of physics: perhaps, for instance, propositions stating how many electrons there are should remain fixed throughout each world proposition. However, it is not even clear that we should draw the line at fundamental physics. For if we really want sets of world precisification pairs to play the proposition role, and in particular the role it plays with respect to propositional attitudes, one ought to take seriously people who are ignorant about the fundamental status of electrons. Throughout history we have often been wrong about which entities are the smallest building blocks – perhaps even electrons are made up of clouds of millions of smaller entities. Even if it turns out to be necessary that electrons aren’t clouds of smaller entities, we should surely be able to accommodate uncertainty about the matter.
Notice, however, that conditional on the hypothesis that electrons are cloudlike, one could have that same distinctive kind of uncertainty towards propositions stating how many electrons there are as one could have towards propositions stating how many real clouds there are, when you know it’s borderline how many clouds there are. (In general, it seems that a proposition, \( p \), is vague if one can find another proposition, \( q \), such that conditional on \( q \), \( p \) has the epistemic profile distinctive of borderline propositions.)

Once we have gone this far it becomes completely unclear what role worlds are playing in our formalism: all contingency can be represented by variation of the precisification coordinate, and the need for more than one world to model modality is no longer present. Moreover, the model theory would have lots of unexplained redundancy. One would be able to find pairs belonging to distinct world propositions that agree about the truth of every sentence; the lines between world propositions no longer seem to cast the divisions we imagined them to. People in the Lewisian mindset, or anyone who already has a metaphysically heavy duty account of worlds in play, will no doubt find this option particularly unattractive for there will be worlds that clearly contain people – for Lewis, particular arrangements of concrete entities looking just like you and me – but which when paired with certain precisifications make the proposition that there are people false. These consequences also come with puzzles of their own: if two distinct metaphysical possibilities can be represented within the same world proposition by varying the precisification coordinate, then we get a failure of things to supervene on the worldly facts – something one intuitively would have thought was essential to any way of giving the worldly/non-worldly distinction an intuitive interpretation.

The puzzle, as I’ve presented it above, relies essentially on the world precisification formalism – the definition of a ‘world’ proposition, and the corresponding operator, would not make sense if the relevant points of evaluation were not ordered pairs of some kind. It is crucial to realise that this particular puzzle can be avoided by adopting an alternative framework that doesn’t theorise in these two dimensional terms. Of course, there may be other paradoxes of higher order vagueness that need addressing, however paradoxes involving the \( S \) operator simply do not arise unless one accepts the supervaluationist formalism.

Let us now turn to the metaphysical interpretation of the worldly/non-worldly distinction. For the theorist who adopts a more inflationary metaphysics of worlds this sort of vagueness over what the world settles, if it could exist, would be particularly worrisome. To accept the present idea that the partition of logical space corresponding to worlds could be vague – i.e. to allow that the boundaries delineating one world from another are not sharp – amounts to permitting vagueness concerning a basic metaphysical kind. Perhaps one could dispense with this particular formalism involving worlds and precisifications and talk of maximal consistent precise propositions as I have suggested. The vagueness concerning where this division lies, however, seems to rule out any metaphysically substantial understanding of this division.

This issue of higher order vagueness therefore generalises to a problem that goes beyond the formalism of worlds and precisifications. Anyone who accepts the ideology of the world ‘settling’ certain questions, however they spell it out, will run into similar worries. Those who theorize in this way tend to grant that the world settles that zero-haired people are bald, so by classical logic it follows that there’s a last man, in a sorites sequence for baldness, whose baldness is settled by the world. Now either it’s vague which guy this is or it isn’t. If it isn’t vague then we run into the paradox discussed above: there is a sharp boundary between being a man whose baldness is settled by the world and a man whose baldness isn’t – a boundary that is settled by the world and that is in principle discoverable due to the absence of vagueness (or any other barrier to knowledge.)

On the other hand, if it is vague at which point the world stops settling whether the members of the sequence are bald – a possibility not available in the supervaluationist semantics – we have another puzzle. Many philosophers share the intuition that there is no such thing as vagueness in the world. Now, it is not entirely clear to me what phenomenon
the expression ‘vagueness in the world’ is supposed to pick out, but the idea that it can be vague which questions the world settles seems to fall into the same category of worries that these philosophers are concerned about. For it to be vague whether the world settles $p$, on a non-metaphysical linguistic conception of vagueness, for example, is for the way we use language to leave it open whether the world settles $p$ or not. But, setting aside extreme anti-realist positions, the way we use language doesn’t determine what is and isn’t settled by the world. Vagueness about what the world settles seems to commit us to a puzzling kind of metaphysical vagueness.

### 3.2.3 The Interaction of Vagueness and Modality

The supervaluational semantics has some interesting consequences for the logic of the interaction of the determinacy and necessity operators. For example in all supervaluational models the following principles are validated:

- **ND**: $\square \Delta \phi \rightarrow \Delta \square \phi$
- **DN**: $\Delta \square \phi \rightarrow \square \Delta \phi$
- **PD**: $\Diamond \Delta \phi \rightarrow \Delta \Diamond \phi$

The validity of each can be easily seen after reflecting a little on the truth clauses for $\square$ and $\Delta$ outlined in section [REF]. It follows by some standard results that the logic generated by the above three principles, plus $S5$ for necessity and $KT$ for determinacy is complete with respect to the class of models described where $S$ is additionally reflexive and $R$ an equivalence relation. (The use of $S5$ and $KT$ for the logics of necessity and vagueness respectively are not required for this completeness argument. Analogous results can be formulated for any pair of the following logics: $KT$, $KTB$, $S4$, or $S5$.)

The last principle $PD$, which I shall refer to as the ‘Church-Rosser principle’, states that possibility commutes with determinacy (on one direction only) and appears to exemplify an asymmetry between the two modalities. This asymmetry is illusory – it is easy to verify that its dual, $\neg \Delta \neg \square p \rightarrow \square \neg \Delta \neg p$, is in fact a consequence of $PD$ by contraposition. What is not a consequence of $PD$, however, is its converse: $\Diamond \Delta \phi \rightarrow \Delta \Diamond \phi$. Indeed the converse is demonstrably false. Suppose there is a bag of 1000 balls of different sizes, ranging from balls that are clearly small to balls which are clearly not small, and with unclear cases as well. Since it is possible for me to pick any particular ball from the bag, it follows that, determinately, for any ball $b$ it’s possible that I have picked $b$ from the bag. Since some ball is the largest small ball it follows that, determinately, it’s possible that I’ve picked the largest small ball. However it is not possible that I’ve determinately picked the largest small ball. To do so I would have to be determinately holding the largest small ball, and since it is indeterminate which ball that is, it would have to be indeterminate which ball I am hold – this is not, I shall assume, a metaphysical possibility.

The interaction between possibility and determinacy is formally analogous to the interaction between the existential quantifier and the universal quantifier: if someone is loved by everyone then everyone loves someone, yet the converse – that if everyone loves someone, someone is loved by everyone – is simply not true.

If we add these principles to a logic, $L_1$, governing $\square$ and another logic, $L_2$, governing $\Delta$ (in our case, $S5$ and $KT$) the result is called the product of $L_1$ and $L_2$. Whether the supervaluationist semantics is adequate will therefore depend on the truth of the theorems of the product logic. However the validity of the product logic is far from obvious.

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12See Gabbay and Shehtman [55], and Kurucz [77]. In Gabbay and Shehtman a class of logics, ‘PTC logics’, are defined for which analogous completeness results can be formulated for their combination.

13Note also that the dual of the converse of $PD$ entails the converse of the final axiom (by contraposition) so the dual of the converse of the final axiom is also false for the same reason.
An important reason for skepticism is that modal operators like $\Box$ and $\Diamond$ themselves appear to be a source of vagueness; a thesis that I think has a great degree of plausibility independently of the plausibility of any putative example. Formulating this idea requires a little care, however. It would not do to simply say that necessity has borderline cases – that for some proposition, $p$, it’s borderline whether $p$ is necessary. If the proposition that it’s necessary that $p$ is a borderline proposition then that’s either due to vagueness in the necessity operator or the proposition $p$. However if necessity had a precise proposition as a borderline case then we would know that necessity is the source of the vagueness:

**Vagueness of Modality:** For some precise proposition $p$, it’s borderline whether $p$ is necessary.

If it’s borderline whether $p$ is necessary, where $p$ is precise, then the source of the vagueness must be the necessity operator. The idea that ‘necessary’ is vague is plausible given both the sparsity of precise words in general, and the plethora of sorites puzzles that arise from ‘Ship of Theseus’ type puzzles (I’ll turn to these in a second).

It is worth noting, then, that Vagueness of Modality cannot be maintained in conjunction with the product logic; specifically, it is not consistent with the principles $\text{ND}$ and $\text{PD}$. For if $p$ is precise then $p$ couldn’t have been borderline, so we know both:

$$\begin{align*}
P1 & \quad \Box (p \rightarrow \Delta p) \\
P2 & \quad \Box (\neg p \rightarrow \Delta \neg p)
\end{align*}$$

Furthermore, $\nabla \Box p$ entails, given excluded middle:

$$\begin{align*}
P3 & \quad (\Box p \land \neg \Delta \Box p) \lor (\neg \Box p \land \neg \Delta \neg p)
\end{align*}$$

But these three principles are inconsistent in the product logic. If the first disjunct of P3 is true, $\Box p \land \neg \Delta \Box p$, then we get a contradiction from P1 and $\text{ND}$: P1 entails $\Box p \rightarrow \Box \Delta p$, which given the first conjunct allows us to infer $\Box \Delta p$. But by $\text{ND}$ this entails $\Delta \Box p$ which contradicts the second conjunct. If the second disjunct is true we get a contradiction from P2 and $\text{PD}$: $\neg \Box p$ entails $\Box \neg p$, which along with P2, entails $\Diamond \neg p$. Assuming $\text{PD}$ this entails $\Delta \Diamond \neg p$ which contradicts $\neg \Delta \neg p$.

Are there any plausible witnesses to Vagueness of Modality? Are there any precise propositions whose necessity is borderline? There are in fact a class of puzzles that arise in the context of moderate forms of mereological essentialism that appear to be a good source of examples. Suppose, for example, that we have a chain necklace, called ‘chainy’, which was constructed by taking 100 links out of a box of links and putting them together. Now according to a moderate form of essentialism (supported by some fairly robust intuitions), it seems as though chainy could have survived the loss of a single link. For example, if chainy broke it would be possible to repair it by replacing one of the links. On the other hand, if we replaced all the links constituting chainy with new links we would have another chain.

Let $X$ be the set of links that chainy is in fact made out of. Note that the above observations provide the ingredients for a sorites sequence. We can formulate the claim that the chain could have survived 1 replacement but not 100 replacements as follows.

(i) $\Box \exists x (x \in X \land x \leq c)$

(ii) $\neg \Box 100 \exists x (x \in X \land x \leq c)$

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\[14\] $\text{DN}$, as far as I can see, is left intact by these considerations.

\[15\] The above argument looks on the surface as though it uses reasoning by cases, however it is possible to formulate the argument so that the only rule appealed to is modus ponens applied to the theorems of $\text{K}$ for determinacy and for necessity respectively.
Here $\exists_n xFx$ means ‘there are at least $n$ $F$’s’ – this is quantifier that can be defined in purely logical vocabulary. $\leq$ represents mereological parthood and $\in$ set membership. (i) states that necessarily at least one member of $X$ is a part of chainy and (ii) says that it is not necessary that every member of $X$ be a part of chainy. 

By classical logic there is an $n$ such that $\Box \exists_n x(x \in X \land x \leq c)$ and $\neg \Box \exists_{n+1} x(x \in X \land x \leq c)$. That is to say: it is necessary that at least $n$ members of $X$ are parts of chainy, although it is not necessary that at least $n + 1$ members of $X$ be parts of chainy. Since this is a paradigm sorites argument, the truths at the boundary will not be determinate truths. This much should, I hope be uncontroversial provided one accepts the moderately essentialist metaphysics implicit in the setup. The next step is to note that $\exists_n x(x \in X \land x \leq c)$ appears to be stated in purely precise vocabulary: it can be formulated only using identity, (unrestricted) existential quantification, set membership, and mereological parthood, all of which are plausibly precise, and two names: $X$ introduced as a name for a particular set, and $c$, a name for a particular object. Since the proposition expressed by this sentence is precise we know that it couldn’t be borderline, and thus we have true instances of P1, P2 and P3. Thus we have a precise proposition that is a borderline case of being necessary.

There are several plausible ways to resist the argument. One might, for example, deny that we succeeded in introducing a precise name for the chain in our example (a similar objection could be leveled at our name for the set of links). This response does not get to the heart of the issue: the premises P1-P3 are just as plausible if we existentially quantify into the position that the name $c$ takes. The existential versions of these premises then rest on the following thought: that there’s at least one chain such that it is borderline whether it could have had less $n$ members of $X$ as parts. The contradiction with ND proceeds just as before, except in the scope of an existential quantifier.

There are, of course, other ways of resisting the argument: one could deny the essentialist metaphysics that we need to get the sorites going, or one could deny that parthood is always precise. The principle that parthood is precise, for example, has a venerable status within philosophy\textsuperscript{17}, however it is not beyond reproach and I myself am sympathetic to vagueness in parthood. It is important not to let the particular example I am using distract from the generality of the problem.

It is, of course, possible to modify the supervaluationist semantics in such a way as to invalidate these inferences. One could, for example, use ternary or quarternary accessibility relations over both worlds and precisifications (I discuss a proposal like this in section [REF]). However these modifications typically weaken the role that the world and precisification distinction normally take. In particular, they typically prevent one from reconstructing the world and precisification propositions from the structure of the accessibility relations (either at all or at least not uniquely) in the way that you can in the simplest version of supervaluational semantics. The result brings us much closer to the view I am urging for: a view in which worlds and precisifications don’t carve out real distinctions.

### 3.2.4 Vagueness and Borderlineness

It is common to classify propositions and properties as being vague or precise.\textsuperscript{18} The distinction between a vague proposition or property and a precise one is importantly different from the distinction between a proposition being borderline and its being definitely true or false, or the distinction between a property having a borderline case and having no borderline cases. To illustrate the difference, consider the property of being a blue swan. This property is vague, not precise – one can imagine a sorites sequence of swans, beginning with a clearly green swan and ending with a clearly blue swan, without much trouble. However the property of being a being a blue swan has no borderline cases because, as it turns out,\textsuperscript{16}

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\textsuperscript{16}$\exists_n xFx$ may simply be defined as $\exists x Fx$. $\exists_{n+1} xFx$ may be defined as $\exists (Fx \land \exists y(y \neq x \land Fy))$

\textsuperscript{17}See, the literature on Lewis’s argument from vagueness, for example [REF].

\textsuperscript{18}Note that some theorists prefer to attribute this distinction instead to sentences and predicates – I will leave it to the reader to mentally make the appropriate substitutions where necessary.
it is a determinate fact that there are no blue swans. A completely analogous point can be made about propositions. The proposition that I am bald and the proposition that I have less than 20 hairs differ in a theoretically important way— one is a vague proposition, the other is precise. However since I am, as of writing, definitely not bald neither proposition is borderline.

It is natural to think that the propositional notion of vagueness and the notion of a property being vague are connected as follows: the property of being an $F$ is vague if and only if for some $x$ the proposition that $x$ is $F$ is vague.\(^{19}\) It is not so clear how the propositional notion of vagueness (and thus the related property notion) relate to the notion of being a borderline proposition (or a property with borderline cases.)

It is clear that these two types of properties that propositions can have— borderlineness vs. vagueness— are related. Indeed it is very natural to think that one type can be reduced to the other: that vagueness and precision can be defined in terms of the notion of being borderline (this is the standard line), or, as I will argue, that the distinction between borderline propositions can be reduced to the vague/precise distinction. But at any rate, given that we have this distinction between vague and precise propositions it is perfectly acceptable to theorise in terms of it whilst remaining neutral on the direction of reduction.

The first thing to point out in this regard is that that the distinction between precise and vague propositions is not itself a precise one. To see this it is sufficient to note that one can construct a sorites sequence of propositions starting with propositions that are clearly precise and gradually changing until we have propositions that are clearly not precise. One such sorites consists of the sequence of conjunctive propositions that includes, for each natural number $n$, the proposition that Harry is bald and has $n$ hairs. Note that the first proposition (when $n = 0$) is precise: it is plausibly a conceptual truth that someone with no hairs is bald, so the proposition that Harry has 0 hairs entails that Harry is bald, and thus the proposition that Harry has 0 hairs and is bald is equivalent to the proposition that Harry has 0 hairs in the sense that they correspond to the same set of world-precisification pairs. The latter proposition is precise and thus, given the sense of equivalence is sufficiently demanding, it follows that the former proposition is precise too. Yet as $n$ increases we eventually arrive at propositions that are vague. This is closely related to the phenomenon of higher order vagueness, which we shall return to in chapter 8.

Given only what we have said so far we can lay out a number of formal conditions that govern the behaviour of precise propositions. For example, it is obvious that the tautologous proposition is precise. Furthermore it is clear that if a proposition is precise, then so is its negation, and similarly that arbitrary conjunctions and disjunctions of precise propositions are also precise. In mathematical terms this just means that the precise propositions form a complete Boolean algebra.\(^{20}\) Our first constraint is thus:

**Boolean:** The set of precise propositions forms a complete Boolean algebra, with the tautologous proposition being the weakest precise proposition.

Another important constraint we will include is that this is an atomic Boolean algebra— that every precise proposition is a disjunction of maximally strong consistent precise propositions, which is equivalent (given the axiom of choice) to stipulating that:

**Atomicity:** Given any jointly consistent set of precise propositions, $X$, there is always a consistent precise proposition that entails each member of $X$.

This guarantees, for example, the platitude that the conjunction of all the precise truths is

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\(^{19}\) Conversely, every proposition corresponds to a vacuous property (for example, the proposition that grass is green correspond to the property of being an $x$ such that grass is green). Thus we can define what it means for a proposition to be vague in terms of a notion of property vagueness: a proposition is vague iff its corresponding property is.

\(^{20}\) Assuming the operations of negation, conjunction, and so forth, are themselves the familiar Boolean operations. This could fail with structured propositions, for example.
Next, we must say how the vague/precise distinction relates to the determinate/borderline distinction:

**Precision to Determinacy:** Every precise truth is a determinate truth.

**Determinacy to Precision:** Every determinate truth is entailed by a precise truth.

The first principle is fairly straightforward; it is equivalent, for example, to the principle that borderline propositions are vague. The converse, of course, is not true – there are determinately true propositions that are nonetheless vague. However, every determinate truth should be grounded in some precise fact: there couldn’t be a determinate proposition stronger than every true precise proposition. (Note that there is a terminological confusion that should be avoided when reading the second principle: some people reserve the word ‘precise’ for propositions that are precise at all orders in my sense, and there can be determinate truths that are not entailed by any truth that’s precise at all orders.)

Finally, we should say how precision interacts with necessity. Here the following seems central:

**Necessity of the Precise:** Precise propositions are necessarily precise.

Along with the above principles this allows us to prove that precise propositions couldn’t have been borderline, or equivalently, are necessarily either determinately true or determinately false: \( \Box(\Delta A \lor \Delta \neg A) \). Note that the above principles do not entail the converse claim: that necessarily determinately true or false propositions are precise.

At any rate, with these preliminaries aside, it is clear that we are in need of an account of the distinction between vague and precise propositions. It is somewhat striking however that most philosophers – including my primary targets, the supervaluationists – choose instead to theorise with the notion of a proposition being borderline, or equivalently (modulo definitions), with the notion of a proposition being determinately true.

More worryingly, these philosophers typically provide an account of what it means for a proposition or sentence to be a borderline truth, but do not explicitly give an account of what it means for a proposition or sentence to be vague or precise. Luckily there is a natural modal way to characterise this distinction which is widely adopted amongst supervaluationists. For example, according to Fine [51] ‘a predicate \( F \) is extensionally vague if it has borderline cases, intentionally vague if it could have borderline cases. Thus ‘bald’ is extensionally vague, I presume, and remains intentionally vague in a world of hairy or hairless men.’ There is a corresponding characterisation of propositional vagueness: the difference between the proposition that I am bald and the proposition that I have 0 hairs consists in the fact that, even though both are determinately false, the former proposition could have been borderline whereas the latter could not. Call this the modal characterisation of precision.

A proposition is vague if and only if it could have been borderline (and is precise otherwise.)

A property is vague if and only if it could have had a borderline case (and is precise otherwise.)

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21 Atomicity is stronger than this however: it additionally guarantees that it’s not merely a truth, but an \( L \) truth that the conjunction of all precise truths is consistent.

22 Although every proposition that’s determinate at all orders is entailed by some proposition that’s precise at all orders. Related to this strong understanding of ‘precise’ is the notion of being a worldly proposition; worldly propositions are also not related to the determinacy operator in the way required by **Determinacy to Precision**. Williamson [132], for example, uses ‘precise’ in a way that corresponds roughly to the notion of ‘worldly’ we introduced earlier.

23 Given that \( \Delta \) satisfies the modal logic \( K \) we can prove that the definition of \( \nabla p \) as \( \neg \Delta p \land \neg \Delta \neg p \) works. Conversely, we can take \( \nabla \) as primitive (see Pelletier [94].)
Is this characterisation adequate? The first order of business would be to show that our above formal constraints are satisfied. In other words, we must show that the tautologous proposition is precise according to this definition, that negations and arbitrary disjunctions and conjunctions of precise propositions are precise, and that every consistent set of precise propositions is entailed by a single consistent precise proposition. It is in fact straightforward to show that these conditions are satisfied by the above definition in the model theory outlined in section [REF]. (Indeed, the fact that negation and finitary disjunction and conjunction preserve precision is provable in the combined logic of vagueness and modality in a way that does not rest on the more contentious principles belonging to the product logic.) Note also that precise truths are always determinate, and every determinate truth is entailed by the conjunction of all determinate truths, and the latter can be seen to be necessarily determinately true or determinately false. Finally, given S4 for necessity, we can see precise truths are necessarily precise according to our definition. So this definitions also meets the other deciderata we listed.

What does the distinction look like in the supervaluationist framework? As I noted earlier in section [REF], due to higher order vagueness about what is precise, the exact structure of the precise propositions will depend on the precisification. However, relative to a given precisification-world pair, \(\langle w, v \rangle\), the picture we got was the following: most of the maximally strong precise propositions are degenerate – singletons of world precisification pairs. The maximally strong precise propositions that aren’t degenerate (the propositions corresponding to the circles in figure [REF]) will all contain pairs that are modally accessible to \(\langle w, v \rangle\).

The fact that the precise propositions have this structure suggests that there is something suspect about this definition. The degenerate maximally strong precise propositions don’t seem to be precise at all: the proposition represented by a singleton of a world-precisification pair will, for example, entail where the cut-off points for all the vague predicates lie. Intuitively no precise proposition should entail things like ‘the cutoff for baldness is 2049 hairs’.

More suspicion is cast on the definition by the observation that we cannot adequately capture the distinctive doxastic features that differentiate vague propositions from precise propositions if we adopt that account of precision. For example, I suggest that the proposition expressed by both the following sentences are vague due to their distinctive doxastic features:

1. Harry is bald.
2. Either Jocasta is the mother of Oedipus or Harry is bald.

Both propositions have the following feature that is quite distinctive to vagueness: in both cases there is some precise proposition, \(p\), such that one is rationally required to be uncertain in the vague proposition conditional on \(p\) (provided there is such a \(p\) consistent with your evidence.) In the former case, for example, one should be uncertain whether Harry is bald conditional on the precise proposition that he has \(N\) hairs, whenever \(N\) belongs to a certain class of borderline cases.

Consider now Oedipus, who believes that Jocasta is not his mother. Suppose Oedipus also knows that Harry is in the borderline region for being bald: for Oedipus both 3.1 and 3.2 are on an epistemic par. He should be uncertain about both propositions in that distinctive kind of way characteristic of uncertainty about borderline matters. What explains the presence of this distinctive kind of attitude in this case? It seems completely obvious that the explanation for the presence of this distinctive kind of uncertainty ought to be the same in both cases, and that the explanation has something to do with both propositions being vague.

Yet note that this conflicts with the modal characterisation of the distinction between the vague and precise. The proposition that Jocasta is Oedipus’s mother or Harry is bald could not have been borderline since it is an *a posteriori* necessary truth that Jocasta is
determinately Oedipus’s mother.\footnote{Certain philosophers may wish to modify the example by substituting the first disjunct for the proposition that Jocasta is Oedipus’s mother if they both exist (although the restriction may not be necessary: see Fine [REF] [necessity and non-existence] and Bacon [5].) If you are not happy with this example other substitutions of a posteriori necessities would do equally well.} In short both 3.1 and 3.2 have features characteristic of vague propositions, yet only 3.1 counts as vague according to the modal characterisation.

This argument rested on a quite general principle, namely that if there’s a (consistent) hypothesis, such that conditional on that hypothesis we should have that distinctive uncertainty about $p$ characteristic of borderline cases, then $p$ is a vague proposition. We are not in a position yet to give an adequate account of what that distinctive kind of uncertainty is. However, we can avoid all mention of this kind of uncertainty by explicitly using the borderlineness operator:

\[ \text{Credence to Vagueness: If there’s some consistent hypothesis, } h, \text{ such that } C_r(\nabla p \mid h) = 1 \text{ for every rational credence function } C_r, \text{ then } p \text{ is vague.} \]

This provides us with another important connection between borderlineness and vagueness.

Thus, for example, there are certain hair numbers such that it is a conceptual truth that people with that amount of hair are borderline bald: according to every rational credence function it’s certain that Harry (say) is borderline bald conditional on his having that many hairs. The same goes for the proposition that either Jocasta is Oedipus’s mother or Harry is bald: conditional on the hypothesis that Jocasta isn’t Oedipus’s mother and that Harry has $N$ hairs we should be certain that that proposition is borderline. The hypothesis in question is surely consistent according to any reasonably fine grained account of propositions. There are, of course, very coarse grained theories of propositions that predict that one cannot be uncertain about who Oedipus’s mother is. However I take it that most people accept enough fine grainedness to make sense of this uncertainty, and these theorists will be in a position to conclude that 3.2 is vague from \text{Credence to Vagueness}.

It should noted that \text{Credence to Vagueness} has some important consequences regarding what we normally consider to be vague: it plausibly entails that the sentences of fundamental physics do not express precise propositions. For example, the role that electronhood plays in thought today is a bit like the role that earth played amongst the ancient Greeks who believed it to be one of the five fundamental elements. We now know that earth is not a fundamental element, and that there can even be things that are borderline cases of being earth. Similarly, even though we presently have good reason to think that electrons are fundamental simple objects, we could consider what things are like conditional on the proposition that our present physics is incorrect and that electrons are actually little clouds of smaller particles. Perhaps, much like an actual cloud, it is also vague where their boundaries lie.

Note, then, that there are physical hypotheses about the world such that conditional on them the proposition that there are electrons has the epistemic profile distinctive to borderline cases; the profile we will be examining later in this book. For example, suppose that $\alpha$ is precise description of a possible arrangement of particles in which it would be borderline whether that arrangement of particles constitute an electron-cloud. Now consider the physical hypothesis that electrons are certain clouds of smaller particles and that there is only one cluster of such particles in the universe and that these particles are arranged in arrangement $\alpha$. Conditional on this proposition we should be uncertain in that distinctive way about whether there are electrons, and indeed we should be certain, conditional on this proposition, that it is borderline whether there are electrons. It follows, by \text{Credence to Vagueness}, that the proposition that there are electrons is a vague proposition, even if it
happens to be necessarily determinately true or determinately false. At any rate, the failure of the modal definition to capture precision, in my opinion, should make one doubtful that the notion can be defined from the determinacy operator at all. If I am right about this, then there is a serious lacuna in existing approaches to vagueness that focus only on analysing the notion of a proposition or sentence being borderline or determinately true. Indeed, the problem I am raising here I think generalises to any theory that takes only the notion of determinacy as primitive. A better theory would take the notion of a proposition being precise as primitive for logical purposes, either as an additional primitive, or an attempt to reduce the notion of determinacy and borderliness to precision could be made. I will return to this project in section [REF].

3.3 Doing Without Worlds or Precisifications

Let us summarize the main points of our discussion of supervaluationism. The primary points I took to tell against it were the following:

(i) The dual distinctions between the worldly/non-worldly and precise/vague seems otiose, at least for the purposes of modeling vagueness.

(ii) The existence of the former distinction is highly contentious, and is hard to elucidate to someone who doesn’t already have those concepts.

(iii) The acceptance of the former distinction commits us to precise boundaries regarding what the world settles.

(iv) The simplest application of the formalism commits us to a controversial logic of vagueness and modality; in the more complex variants it is less obvious what work the world/precisification division is doing.

(v) While the supervaluationist formalism adequately models determinacy and borderliness a more sophisticated formalism is required to model precision and vagueness.

So, what might an alternative look like?

One way you might try to justify the supervaluationist formalism whilst accommodating the above points is to take a more instrumentalist approach to the model theory. One could have the attitude that while the world-precisification formalism gives a particularly simple way of modeling necessity and vagueness, the the use of worlds and precisifications, and the subsequent distinctions between worldly and non-worldly propositions, is merely a formal artefact of this model. On this view one could potentially have two models which disagreed about which propositions are worldly whilst being equally adequate regarding the things they are intended to model, such as which propositions are vague and precise.

I think this is definitely a step in the right direction. However, once we have acknowledged that the worldly/non-worldly distinction doesn’t correspond to real non-arbitrary division between propositions, it becomes important to ask whether use of worlds and precisifications has an important formal role in modelling vagueness. From a purely formal perspective anything can play the role of indices provided there are enough of them – they could be numbers, ordered pairs or even bananas! This attitude therefore quickly runs the risk of making the vocabulary of ‘worlds’ and ‘precisifications’ completely redundant – one needs to show that the fact that the indices are ordered pairs of entities (as opposed to entities that are not ordered pairs such as, say, numbers) is playing some important formal role or metaphysical role. Since the view under consideration denies that there is an important metaphysical role, they must demonstrate that there is something formally insightful about thinking of indices as ordered pairs.

26Although the proposition that there are electrons is vague, there are closely related propositions that plausibly are precise, such as the conjunction: there are electrons and electrons are fundamental.
The most obvious point at which the ordered pair structure becomes important is in validating the product logic, which allows $\Omega$ and $\Delta$ to commute with one another in certain ways. However we noted that it is far from clear whether the product logic is even something that we want.\footnote{Note that even with the product logic, we do not need to think of indices as ordered pairs. There are models of the product logic that cannot be represented by a product model (the kind of model described in section [REF]). However, it is clear that the product models are the simplest and most natural way to ensure the product logic is validated.} Without the product logic we lose the primary justification for modeling things with ordered pairs that is not given by appeal to an independent realism about the worldly/non-worldly distinction.

An alternative view simply rejects the use of possible worlds and precisifications altogether in favour of evaluating propositions relative to single indices. Supervaluationism encourages a kind of ‘two ingredient’ picture of truth, in which two fundamentally different things are needed to determine the truth value of a proposition: the unvarnished worldly facts and the remaining free-floating borderline truths, which are each of a fundamentally different kind. On the alternative picture, there is no sharp distinction between two kinds of ingredients, they are rather two ends of a spectrum: the former kind of fact merges seamlessly into the latter kind, presumably with borderline cases in between. The natural formalism for this picture will be one in which we evaluate propositions at simple unstructured indices. In the rest of this chapter I shall sketch a view that falls under this kind of description.

This view is more radical that it might initially seem. On this view possible worlds, as they are ordinarily conceived, simply do not exist and nor can they be reconstructed in a straightforward way. Similarly, the notion of a precisification that tells us where the boundaries are once the possible world coordinate has been provided no longer makes much sense. Consequently you might think a certain amount of revision to philosophical concepts that rely on the notion of a possible world (and that rely on the notion of a precisification) is required.

Although the world role simply isn’t satisfied we can divide the role up into two consistent halves. There is the use of possible worlds in philosophical logic and formal semantics. In these contexts the notion of an index, or perhaps a metaphysically possible index, can play the role that is required: few formal results in modal logic, for example, require that we interpret the indices they quantify over in any particular way (see, for example, our discussion of rigid designators and the actuality operator later). On the other hand, we saw that supervaluationists also employ a more metaphysical notion of a possible world. According to that conception possible worlds don’t settle vague matters, so indices cannot play the metaphysical role. In this case the notion of a maximally strong precise proposition seems to do better. Getting a clearer picture on the latter conception of world will be my main focus in what follows.

### 3.3.1 Individuating Propositions

Rather than take certain theoretical entities, such as worlds and precisifications, as our starting point, my approach will be to instead start by taking the relatively concrete notion of a proposition as given. For the most part I shall not assume much about them except that they play the proposition role outlined in section [REF], and that they form a complete Boolean algebra under the usual logical operations. The Boolean assumption means that we are ignoring any fineness of grain due to structure – structured propositions do not form a complete Boolean algebra. It may be possible to add in extra fineness of grain later if we wanted to, but for now it is useful to abstract away from that. This seems harmless to me since the fine grainedness due to vagueness is certainly not due to structure. Once we have the notion of proposition at hand my approach will be to define up the more theoretical notions.
The supervaluationist picture gave us a clear picture of how propositions are individuated. What do we get if we reject that picture? The proposition role certainly helps set lower bounds on individuation. However, if we want to set out thinking about propositions without conceiving of them as sets of independently understood entities, whether world-precisification pairs or not, it is quite natural to take the notion of individuation as primitive and to reason from there.

Thus my approach will be to introduce a primitive individuation connective, $A \equiv B$, to be read as saying that the proposition that $A$ is identical to the proposition that $B$. For the time being, at any rate, we shall treat this connective as primitive; I will present a theory of propositions, with clear individuation conditions, in chapter 7. The most fundamental principle governing this connective is a variant of Leibniz’s law – that equivalent things are substitutable in all contexts:

**Substitution:** $A \equiv B \rightarrow (\phi \rightarrow \phi[A/B])$

We can see straight away that the notion of necessary equivalence, while a useful approximation, cannot be the notion we are after because necessarily equivalent things cannot always be substituted within the scope of a determinacy operator. Similarly the notion of determinate equivalence will not do. Less obvious is the fact that determinate necessary equivalence and necessary determinate equivalence will not do either. The reason is that second order indeterminacy can prevent the intersubstitutability of necessary determinate equivalents and determinate necessary equivalents within determinacy contexts; similar problems arise for other finite combinations of ‘necessary’ and ‘determinate’ and vagueness at higher orders.

Two other principles governing individuation provide a more complete picture, although they play less of a role in what follows:

**Identity:** $A \equiv A$.

**Rule of Equivalence:** If $\vdash A \leftrightarrow B$ then $\vdash A \equiv B$

If we were in a higher order logic, which allowed quantification into the position that a sentence operator takes, then we could define $A \equiv B$ as $\forall O (OA \rightarrow OB)$. For this definition all of the above principles would be provable.

Given the notion of equivalence above we can introduce extremely broad notions of necessity and consistency. The broad notion of necessity, which I will write $L$ and is defined by $A \equiv (A \equiv A)$:28 From this notion analogous notions of consistency and strict implication can be introduced in the obvious ways: consistency, written $M$, can be defined as $\neg L \neg A$, and strict implication/entailment can be defined by $L(A \rightarrow B)$.

Given the identity axiom we can prove the $T$ axiom, $LA \rightarrow A$, since by substitution we get $A \equiv (A \equiv A) \rightarrow (A \equiv A \rightarrow A)$, which given $A \equiv A$ allows us to infer $A \equiv (A \equiv A) \rightarrow A$. Similarly we can prove the $K$ axiom, $L(A \rightarrow B) \rightarrow (LA \rightarrow LB)$, and the necessitation principle which says that if you can prove $A$ in this system you can prove $LA$. Thus $L$ represents a reasonable notion of necessity.

What is perhaps most striking about the substitution axiom is that it guarantees that the notion of $L$-necessity is the always broadest kind of necessity in your language. Let us say that an operator $O$ is necessity-like in a theory if that theory contains every instance of the schema $O(A \equiv A)$. Being ‘necessity-like’ is a necessary condition on representing some kind of necessity operator, and is satisfied by all the candidates usually discussed when comparing the breadth of necessity operators: ‘it’s a conceptual truth that’, ‘it’s metaphysically necessary that’, ‘it’s known a priori that’, ‘it’s determinate that’, ‘it’s a logical truth that’ and so on. The following theorem guarantees that the $L$ operator is at least as broad as all of these operators:

28You can also define it as $A \equiv \top$ for any tautology $\top$ however my definition makes for simpler proofs in many places.
Let $T$ be any theory containing Substitution in a language contain $\equiv$, and let $O$ be any necessity-like operator in $T$. Then every instance of $LA \rightarrow OA$ is a theorem of $T$.

That is to say, $L$ is the broadest necessity operator. The proof here is quite straightforward: $A \equiv (A \equiv A) \rightarrow (O(A \equiv A) \rightarrow OA)$ is an instance of substitution. Given that $O$ is necessity-like it follows that we have $O(A \equiv A)$, and so we can derive $A \equiv (A \equiv A) \rightarrow OA$, which is $LA \rightarrow OA$ modulo definitions.

Some applications of this theorem are noteworthy. For example, we can immediately infer that if we add $\Delta$ and $\square$, then we can infer the following two theorems: $LA \rightarrow \square A$ and $LA \rightarrow \Delta A$. Indeed we can infer $LA \rightarrow \pi A$ where $\pi$ is any sequence of $\Delta$s and $\square$s for the sequence $\pi$ also represents a complex operator that is also necessitatable. Thus we see that $L$ is broader than anything definable with a finite string of $\square$s and $\Delta$s.

Another upshot of this theorem is that $L$ satisfies the characteristic $S4$ axioms, $LA \rightarrow LLA$. Note that since $L$ is necessitatible – something we showed earlier – and since we can prove $A \equiv A$, we can also prove $LL(A \equiv A)$ by applying necessitation twice. Thus $L$ is a possible substitutend of $O$ in the theorem, delivering $LA \rightarrow LLA$ as required. Curiously, one cannot prove within this system the characteristic $S5$ axiom, $\neg LA \rightarrow L\neg LA$. This is perhaps a good thing: this principle would rule out the possibility of borderline $L$-necessities, which is something we might want to countenance.

The interaction between $L$ and $\square$ and $\Delta$ was quite indicative, as it suggests that within the present system it is possible to provisionally think of $LA$ as simply conjoining the result of prefixing $A$ by any finite string of determinacies and necessities. If we have infinite conjunction in the language we can define an operator $LP := \bigwedge \pi P$ where $\pi$ ranges over arbitrary finite sequences consisting of $\square$ and $\Delta$ symbols; once $L$ is defined we can introduce $A \equiv B$ by the definition $L(A \leftrightarrow B)$. Of course, in a language with operators other than just $\Delta$ and $\square$, this definition may not be adequate, however it is a concrete way to think about it that will guide us in what follows. At any rate, the main point here is that the appeal in this context to an operator behaving like $L$, or like the interdefinable $\equiv$ connective, doesn’t require any substantive commitments: it is available to anyone who can make sense of determinacy and necessity operators and infinitary conjunction. Indeed, with a suitably rich logic of infinite conjunction, determinacy and necessity it is possible to show that $\equiv$, defined as above, satisfies the principle Substitution, Identity and The Rule of Equivalence.

### 3.3.2 The General Kripke Semantics

It is worth asking whether we can give some kind of model theory for this language – the language of determinacy, necessity, and the individuation connective – that does better with respect to the kinds of problems we levelled at the supervaluationist model theory. I shall divide up my discussion into two parts. The first part shall be devoted to semantics that treat the operator ‘it’s determinate that’ $A'$ in a in setting where we can talk about necessity and $L$-necessity. In section [REF] I shall turn to the slightly more general question of giving a semantics for the operator ‘it’s precise that’ $A'$, roughly meaning that the proposition that $A$ is precise, in setting containing necessity and $L$-necessity. The second question would be subsumed by the first if one could define precision in terms of determinacy and necessity, however I have already suggested that it cannot, so a slightly more general model theory will be needed. I should also mention that the approaches to modelling these notions I will discuss are fairly abstract, and unlike the supervaluational semantics have little heuristic value; this is an issue I shall take up in chapter 7. However the current discussion is important for establishing the overall logical picture.

Here I shall argue that a fairly ordinary Kripke semantics, of which the supervaluational semantics is a special case, provides a perfectly general way to model this language without
committing us to the notion of a world proposition, or a parallel notion of a precisification proposition. Indeed I suggest that the paradoxes of higher order vagueness and the other puzzles I raised are completely avoidable in this setting.

A pointed Kripke frame for the language described above is given by a quadruple \((\mathcal{I}, R, S, i)\). Here \(\mathcal{I}\) is supposed to represent a set of indices of some kind; these can be any kinds of objects we like, and their sole purpose is to be points of evaluation relative to which formulae will be assigned truth values. \(R\) and \(S\) are simply relations on \(\mathcal{I}\) – they will be used to interpret \(\Box\) and \(\Delta\), much like the analogous relations were in the supervaluationist semantics. An important disanalogy, however, is that \(R\) and \(S\) act over the same domain of indices in the Kripke semantics, while their domains are disjoint in the supervaluational case. Finally \(i\) is a member of \(\mathcal{I}\); when we ask what is true in a model we are simply asking what is true relative to \(i\). If there is such a thing as an intended model, truth in the intended model would correspond to the disquotational notion of truth (as opposed to supertruth, for example).

The crucial difference between the general Kripke model theory and the supervaluational model theory is that we only have one set of indices over which the accessibility relations are defined over. In the supervaluational setting each accessibility relation had its own domain, and each relation is completely independent of the other in the sense that \(R\) completely ignores the precisification coordinate, and \(S\) ignores the world coordinate when we evaluate a formula at a pair of a world and a precisification. In what follows it will be natural to restrict ourselves to rooted frames: a rooted frame is one in which every element of \(\mathcal{I}\) is reachable from \(i\) by some finite number of transitions along either \(R\) or \(S\). In a rooted frame a sentence \(\phi\) is true at every index in the frame iff \(L\phi\) is true in the frame (i.e. true at the distinguished index).

As in with supervaluational semantics, we can assign truth values to each atomic sentence relative to each world, and we can extend truth to arbitrary sentences relative to each index \(i \in \mathcal{I}\) in the standard way. The crucial clauses for \(\Box\), \(\Delta\) and \(L\) are given as follows:

\[
\begin{align*}
(\Box) & \quad i \models \Box \phi \text{ if and only if } j \models \phi \text{ whenever } Rij \\
(\Delta) & \quad i \models \Delta \phi \text{ if and only if } j \models \phi \text{ whenever } Sij \\
(L) & \quad i \models L \phi \text{ if and only if } j \models \phi \text{ for every } j \text{ which can be gotten from } i \text{ by following some finite chain of } S \text{ or } R \text{ relations.}
\end{align*}
\]

A sentence is true in a model iff it is true at the distinguished index.

\(A \equiv B\) is introduced by the formula \(L(A \leftrightarrow B)\) (alternatively, we could have taken \(\equiv\) as primitive and defined \(L\) from it as in the last section). Provided either \(R\) or \(S\) is reflexive, all of the principles we have listed for \(\equiv\) will be validated in a Kripke frame of this sort.\(^{29}\)

In accordance with our earlier observation, the \(S5\) axiom for \(L\) will not be satisfied unless we additionally stipulate that both \(R\) and \(S\) are symmetric.

To ensure a reasonable logic of necessity and determinacy we must place additional constraints on \(R\) and \(S\). It’s natural to want a modal logic of \(S5\) and a logic of determinacy of at least \(KT\). The distinctive feature of the former logic is that it rules out all kinds of higher order contingency – if something’s necessary it’s necessarily necessary, and if it’s not necessary it’s necessarily not necessary. Analogous principles for the logic of determinacy are not appropriate, and the logic \(KT\) reflects this; the only distinctive principle in this logic is the factivity of determinacy. To ensure that the theorems of these logics are \(L\)-necessary we must stipulate that \(R\) is an equivalence relation and \(S\) is a reflexive relation.

A point of departure from the supervaluational semantics should be emphasised at this point: according to that semantics we modelled \(R\) with the universal accessibility relation, the relation that relates every possible world to every other possible world. In the present

\(^{29}\)If neither \(R\) nor \(S\) is reflexive it would then be necessary to add \(A \equiv B\) as an additional primitive, given by the conditions that \(A\) and \(B\) have the same truth value at every index accessible via some finite chain of \(Rs\) and \(Ss\). However, this possibility will not figure in our discussion from here on out.
If we additionally impose, for each $n$ into sentence position, and we want to make certain supervenience claims true, then we far as I can see it is still open which of the interaction principles is true.

These constraints ensure certain features of the pure modal logic, and the pure logic of determinacy. Further constraints need to be added if we want the two operators to interact in the right way. In section [REF] I argued that at least some aspects of the product logic are controversial. However if we wanted to add some of the principles, in a way that allows the necessitation rule to be applied to them, this can be done by imposing the last three constraints listed in table 3.1.\(^3\) I shall treat these as strictly optional in what follows; as far as I can see it is still open which of the interaction principles is true.

If we don’t want the product logic, are there any further constraints we should put on $R$ and $S$? I shall later argue that if we want to expand the language to contain quantification into sentence position, and we want to make certain supervenience claims true, then we should additionally impose, for each $n$, the constraint labeled Supervenience\(^n\) in table 3.1.

If you are looking at models in which $R$ is an equivalence relation and $S$ reflexive, this constraint makes no difference to the logic in the language without propositional quantifiers, although it is important to have the constraint in mind. So the constraints we are interested in are given by the first five principles in table 3.1, with the last three being optional.

Note that the supervaluational semantics is a special case of this type of semantics. For given a set of worlds $W$, precisifications $V$, a distinguished pair $\langle w, v \rangle$ and relation $R'$ on $W$ and $S'$ on $V$, we can turn this into an ordinary Kripke model by the following process. We begin by setting our indices, $I$, to be the set of ordered pairs from $W$ and $V$, $W \times V$, and our distinguished index, $i$, to be $\langle w, v \rangle$. Then we introduce two relations, $R$ and $S$, on $I$ defined so that $R(u, v)\langle x, y \rangle$ if and only if $v = u$ and $R'\langle x, u \rangle$, and $S(v, w)\langle x, u \rangle$ if and only if $w = x$ and $S'\langle u, v \rangle$. The result is a Kripke frame that satisfies all the conditions in table 3.1, but it is easy to see that the conditions for truth at a pair are exactly the same as the conditions for truth at a pair in the supervaluational model we started out with. Note, as we pointed out earlier, that even if $R'$ is the universal relation on the set of worlds, the $R$ relation defined above will be an equivalence relation that partitions the indices (i.e. the world precisification pairs) in to a number of equivalence classes equal to the number of precisifications.

The converse to this result, it should be mentioned, is certainly not true: there are plenty of Kripke models of the sort described that are not isomorphic to any supervaluational model. This fact remains true even if we insist that the Kripke models in question satisfy all the conditions in table 3.1 corresponding to the product logic. Let us look at one way in which this can happen. In any finite supervaluational model based on ordered pairs of worlds and precisifications the total number of ordered pairs is just the result of multiplying the number of possible worlds by the number of precisifications. So if the total number of pairs is a prime number then either there is only one world or one precisification, and it

\[^3\]The condition: $(Rxy \wedge Sxz) \rightarrow \exists w(Syw \wedge Rzw)$ is sometimes called the ‘Church-Rosser’ property. $(R \circ S)xy$ means that $x$ is an $R$ of an $S$ of $y$.  

---

| $\Delta \phi \rightarrow \phi$ | $S$ reflexive |
| $\square \phi \rightarrow \phi$ | $R$ reflexive |
| $\square \phi \rightarrow \square \phi$ | $R$ transitive |
| $\phi \rightarrow \square \phi$ | $R$ symmetric |
| Supervenience\(^n\) | If $Rij$, $Rik$, $S^o ju$ and $S^o ku$ then $j = k$ |
| $\square \Delta \phi \rightarrow \Delta \phi$ | $S \circ R \subseteq R \circ S$ |
| $\Delta \square \phi \rightarrow \Delta \phi$ | $R \circ S \subseteq S \circ R$ |
| $\Diamond \Delta \phi \rightarrow \Delta \Diamond \phi$ | If $Rxy$ and $Sxz$ then there’s a $w$ such that $Syw$ and $Rzw$ |

Table 3.1: Interaction principles for vagueness and modality
is thus a model in which either there is no indeterminacy anywhere in the model or no
contingency anywhere in the model. On the other hand for any Kripke model there is an
operation that increases the size of the model by any finite number but makes exactly the
same modal formulae true.31 Thus take any finite model of your logic in which makes the
formula $\forall A \wedge \neg\Box A \wedge \neg\Box \neg A$ true somewhere in the model (since this formula is clearly
consistent, it is natural to think that whatever constraints you put on your Kripke models
there will be a model like this). Now find a finite model which makes the same formulae
true at the distinguished index but has a prime number of worlds (noting that whatever
the size of the original model there’s always a prime bigger than it.) This model cannot be
represented by ordered pairs of worlds and precisifications since, as noted above, any such
model with a prime number of indices is one in which there is either no contingency or no
indeterminacy at any index, but by stipulation, the formula stating that $A$ is contingent
and determinate is true somewhere in this model. 32

Without imposing the product logic (the last three constraints from table 3.1) matters
are even more dire. For example, consider the following model which I’ll call the ‘one-way
roundabout’ . In this diagram the dotted lines represent the $S$ relation, and the solid lines
the $R$ relation; each index is related to itself by both relations, although to reduce clutter
I shall not represent this on the diagram.

This model satisfies all but the last three principles of table 3.1. Notice that cardinality
does not seem to be a barrier to our representing this Kripke frame. It looks as though it
could be modeled using two worlds and two precisifications giving us a total of four pairs.
But when we try to define the accessibility relations over these two precisifications we find
that we cannot because the determinacy accessibility relation goes in opposite directions
on the top pair than on the bottom pair.33 Similar points apply to any Kripke frame that
contains the one-way roundabout as a subframe.

3.3.3 A Representation Theorem?

The Kripke semantics is general enough to ensure that the intended model will be among
the set of Kripke models described above. For given some assumptions about propositions,
specifically the assumption that they form a complete atomic Boolean algebra under $L$-
entailment34, one can think of indices as maximally strong propositions. The accessibility
relations can be informally introduced as follows: $Sij$ holds if and only if $i L$-entails $\neg\Delta j$

31Given a Kripke model $(W,R,S,i,\{\})$ such that $x \in W$ and $x' \notin W$, and such that $R$ and $S$ are reflexive,
one can construct a new model whose cardinality is one bigger: $(W',R',S',i,\{\})$; $W' = W \cup \{x'\}$, $R'y$z
iff $Ryz$ or $[y = x'$ and $Rzx]$ or $[z = x'$ and $Rxy]$ (and similarly for $S'$), and $[P]_y^z = [P]_y$ if $y \in W$ and
$= [P]_x'$ if $y = x'$. Repeat this operation as required to increase the cardinality by other finite numbers.

32This argument does not depend on whether we model vagueness and modality with binary or ternary
accessibility relations that we introduced in section [REF]. This argument doesn’t obviously generalize to
infinite models, although one can establish the same point through different arguments there.

33Any supervaluational model that has four pairs must either have two worlds and two precisifications,
four worlds and one precisification, or one world and four precisifications; only the first kind of model
can represent the one-way roundabout. In fact, this argument works even if we try to represent it with
a subframe of a supervaluational model: we know that the two left most indices have the same world
coordinate, and analogously for the two rightmost indices, because they are related by $R$. Similarly, the
top indices share the same precisification coordinate, and the bottom indices share the same precisification
coordinate. So the indices of our model are of the following form: $\{u,x\} \times \{u,v\}$. Finally note that we
cannot define $S$ over $\{u,v\}$ to produce the one-way roundabout model, due to the differing direction of
accessibility between the topmost and bottommost indices.

34This assumption entails that there are set many propositions; this is perhaps a controversial assumption.
However it is possible to generalise the Kripke semantics to deal with proper class many propositions by
invoking plural quantification or some similar device.
and $Rij$ iff $i$ $L$-entails $\diamond j$. Finally the distinguished index should be identified with the (unique) maximally strong proposition that is true.\textsuperscript{35}

The fact that the intended model has the structure of a Kripke frame is pretty much unavoidable given the logical assumptions above; unfortunately there is no straightforward argument that the intended model must have the structure of a supervaluationist frame. A supervaluationist who wanted to put the worldly/non-worldly distinction in good standing might want to have some kind of general guarantee of the following form: whatever the intended model looks like, there is some canonical way of associating each maximally strong proposition (i.e. each index) in the intended model with an ordered pair from a fixed pair of sets, $W$ and $V$, and to reconstruct the accessibility relations purely from two independent relations on each of those two sets respectively. The guarantee that the supervaluationist is looking for might be called a ‘representation theorem’. For a supervaluationist, it would be extremely nice to have a representation theorem, because it would suggest that the distinction between worldly and non-worldly propositions isn’t a peculiar feature of their preferred semantics, but is forced on us merely by the acceptance of certain logical principles determinacy, necessity and $L$-necessity. For given those logical assumptions the intended model has the structure of a general Kripke model, as noted above, and given the representation theorem it follows that the intended model has the structure of a supervaluational model and thus the notion of a world proposition and a precisification proposition is in good standing.

What the above considerations seems to suggest is that no such representation theorem will be forthcoming. There can be no guarantee that we can associate isomorphically each Kripke model with a supervaluationist model. Reflection on the one-way roundabout model also seems to suggest that there can’t even be a way to canonically transform (non-isomorphically) an arbitrary Kripke model into a supervaluationist model; in that case it is totally unclear which representable model the one-way roundabout model would get transformed to, or what the significance of the transformation would be if there was one.

One might try to get around this by weakening our definition of a supervaluationist model. One of the biggest constraints those models imposed was the constraint that the accessibility relations over ordered pairs had to be generated by two independently moving accessibility relations over $W$ and $V$ respectively: the modal accessibility relation, when given two pairs, only looks at the first coordinate of both pairs to decide whether it holds between them, and the determinacy accessibility relation only looks at the second coordinates. The most natural way to relax this constraint would be to have instead two quaternary relations over both domains both relating a world and a precisification to another world and another precisification where they both, as it were, can ‘see’ both coordinates of both pairs. Thus, for example the modal truth clause, would become: $x, u \models \square \phi$ if and only if $y, v \models \phi$ for every $y$ and $v$ such that $Rxuyv$. I will not go into the resulting theory in too much detail. Suffice it to say that it does allow one to prove a representation theorem – indeed it is no more than a notational variant of the Kripke semantics. This comes at the cost of having the representation in the quaternary semantics be radically non-unique. Indeed, to see how inadequate many of these representations are, note that for any Kripke model whatsoever there is a corresponding model of this sort in which there’s only one possible world, and by a parallel argument, another corresponding model in which there’s only one precisification. This is all because the ordered pair structure isn’t playing any role in this semantics whatsoever – constancy over the world coordinates needn’t correspond to necessity, and constancy over the precisification coordinates needn’t correspond to determinacy. We might as well have been modelling everything with unstructured entities and a pair of binary accessibility relations, as one does in the Kripke semantics.

\textsuperscript{35}Although this argument guarantees that there is a set theoretic structure isomorphic to the intended model, there is no guarantee that any set theoretic structure that it is determinately isomorphic to the intended model. It is, for example, always indeterminate which maximally strong proposition is true so it is borderline which the distinguished index of the intended structure is. If we are restricting ourselves to rooted frames, this can even mean that it’s indeterminate how many indices are in the intended model.
The distinction between worlds and precisifications is completely redundant in the semantics with quaternary accessibility relations, but perhaps there is something like a happy medium in which some of the ordered pair structure plays a role in the semantics but not enough to prevent a representation theorem. If we replaced one or both of the binary accessibility relations in the supervaluational model with a ternary relation, for example, we would be have something intuitively inbetween the binary and quaternary proposal. In what follows I shall consider replacing the binary determinacy accessibility relation with a ternary relation $S \subseteq W \times V \times V$, which when given two world precisification pairs would look at both coordinates of the first pair, but only the second coordinate of the second pair, to work out whether it held between them. The effect is to allow that what is $S$-accessible to what depends on the possible world – one could think of this as allowing the notion of a precisification being ‘admissible’ to be contingent: $u$ can be admissible relative to $v$ according to some worlds but not others. The semantics would be the same as the supervaluational semantics, except that the clause for the determinacy operator would be: $w, v \vDash \Delta \phi$ if and only if $w, u \vDash \phi$ for every $u$ such that $Swvu$. In order to get a representation theorem we must also relax the requirement that once we have chosen $W$ and $V$, every ordered pair in $W \times V$ must correspond to the index of a model. In other words, in the clause for $\Delta I$ shouldn’t quantify over every $u \in V$, only those $u$ such that $\langle w, u \rangle$ is in a predetermined subset of $W \times V$. For as before, if we are trying to represent a Kripke model with a prime number of indices, if we required all elements of $W \times V$ to be in the model, there would either be exactly one precisification or one world, and this would cause problems even in the binary/ternary representation.

Unfortunately these modifications alone do not suffice for a representation theorem. However if we restrict attention to Kripke models with certain structural features then a representation theorem the modified supervaluational semantics is provable:

**Theorem 3.3.1.** Suppose that $\langle I, S, R, i \rangle$ is a Kripke model. Let $S^+$ and $R^+$ denote the transitive symmetric closures of $R$ and $S$ respectively.

If $S^+ \cap R^+ \subseteq= \{i \in I : \text{ for every } i \in I \text{ then } \langle I, S, R, i \rangle \text{ is isomorphic to a modified supervaluational model } (W, V, I, R', S', w, v) \text{ where } R' \subseteq W \times W \text{ is a binary relation and } S' \subseteq W \times V \times V \text{ is ternary relation and } I, \text{ the set of indices for the model, is a subset of } W \times V.$

**REF.** [For appendix. The following is just to remind me how it’s done.]

- Worlds are equiv classes under $S^+, prec$ equiv. classes under $R^+$.
- $\langle [i]_{S^+}, [j]_{R^+} \rangle R' \langle [k]_{S^+}, [l]_{R^+} \rangle$ iff $[i]_{S^+} = [k]_{S^+}$ and $R^l k$. 
- $\langle [i]_{S^+}, [j]_{R^+} \rangle S' \langle [k]_{S^+}, [l]_{R^+} \rangle$ iff $[j]_{R^+} = [k]_{R^+}$ and $Spq$ where $\{p\} = S^+(i) \cap R^+(j)$ and $\{q\} = S^+(k) \cap R^+(l)$.

Does this representation theorem put the supervaluationalist’s distinction between worldly and non-worldly propositions, and the analogous distinction between precisifications, in good standing? I think the supervaluationalist should find this representation theorem unsatisfactory for a few reasons. The first is that, although not as bad as the quaternary proposal, we have no general guarantee that this representation will be unique. There could be two equally good ways of partitioning the sets of indices into world propositions and precisification propositions, such that the two binary relations of the Kripke model can be reduced to a binary and a ternary relation defined on the two respective partitions. In such cases there is nothing to uniquely bearing the title ‘world’ and ‘precisification’ can be recovered. 

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36Another theorem along these lines is proved in Kurucz [78], Lemma 2.1: if $\langle I, R, S \rangle$ is a frame for
Another problem, which I think is much more pressing, is that the assumptions needed to prove this theorem (i.e. that \( S^+ \cap R^+ \subseteq = \)) are incredibly strong. They are not guaranteed even by imposing all of the conditions listed in table 3.1. The very simple frame below, for example, satisfies all those properties, but it is not possible to represent it even in the binary/ternary way described earlier:\(^\text{37}\)

\[
\begin{array}{c}
\bullet \\
\end{array}
\]

Lastly there is the issue that for this representation theorem to work, some ordered pairs must be left out of the model. But intuitively, a precisification is supposed to tell you not only where the cut-off points in fact are but where they would have been had the unvarnished facts been different. For every way the world could have been, the precisification tells us where the cut-off points would have been, and so every way of combining a world with a precisification produces a coherent description how things are. But to make the representation theorem work we have to drop some of these combinations from the model, which is to deny some of the heuristic value of the formalism that made it attractive in the first place.

### 3.3.4 Recovering Possible World and Precisification Talk

The picture I am proposing, then, rejects the notion of a possible world and of a precisification as they are standardly conceived. One might have initially thought that these concepts are so basic to the understanding of modality and vagueness that any account of modality and vagueness would be able to recover these notions, perhaps as certain kinds of propositions. But as we have seen in the last section, not even this is true in general.

That said, the notion of a precisification or a sharpening of a language has been incredibly useful in the philosophy of vagueness, much as the notion of a possible world has been fundamental in the philosophy of modality. Their use has become ubiquitous if only due to their considerable heuristic value. One might think, therefore, that it would be worth having some sort of story about what is going on when philosophers talk about precisifications and a similar story for when they talk about worlds, even if it is not quite how these philosophers usually conceive of what they’re doing.

Indeed, in the case of modality something like this project has been carried out already. According to one position, modalism, it is the ordinary modal adverbs and corresponding operators that are primitive. A modalist can accept the point that ‘possible worlds’ talk has heuristic value, but insist that this talk is ultimately going to be spelt out in terms of the modal operators and other more commonplace entities such as propositions. While David Lewis’s modal realism immediately springs to mind when one thinks of views that deny modalism, modal realism is not the only alternative to modalism. Views that take the modal idioms to be reducible to talk about truth relative to abstract entities are also possible (see, for example, chapter 3 of Stalnaker [120].)

There is a completely analogous question in the philosophy of vagueness. Should one take the idioms of being borderline and determinate, or even better, of being vague and precise, as primitive? That is, should we state things in terms of familiar entities, propositions, \(S^5\) and \(KT\) respectively then it is the image via a bounded morphism of a subframe of a product frame. Unlike the representation theorem we do not get a structure isomorphic to the original. Moreover, like the representation theorem, there will typically be several product frames which stand in this relation to the original, so there will be no unique way of assigning world propositions and precisification propositions to a frame like this.

\(^{37}\)The top two points must have the same precisification coordinate, because they are modally related. Also the ternary relation guarantees that if a pair \(Ss\) another pair, then it \(Ss\) any other pair with the same precisification coordinate. Since the top left point \(Ss\) itself and has the same precisification coordinate as the top right, it must \(S\) that too which contradicts the diagram.
and more properties of these entities, like being vague and precise? Or should one take less familiar abstract entities – precisifications – as primitive and explain borderliness, determinacy, vagueness and precision in terms of these objects and the notion of a precisification being ‘admissible’?

Whilst the latter view seems to be the dominant one, at least among classical approaches to vagueness, I prefer latter approach. That said, one might hope to carry out something analogous to the modalist’s project of recovering possible worlds talk. Given that talk involving precisifications is so pervasive, one might hope to be able to recover some of this way of talking in a framework in which precision or determinacy operators are taken as primitive. In what follows I’ll show that one can indeed recover some of these ways of talking, both in terms of possible worlds and precisifications, however they will not behave in the way that these entities are standardly assumed to behave. In particular I take it that when the supervaluationist is invoking possible worlds in their model theory, they take these entities to be completely determinate. I.e. it is determinate which entities are the possible worlds, determine how many of them there are, and so on and so forth.

Possible Worlds are Determinate: For any entity, \( x \), it is always a determinate matter whether \( x \) is a possible world or not, and for any proposition, \( p \), it is always determinate whether \( p \) is a world-proposition.

I will also show how one can recover something a bit like the notion of a precisification, and an accessibility relation between precisifications. It will also turn out to be vague which propositions are precisification propositions, although to a lesser degree. However, the more important shortcoming is that we will not be able to understand borderlineness in terms of an accessibility relation between precisifications so defined. That is, defining precisifications as a type of proposition, and defining accessibility as a certain relation between precisifications, one would hope that the following would be \( L \)-necessary:

Precisifications and Determinacy: The conjunction of a precisification proposition, \( v \), and a world proposition, \( w \), \( L \)-entails that it’s determinate that \( P \) if and only if it \( L \)-entails that \( P \) is \( L \)-entailed by the conjunction of each accessible precisification proposition with \( w \).

This principle will not turn out to be true.

Prior, perhaps the archetypal modalist, wanted to do away with worlds and instants of time by defining them in terms of the modal and tense operators as certain kinds of propositions – roughly the conjunction of all the propositions that would be said, by the anti-modalist (or anti-temporalist), to be true at that world or instant. How one carries this out is actually surprisingly tricky. According to Prior a proposition is a \( W \)-proposition if and only if it is possible both that it is true and strictly implies all truths. The way he does this is to introduce a device for quantifying into sentence position; this can be added to our model theory fairly straightforwardly.\(^{38}\) With that in place one can introduce an operator, \( W \), for talking about world propositions as follows \( Wp := \lo (p \land \forall q(q \rightarrow \lo(q \rightarrow p))) \). A proposition, \( q \), is true at a world proposition \( p \), iff \( p \) strictly implies \( q \) (\( \lo(p \rightarrow q) \)). A similar strategy is applied to instants (see Fine appendix [98] and Prior chapter XI [96] for details.)

If we assume a fairly boring logic of necessity no definition of accessibility is required. If necessarily equivalent propositions are identical, then these definitions are adequate. However in the present setting, in which we are making a distinction that at least vagueness contributes to fineness of grain, the definition fails to uniquely single out a proposition for each thing the anti-modalist would call a ‘possible world’. Distinct world propositions can make exactly the same propositions true. Indeed, there are even distinct world propositions corresponding

\(^{38}\)One firstly introduces countably many propositional variables into the language, \( p_n \). An assignment is then a function from propositional variables to subsets of \( I \), and we can write \( a[p_n]b \) to mean that assignments \( a \) and \( b \) agree every where except, possibly, at the subset assigned to \( p_n \). Finally we relativise truth at each index to an assignment. The crucial clause is: \( i, a \models \forall p_n \phi \iff i, b \models \phi \) for every \( b[p_n]a \).
to the actual world. The reason for this, roughly, is that the result of disjoining any world proposition with something that’s necessarily false, but not determinately false will also be a world proposition, and these propositions will differ when embedded in determinacy operators. The disjunction will, more generally, strictly imply exactly the same things as the original proposition.\textsuperscript{39} For in general, if \( q \) is necessarily false, then \( p \) and \( p \lor q \) are necessarily equivalent, and will strictly imply the same things. Moreover, if \( p \) is a world proposition so is everything necessarily equivalent to it. In many cases the disjunction will be different from the original world proposition, so distinct world propositions can have exactly the same propositions true at them.\textsuperscript{40}

With a little bit of experimentation one will quickly realise that other purely modal attempts to characterise worlds in terms of propositions fail for similar reasons. An approach that employs the precision or determinacy operators might do better. Here is an extremely natural notion that captures the basic thought that worlds settle all (and only) the determinate questions, and leaves room for precisifications to play a distinctive role. This is the conception of worlds as \textit{maximally strong precise propositions}. Here I introduce another operator, \( \sharp p \), which is supposed to be read as ‘it’s precise that \( p \)’. Later I will give a more satisfactory account of it. If you wished, you could also substitute the modal characterisation of precision, \( \Box (\Delta p \lor \Delta \neg p) \), in its place without affecting the following discussion.

\begin{align*}
\text{(MSP)} \quad & \text{A proposition } p \text{ is maximally strong precise if and only if it is precise, } L\text{-consistent and } L\text{-entails every precise } L\text{-consistent proposition that entails it. In symbols: } \sharp p \land Mp \land \forall q (\sharp q \land Mq \to (L(q \to p) \to L(p \to q))).
\end{align*}

The rationale for this definition is quite straightforward: an MSP is a complete description of how the totality of all of the precise facts could have been. MSP’s are, I think, the best candidates for what many theorists refer to as ‘possible worlds’; although, as is inevitable, they do not play all the roles wanted of them. One oddity of our definition, which is easily fixed, is that there is no guarantee that a MSP be a metaphysically possible description of a precise state of affairs. There is surely an epistemic possibility in which Hesperus is not the same as Phosphorus, and that this is moreover a precise fact. Thus there are MSP’s that entail that Hesperus is not Phosphorus, and therefore, which cannot be metaphysically possible. To get a better correspondence with possible world talk one should restrict attention to MSP’s that are metaphysically possible, however the notion of an MSP is much more basic and important to theorizing about vagueness (I have already appealed to the notion many times throughout this book, for example.)

Intuitively, relative to each index of our model, there will be a partition of the space of all indices into cells consisting of propositions that are maximally strong and precise according to that index. If we recall figure [REF] from section [REF], the maximally strong precise propositions corresponded to the four circles and the singletons of the pairs that were outside of the circles. The peculiarity that most MSPs are represented by singletons in that model is, I think, a symptom of our failing to taking the notion of a proposition being precise as primitive and of attempting to reduce it to necessity and borderlineness. The more general framework developed in the next section agrees with the basic picture of partitioning the space of indices into cells relative to each index, although it doesn’t inherit the peculiarities of the modal definition of precision. Both frameworks agree, however, that whether a proposition is precise or not, and consequently the partitioning of the space into cells, depends on what index you are at. This is because the concept of precision itself

\textsuperscript{39}So for example, if \( N \) is the cutoff for bald, then (given the simplifying assumption that baldness supervenes on hair number) the proposition that Harry has \( N + 1 \) hairs and is bald is necessarily false but not determinately false. And the result of disjoining this with any world proposition will also be a world proposition and will also strictly imply exactly the same propositions as the original.

\textsuperscript{40}Uniqueness could be restored by identifying by disjoining equivalence classes of necessarily equivalent world propositions and using these propositions in stead. However these are very unnatural objects, and it’s almost always vague whether a given proposition is true at (strictly implied by) such propositions.
admits borderline cases – there are propositions such that it is borderline whether they are precise, and so there must be accessible indices at which they are precise, and other accessible indices at which they are not.

I suggest, then, that if it is possible to recover a notion of a possible world at all, taking only modal notions and the notion of a precise proposition as primitive, it is one in which the ‘boundaries’ between the possible worlds are vague. This should not be too surprising. One thing I have been stressing throughout this chapter is that what people refer to as the ‘worldly’, ‘metaphysically first rate’ facts, such as facts about hair number, either commit us to problems involving higher order vagueness, or they are the kinds of facts that segue continuously and soritesably into the vague truths, such as the truths about who is and isn’t bald. If there any distinction like this is to be found it is hardly surprising that it is a vague one.

However it does mean that any formalism employing ordered pairs of worlds and precisifications is misleading. Worlds, as they are employed in that formalism, are not vague in the way that MSPs are. The formalism assumes a clean split between worlds and precisifications, whereas if worlds are MSPs then it’s vague which things (i.e. which propositions) fit the bill and thus vague which questions are settled by the world and which by the precisification. Certain naïve ways of talking about possible worlds, then, require modification. For example, talk of the ‘actual world’, as though it matches up perfectly with some precisely bounded concrete object – ‘us and our surroundings’ – is misguided. The actual world, according to our definition, is simply the conjunction of all the precise truths. Since there is vagueness concerning which truths are precise, it is vague which of a number descriptions of our world capture all the precise facts about it.

Many other concepts that explicitly invoke possible worlds need rethinking. For example, a ‘rigid designator’ is supposed to be a designator that denotes the same individual relative to every possible world. The name ‘Mt. Everest’ is presumably supposed to fit the description of a rigid designator, although we shouldn’t expect it to denote the same individual relative to every index – there is vagueness concerning which fusion of particles is identical to Mt. Everest so the semantic value of ‘Mt. Everest’ shouldn’t be constant across indices. It is also hard to say what the denotation of a term is relative to an MSP: a single maximally strong precise proposition will typically be represented by a set of indices at which the denotation of ‘Mt. Everest’ will vary. However the distinction Kripke and others were after, I think, can be captured perfectly well by simply using the formal concept of a metaphysically possible index. Accordingly a name, ‘a’, is rigid if it denotes the same individual relative to every metaphysically possible index. This definition guarantees that if ‘a’ is rigid the sentence ‘∃x∀a = x’ is true. The philosopher’s actuality operator is another notion that might need revising on this picture (although here I am uncertain whether the notion is one I want accept). So the picture I am describing certainly involves some revision, I think the costs mainly concern theoretical terms that were on less than firm ground in the first place.

Before we move on, let me briefly mention another contender for being the replacement for the notion of a world. Recall that a proposition is precise* if it is precise, precisely precise, precisely precisely precise, and so on, at all finite orders. One might hope to limit the vagueness in our theoretical entities by theorizing in terms of maximally strong precise* propositions, or MSP*s. According to some theorists there’s only one maximally strong possible precise* proposition the tautologous proposition, because on those views very little counts as precise at all orders. So this proposal is not open to those philosophers. However even for theorists who accept a variety of precise* propositions there are no good reasons to think the boundaries between the possible maximally strong precise* propositions

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41The actuality can be represented in a Kripke model by a function, f, that maps each equivalence class under R to a representative in that equivalence class, with the restriction that if Sij then Sf(R(i))f(R(j)). In some models there may be several such functions, and in others there may not be any.

42Dorr [32] defends this view, although Dorr’s account is even more radical: even tautologies aren’t precise*.
are any more precise. One can see that in the four index model depicted below that relative to the left two indices the singleton of the top right index is not a maximally strong precise\(^*\) proposition, but relative to the right hand precisifications it is. [On the left the partition, looks as it does, and in the right. it is vague at the left hand nodes which propositions are possible maximally strong precise\(^*\) propositions: [WORK OUT HOW TO PUT THESE INSIDE BOXES to distinguish the partitions relative to the right and left indices. EXPLAIN DIAGRAM. Note also that this model validate the product logic.]

\[\begin{array}{c}
  \bullet \\
  \downarrow \\
  \bullet \\
  \downarrow \\
\end{array} \quad \begin{array}{c}
  \bullet \\
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  \downarrow \\
\end{array} \quad \begin{array}{c}
  \bullet \\
  \downarrow \\
  \bullet \\
  \downarrow \\
\end{array}\]

The notion of an MSP\(^*\) is therefore not any more precise than an MSP, and I am doubtful that it is a more theoretically fruitful notion so I shall set it aside for now.

Let us now turn to the notion of a precisification. Can something analogous to the modalist project of recovering precisification talk in terms of propositions be carried out here?

Intuitively a precisification is supposed to be a non-contingent proposition describing where all of the cut-off points would have been depending on the different ways things could have been. This definition looks suspiciously like it assumes the two dimensional supervaluationist picture that we are rejecting. However the model theory described in section [REF] does provide us with a precise way to cash it out: the \(R\) relation employed for modelling modality will divide the space of worlds into equivalence classes and so we can naturally identify each one of these equivalence classes with a precisification proposition. Such propositions are characterised by the following definition:

**Precisification Proposition** A precisification proposition is an \(L\)-consistent, \(L\)-necessarily non-contingent proposition, which \(L\)-entails every \(L\) consistent and \(L\)-necessarily non-contingent propositions that entails it. In symbols, \(p\) is a precisification proposition when:

\[
M_p \land L(\Box q \lor \Box \neg q) \land \forall q (L(\Box q \lor \Box \neg q) \rightarrow (L(q \rightarrow p) \rightarrow L(p \rightarrow q)))
\]

There is also vagueness concerning which propositions are precisification propositions, as the simple model above witnesses.

However, unlike the modal case, the accessibility relation over precisifications plays a bigger role. If we want to properly recover precisification talk, we need to be able to say what it means for one precisification to be admissible relative to another in a way that is \(L\) necessary. A standard definition of accessibility could be adopted: \(u\) is accessible to \(v\) iff everything determinate at \(v\) is true at \(u\) (in symbols \(\forall p(L(v \rightarrow p) \rightarrow L(u \rightarrow p))\). However, if we do not assume the product logic, it can be contingent whether one precisification is accessible to another – this is demonstrated by the one-way roundabout. It is doubtful that there will be a better way to define the accessibility relation in such a way that it is \(L\)-necessary what the structure of that relation looks like.

### 3.4 The relation between precision and necessity

In section [REF] we raised some troubles for the idea that we can take the notion of borderlineness and necessity as primitive, and define the notion of a propositions being precise from those notions. The primitive notion of my account of vagueness, to be spelled out in more detail in chapter 7, is precision, and not borderlineness. Here I want to spell out in abstract terms a theory which takes precision as a primitive notion, rather than borderlineness.

We made two stipulations about the abstract structure of precision:
**Boolean:** The set of precise propositions form a complete Boolean algebra, with the tautologous proposition being the weakest precise proposition.

**Atomicity:** Given any jointly consistent set of precise propositions, \( X \), there is always a consistent precise proposition that entails each member of \( X \).

The first order of business, of course, is to show that you can recover the notion of borderlineness and determinacy once you’ve taken vagueness and precision as your primitives; unless we could do this, we would be no better off than those who take borderlineness as primitive. Luckily the reverse definition is straightforward:

It’s determinate that \( p \) if and only if the strongest true precise proposition (i.e. the conjunction of all the precise truths) \( L \)-entails \( p \).

It’s borderline whether \( p \) iff the strongest true precise proposition is \( L \)-consistent with \( p \) and with \( \neg p \).

The definition of determinacy is equivalent to the following simpler formulation: a proposition is determinate if it is \( L \)-entailed by some precise truth. Thus these definitions can be formalised in the object language, given enough expressive resources: \( \Delta p \), for example, just becomes \( \exists q (q \land \#q \land L(q \rightarrow p)) \).

We can understand this as a general principle connecting preciseness to the determinacy operator. As a principle it is unexceptional. A supervaluationist who endorses the modal characterisation of precision will be able to interpret and derive the above principle – propositions can be identified with sets of world precisification pairs, entailment with set inclusion, and precision with the impossibility of borderliness.\(^{43}\) The view I prefer takes the notion of a precise proposition to be the theoretically interesting notion, thus the above allows us to connect this with more familiar talk of determinacy operators, from which we can say what it means for a proposition to be borderline.

The modal characterisation – that a proposition is precise if and only if it couldn’t have been borderline – I have argued, cannot serve as a reduction because there are imprecise propositions that couldn’t have been borderline. This is enough to show that it cannot be a definition of ‘precise’ and that the general biconditional claim is false. But even if the biconditional is false, it is hard to deny that one of its directions is true: the direction that asserts that no precise proposition could have been borderline. Even if we accept the precise/vague distinction as basic we still ought to have a theory of how precision and vagueness relate to the possibility of a borderline case. The most natural way to ensure the true direction of the modal characterisation of precision is to maintain that it cannot be contingent which propositions are precise:

**The necessity of the precise:** Necessarily, if \( p \) is precise then it’s necessary that \( p \) is precise.\(^{44}\)

This allows us to prove that no precise proposition is possibly borderline from our definition of ‘borderline’. Suppose that \( p \) is precise, and thus, by **Boolean**, that \( \neg p \) is precise. These propositions are therefore both necessarily precise. Thus necessarily if \( p \) is true it’s a precise truth, and thus it’s entailed by the conjunction of the precise truths which by our definition means it’s determinately true. By symmetrical reasoning, it’s necessary that if \( \neg p \) then it’s determinate that \( \neg p \) – thus if \( p \) is precise, it could not have been borderline whether \( p \).

\(^{43}\)For such a supervaluationist the conjunction of all precise truths at a world precisification pair, \( \langle w, v \rangle \), is just the set \( \{ \langle w, u \rangle \mid Svu \} \), and this is clearly a subset of a proposition \( p \) iff \( p \) is determinate at \( \langle w, v \rangle \).

\(^{44}\)Assuming that necessity obeys a modal logic of \( S5 \) one can prove from this principle that necessarily, if \( p \) is not precise then necessarily \( p \) is not precise. If we are not assuming \( S5 \) this consequence should be assumed to be part of what is meant by the claim that it’s necessary which propositions are precise.
3.4.1 A logic and semantics for precision

Neither the supervaluational semantics, nor the more general Kripke semantics for determinacy and necessity is capable of modelling the distinction between precision and vagueness. A more general model theory is therefore needed. The basic idea is that just as the Kripke and supervaluational semantics tell us what is determinate and borderline relative to each index, the more general semantics will tell us which propositions are precise and vague relative to each index.

The second important insight is captured by the principles BOOLEAN and ATOMICITY. The set of precise propositions relative to each index must form a complete atomic Boolean algebra. The best way to specify such an algebra is just by saying which propositions are the atoms of that algebra, i.e. by saying which propositions are the maximally strong precise propositions according to that index. Once these have been fixed, the precise propositions determined by arbitrary disjunctions of the maximally strong precise propositions along with the inconsistent proposition. A specification of the maximally strong precise propositions is simply a partition of the indices into cells: the intuition is that two indices in the same cell will agree about all precise matters, and differ from one another only about truth of vague propositions.

Putting this all together, we can think of a function, \( e \), mapping \( I \) into partitions over \( I \), \( e(i) \) then represents the partition of propositions that are maximally strong precise propositions according to \( i \), and disjunctions of these, the precise propositions according to \( i \). The notion of a pointed frame can be introduced in this setting in a way analogous to a pointed Kripke frame: it’s a quadruple \( \langle I, e, R, i \rangle \) where \( I \) is a set of indices, \( e \) is a function from \( I \) to partitions over \( I \), \( R \) a relation representing the modal accessibility relation, and \( i \) a member of \( I \).

In general we shall write \( e(i)(j) \) for the unique member of \( e(i) \) that \( j \) is a member of. Given our semantics for precision and our definition of determinacy from precision, it is possible to introduce the accessibility relation for the determinacy operator from the \( e \) function. Intuitively an index is accessible from \( i \) if it is consistent consistent with the maximally strong precise proposition containing \( i \), thus: \( S_{ij} \) if and only if \( j \in e(i)(i) \).

The language that this language models is just the result of adding two operators to the propositional calculus: the modal operator, \( \Box A \), and the precision operator, \( \#A \), read as ‘the proposition that \( A \) is precise’. As usual, given a frame, a model for this language can be gotten by assigning to each atomic sentence, \( \phi \), a set of indices, \( [\phi] \) and \( 2 \) and \( \# \) is a function from \( I \) to partitions over \( I \), \( R \) a relation representing the modal accessibility relation, and \( i \) a member of \( I \).

Intuitively \( \Box \) is accessible from \( i \) if it is consistent consistent with the maximally strong precise proposition containing \( i \), thus: \( S_{ij} \) if and only if \( j \in e(i)(i) \).

The language that this language models is just the result of adding two operators to the propositional calculus: the modal operator, \( \Box A \), and the precision operator, \( \#A \), read as ‘the proposition that \( A \) is precise’. As usual, given a frame, a model for this language can be gotten by assigning to each atomic sentence, \( \phi \), a set of indices, \( [\phi] \) and \( 2 \) and \( \# \) is a function from \( I \) to partitions over \( I \), \( R \) a relation representing the modal accessibility relation, and \( i \) a member of \( I \).

Intuitively \( \Box \) says that logical equivalence preserves precision, and the remaining axioms say that precision is preserved under the finitary Boolean operations (our language is not expressive enough to express the infinitary Boolean operations).

Say that a proposition is precise\(^2\) if it is both precise, and the proposition that it is precise is itself precise. Let me note that, although we can prove that a disjunction of precise things is always precise, we can’t prove in this theory that a disjunction of precise\(^2\)
things is precise\textsuperscript{2}.\textsuperscript{45} This is an interesting difference between the theory that I have outlined and modal characterisation of precision: if we assume the modal characterisation then the disjunction of two precise\textsuperscript{2} propositions is precise\textsuperscript{2} (this argument relies on standard principles governing $\Delta$ and $\Box$, but does not require the product logic). If one wanted to have this principle one would have to add it manually and impose corresponding constraints on the $e$ function. Similar principles governing precision at higher orders can also be formulated.

To cater for the modal operator we also need to add some axioms governing that; no most natural candidate being the modal logic $\mathbf{S5}$. However, it is natural to ask whether there are any principles governing the interaction of $\Box$ and $\sharp$ analogous to the product logic in the supervaluational case. The principle The necessity of the precise, mentioned in the previous section, can be formalised in the present system as follows:

$$\sharp A \rightarrow \Box \sharp A$$

and corresponds to one such constraint. This clearly corresponds to the condition:

If $Rij$ then $e(i) = e(j)$

Another interaction constraint, whose motivation I will turn to now, is that we shouldn’t allow two indices modally accessible to one another, $i$ and $j$, to appear in the same cell of either $e(i)$ or $e(j)$. Thus:

If $X \in e(i)$ then $|R(i) \cap X| \leq 1$.

To suppose otherwise would be to allow two distinct metaphysically possible states of affairs to agree about all precise matters: this would constitute a failure of the facts to supervene on the precise facts. It is to this principle I shall now turn.

### 3.4.2 The supervenience of the vague on the precise

According to a popular slogan ‘the vague supervenes on the precise’. Untangling this slogan, however, is a tricky business. Supervenience is a modal notion relating one set of true propositions to another – the $A$ facts supervene on the $B$ facts iff, necessarily, every $A$ fact is necessitated by the truth of a $B$ fact. Yet according to some theories of vagueness the objects of vagueness and precision are not facts or propositions but representations.

It is clear that one must take care in formulating this principle.\textsuperscript{46} Fortunately the details of its formulation do not matter as much for the specific applications of the idea that motivate the underlying thought. The claim we are interested in concerns whether the facts about baldness supervene on facts about hair number, distribution and so on, or whether the facts about tallness supervene on facts about exact numerical height, and other precise measurements, and so on – it does not matter to this question, at least, whether facts or propositions can correctly be thought to be the objects of vagueness and precision since I did not phrase the question in these terms.

What does it mean to accept or reject these supervenience theses? To deny the latter supervenience claim, for example, would be to countenance a metaphysical possibility in which someone has a certain precise height, and other precise physical measurements $m$ and is tall, and another metaphysical possibility in which someone has exactly the same measurements but is not tall. Many philosophers share the intuition that this these two scenarios are not both possible and are motivated to endorse a strong supervenience claim: that there couldn’t be two metaphysically possible scenarios $i$ and $j$ that agree that objects

\textsuperscript{45}Countermodel: $e(i) = \{\{i\}, \{j\}, \{k, l\}\}, e(j) = \{\{i\}, \{j\}, \{k, l\}\}, e(k) = \{\{i\}, \{j, k, l\}\}, e(l) = \{\{i, j, k\}, \{l\}\}$. At $i$ the following is true: $\sharp p, \sharp q, \sharp \sharp p, \sharp \sharp q, \sharp (p \lor q) \rightarrow \neg \sharp \sharp (p \lor q)$.

\textsuperscript{46}See chapter 9 of Williamson [131] for a good discussion from the perspective of a linguistic theorist. Here things are set up in terms of the supervenience of the vaguely describable facts on the precisely describable facts.
\( x \) and \( y \) have the same precise height and measurements, but differ in that according to \( i \) \( x \) is tall and according to \( j \) \( y \) is not tall. A weak supervenience claim would state instead that for any possible scenario \( i \) in which objects \( x \) and \( y \) have the same precise height and measurements is one in which \( x \) and \( y \) are both tall or both not tall. The weak supervenience claim is a straightforward consequence of the strong one.

It is important to distinguish supervenience, weak or strong, from a related epistemic claim. When Harry is borderline tall I can know his exact physical measurements are \( m \) but still fail to know whether he is tall or not. In this case it is epistemically possible for me that Harry is tall and has measurements \( m \), and also epistemically possible for me that he is not tall and has measurements \( m \). It would be a mistake to conclude that these epistemic possibilities are metaphysically possible. One of the two possibilities is metaphysically impossible, although it is indeterminate which. Thus strong supervenience is not undermined, even though a principle formally analogous to strong supervenience involving epistemic notions will presumably be false. It is interesting to note, however, that it is extremely natural to think that the epistemic analogue of weak supervenience is true. In general, we can know that if \( x \) and \( y \) have exact the same precise measurements \( m \) they are either both tall or both not tall (even if we don’t know which), due to the existence of a penumbral connection between the tallness of \( x \) and of \( y \).

The weak/strong distinction is only relevant when we are talking about one set of properties supervening on another. In the framework I have been developing it is possible to state, using an object language formula, what it means for the vague facts to supervene on the precise facts, making use now of propositional quantification. Indeed, we can state that all facts supervene on the precise facts. This is not stronger than the preceding claim since every fact is either precise or vague, and the precise facts trivially supervene on the precise facts. The principle in question is:

\[
\text{Supervenience} \text{ Necessarily for every truth there is a precise truth that necessitates that truth.}
\]

\[
\Box \forall p (p \rightarrow \exists q (\sharp q \land q \land \Box (q \rightarrow p)))
\]

Note that the supervenience of everything on the precise is entailed by theses such as physicalism, the thesis that everything supervenes on physical facts, in conjunction with the claim that physical propositions are precise. However, physicalism is strictly stronger than we need. Even if there were non-physical fundamental facts, perhaps there are ghosts whose positions do not supervene on the physical facts, it is still natural to think that whatever the fundamental facts upon which all truths supervene are, they must be precise facts. The truth of \text{Supervenience} depends only on the existence of some precise supervenience base, not the claim that any particular set of precise propositions is a supervenience base. A useful consequence of \text{Supervenience}, given \text{Boolean}, is that every proposition \( p \) is necessarily equivalent to a precise proposition, namely, the disjunction of precise propositions that necessitate \( p \) (in the formal language: \( \forall p \exists q (\sharp q \land (p \leftrightarrow q)) \)).

What further constraints does the validity of \text{Supervenience} place on the \( e \) function, and the modal accessibility relation, \( R \)? The relevant constraint, that ensures that \text{Supervenience} holds at an index \( i \), is that for every \( Z \in e(i) \), \( |R(i) \cap Z| \leq 1 \) (and we can say that supervenience is valid in a frame if this condition holds for every index.) In other words, every maximally strong consistent precise proposition either contains exactly one metaphysically possible index, or none. The question can be formulated and proved rigourously in terms of frames and validity, however the rationale behind this constraint is quite intuitive: if some maximally consistent precise proposition, \( Z \), was consistent with two distinct metaphysical possibilities, then there are two different ways things could have been with all the same precise facts obtaining, and this is tantamount to denying supervenience.

This constraint also has upshots for the interaction between the accessibility relations for the modal and determinacy operators. By our earlier condition we have that \( e(i) = e(j) \) whenever \( R_{ij} \), thus \( S(j) \in e(i) \) and so \( |S(j) \cap R(i)| \leq 1 \) whenever \( R_{ij} \). Since \( j \in S(j) \cap R(i) \)
this just becomes \( S(j) \cap R(i) = \{ j \} \). Furthermore, if we assume that \( R \) is an equivalence relation, we can infer that \( R(i) = R(j) \) whenever \( Rij \), so we can simplify this to constraint to \( S(i) \cap R(i) = \{ i \} \) for all \( i \). Putting this together with our earlier discussion we can derive the following principle:

If \( Rij, Rik, Sju \) and \( Sku \) then \( j = k \).

For suppose that \( Rij, Rik, Sju \) and \( Sku \). Since we required that it never be contingent what is precise, \( e(i) = e(j) = e(k) \), so either \( S(j) \cap S(k) = S(j) = S(k) \) or \( = \emptyset \). Since \( u \in S(j) \cap S(k), S(k) = S(j) \). But since \( R(i) \cap S(j) = \{ j \} \) and \( R(i) \cap S(k) = \{ k \} \) it follows that \( k = j \). Note that this condition rules out the possibility that some false but metaphysically possible index is not determinately false and other similar pathological scenarios, but it does not rule out the possibility that some false but metaphysically possible index is not determinately determinately false.

To rule out these apparently pathological situations we might appeal to something stronger than the supervenience of everything on the precise. As we have noted already, the property of being a precise proposition has borderline cases; we cannot therefore infer from the fact that \( p \) is precise that the proposition that \( p \) is precise is itself a precise proposition. There are of course precise propositions which are also such that the proposition that asserts their precision is also precise, but not all precise propositions are like this. Call propositions of this special kind ‘precise\(^2\): a proposition which is precise, and is such that the proposition that it is precise is itself precise. This idea iterates: we can say that a proposition is precise\(^{n+1}\) if and only if it is both precise and precise\(^n\). This notion is definable in the object language by the iterative definition \( \sharp p := \sharp p \land \sharp \sharp p \).\(^{47}\) Finally we may say something is precise\(^n\) if and only if it is precise\(^n\) for each natural number \( n \) (there is no analogue of this notion in the formal language we have been considering that permits only finite conjunctions.) These definitions can be continued into the transfinite, although for my discussion this will not be necessary. (Indeed, in chapter 8 I will argue (a) that there is no difference between being being precise at every finite ordinal (being precise\(^n\)) and being precise at all ordinals, finite or transfinite, and (b) that even the distinction between being precise at all orders and being imprecise at some order is an imprecise distinction; this discussion will have to wait.)

With these distinctions at hand we can then introduce stronger forms of supervenience as follows:

\[ \text{SUPREMEVENIENCE}^n: \] Necessarily for every truth there is a precise\(^n\) truth that necessitates that truth.

\[ \nabla p (p \rightarrow \exists q(\sharp^n q \land q \land \nabla (q \rightarrow p))) \]

Here \( \nabla \) can be substituted for any numeral (or for the \( \sharp^* \) symbol to get the strongest form of the principle.) It is worth noting that some of the considerations that motivated accepting the supervenience of everything on the precise extend straightforwardly to these strengthenings too. According to a natural thought, propositions about the physical – about the locations, velocities and fundamental properties of particles and fields and so on – are not only precise, but precise\(^n\) for any number \( n \). If this thought is correct then physicalists, for example, should accept the strengthened supervenience claims. Analogous cases can be made that other putative supervenience bases are precise\(^n\).

Note, however, that some philosophers argue that very little is precise\(^*\). Williamson, for example, suggests that not even propositions stateable in the language of fundamental physics are perfectly precise ([131] chapter 6) and Dorr [32] argues that even the logical truths fail to be precise\(^*\). Such philosophers will not endorse the strengthened supervenience claims. Perhaps there are precise supervenience bases, and precise\(^2\) supervenience bases,

\(^{47}\)Note that the first conjunct is required to ensure that every precise\(^n\) proposition is precise. The formula \( \sharp \sharp p \) is consistent with \( \neg \sharp p \) since it can be a completely precise fact that a proposition is not precise.
but as $n$ increases the number of supervenience bases that are precise$^a$ will decrease until, presumably, none are left. I shall consider these philosophers in chapter 8, however for now I shall remain neutral on this issue, and consequently neutral on the question of whether the strengthened supervenience theses are true. The truth of $\text{Supervenience}^n$ at an index $i$ requires the following condition:

$$\text{If } R_{ij}, R_{ik}, S^n_{ju} \text{ and } S^n_{ku} \text{ then } j = k.$$  

Where $S^n_{ij}$ holds iff there is a sequence of $n$ indices, $x_1,x_2...,x_n$, with $i = x_1$, $j = x_n$ and with $S_{x_kx_{k+1}}$.  

What are we to make of the supervaluationist who does not have a primitive distinction between the precise and imprecise at hand? For these philosophers the relevant principle is that everything supervenes on the facts that couldn’t have been borderline.

Necessarily every truth is necessitated by a truth that couldn’t have been borderline.

$$\Box \forall p(p \rightarrow \exists q(\Box(q \lor \Delta q) \land q \land \Box(q \rightarrow p)))$$

This principle is a consequence of $\text{Supervenience}$: since everything supervenes on the precise, and each precise fact couldn’t have been borderline, everything supervenes on facts that couldn’t have been borderline. Perhaps more surprisingly, the above principle true in a model, defining the $\Delta$ operator from the $\sharp$ operator, only if $\text{Supervenience}$ is, removing any logical difference between my general statement and the above one. Thus $\text{Supervenience}$ is a equivalent to a formula in the pure $\Box,\Delta$ language. The importance of this observation is that what seemed initially to be an principle only available to those who take precision as primitive can be stated equivalently by someone taking an determinacy operator as primitive instead.

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$^a$Unlike in the $n = 1$ case, it is unclear whether this condition is sufficient for the truth of $\text{Supervenience}^n$ at an index $i$ when $n \geq 2$. It is possible that this question is more tractable if we assume that the precise propositions form a complete Boolean algebra (a hypothesis that does not follow from the claim that the precise propositions form a complete Boolean algebra [see section [REF].])

$^b$Proof sketch: for each truth $p$, at $i$, there is a truth $q$, would couldn’t have been borderline at $i$. To get a precise truth that necessitates $p$ simply pick the proposition $q' = \bigcup \{S(j) \cap q \mid R_{ij}\}$. 

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Chapter 4

Vagueness and Evidence

In chapter 2 we argued against linguistic accounts of vagueness on the grounds that if it is true that we cannot know whether Harry is bald, then this is the sort of thing that would be true no matter how we happened to use the sentence ‘Harry is bald’. Whether a belief that Harry is bald constitutes knowledge or not, I suggested, is just not sensitive to facts about your linguistic environment.

Whether or not vagueness is linguistic, there is a quite general puzzle as to how we acquire vague beliefs at all, and as to what role, if any, language plays in the acquisition of these beliefs. Speaking on their behalf, how might a linguistic theorist respond to this challenge? A natural and reasonably common thought go something like this: in order to have a belief that Harry is bald you need to be able to internally token a certain kind of sentence in your mental language; one that is synonymous with, or has the same conceptual role as the English sentence ‘Harry is bald’.

Could a mentalese sentence have this kind of role without the thinker standing in some kind of important relation to a public language? If vagueness is a linguistic phenomenon it is hard to see how this story could work unless the thinker did stand in such a relation. According to the analysis of vagueness in terms of semantic indecision, for example, the feature that distinguishes sentences like ‘Harry is bald’ from sentences like ‘Harry has at most \( n \) hairs’ is to be spelled out in terms of the different ways in which these sentences are used to coordinate beliefs between members of a linguistic community. For this talk of coordination to even make much sense we must be talking about a community that contain at least two members.\(^1\) Thus whether one has vague beliefs at all, according to this proposal, is dependent on one speaking a public language. Indeed this kind upshot is already partially anticipated by conclusions of Burge [19], that purportedly show that whether one has beliefs involving arthritis and other ‘deferential’ concepts can be sensitive to your linguistic environment. While these arguments have enjoyed a moderate amount of acceptance among philosophers, the present proposal is far more radical and far reaching. Even semantic externalists wouldn’t argue that it is impossible to have beliefs about arthritis without speaking a public language. More importantly, pretty much all of our beliefs are vague: the exceptions seem to be beliefs about logic and mathematics, and possibly fundamental physics as well. An argument that purports to show that one cannot have vague beliefs unless one speaks a public language would threaten to show that we could have barely any beliefs at all unless we spoke a public language.

There are plenty of other things to say about this proposal. My purpose in this chapter is rather to highlight other mechanisms by which we acquire vague beliefs which the linguistic theorist cannot account for. I shall argue that most of our beliefs do not require any familiarity with a public language; most of our vague beliefs are acquired via our sensory faculties, vision, smell, proprioception, or other non-linguistic faculties such as memory.

\(^{1}\)See the discussion of this point in Dorr [31].
Sometimes public language sentences express vague propositions and these sentences can be used to communicate vague beliefs, but the class of vague propositions which satisfy the evidential profiles outlined vastly outstrips the class of propositions expressed in any given language and the way in which language features in our acquisition of vague beliefs is minimal.

In section 4.1 I introduce the idea of one’s evidence being ‘inexact’ and argue that there are important connections between these cases and cases in which one’s evidence consists of vague propositions. More specifically, I argue that updating on vague evidence is a particular instance of a more general account of updating on inexact evidence: Jeffrey updating relative to a partition – in this case relative to a set of ‘precisifications’. This observation leads to a couple of insight into the evidential relation between vague propositions and precise propositions that will form the basis for the account of vagueness I am advancing in this book. One of these, which we will develop in chapter 5, is that your beliefs in the vague propositions are completely fixed (in a sense to be spelled out) by your beliefs in the precise. Thus one’s beliefs in a particular vague proposition can be uniquely determined by your beliefs in the precise and a particular evidential relationship to the precise propositions. The way in which credences in that proposition depend on your credences in the precise propositions we shall call that vague proposition’s ‘evidential role’. The second insight, which I defend in this chapter, is a principle of plenitude: that for any possible evidential role in thought there is some vague proposition which has that role.

4.1 Inexact evidence

Suppose you are looking out of your window at a tree in the distance. In doing so you obtain some knowledge about the tree. However, your knowledge is not exact; for example, while you now know that the tree is larger than 10cm and less than 1000cm, based on what you can see the exact height of the tree in cm is still unknown to you.

In [130] Timothy Williamson introduced the term ‘inexact knowledge’ for the kind of epistemic state you would be in situations like the one above. A parallel distinction arises with respect to your evidence in such situations. Given that you have only seen the tree from a distance, and your eyesight is not perfect, your evidence includes some propositions – that there is a tree, that it’s larger than 10cm and smaller than 1000cm – but does not include the proposition stating the trees exact height. We may call your evidence in such cases inexact evidence.

Being inexact is not the same as being inaccurate; if the tree appeared to be less than 500cm when it was in fact greater than 600cm then my evidence, or my apparent evidence, would be inaccurate, but in the case described in the opening paragraph my beliefs are not inaccurate. All of my evidence, or apparent evidence, is accurate. On the other hand, if I went out and measured the tree, the relevant source of inexactness in my evidence about the tree’s height would eliminated. But the phenomenon at hand is not ignorance or lack of evidence about the tree’s exact height. If I had never seen the tree, but had been informed by a highly reliable person that the tree was less than 600cm then my evidence would be exact, even though I still do not know the height of the tree. Nor is the relevant sense of exactness specificity: if I have been told that the tree is between 400cm and 500cm then my evidence is more specific than it would have been if I had only been told that the tree was between 300cm and 600cm, but the latter case is not closer in kind to the state of evidence you would find in the opening example.

I hope it is clear that the evidential status of someone who has seen a tree from a distance is relevantly different from any of the preceding examples. While I cannot give an uncontroversial definition of what must be involved in cases where one’s evidence is inexact, I hope the preceding examples have elucidated it enough to provide a good enough handle on the kind of situation I am interested in to be worth discussing further.

Evidence obtained via imperfect sensory faculties are paradigm examples of inexact
evidence. However, it is not clear that it is always the case that when your evidence is inexact, some sensory faculty is at fault. Having taken a cursory look at my bookcase, I gain inexact evidence about the number of books I own; I have a general feel for how many books there are but I do not know exactly how many books there are without having counted. However it is not completely obvious that my evidence would be significantly better if I had perfect vision. It seems more natural to say that it was not my visual experience, but rather my ability to process it, which was to blame.

I take it that most ways of obtaining evidence leave room for the possibility that evidence obtained in that way is inexact. I also take it that most of our evidence is inexact. The main thesis of this chapter is the claim that inexact evidence is vague evidence; that when we find ourselves with inexact evidence the strongest proposition we ought to be certain of on the basis of this evidence is a vague proposition. This partly vindicates the claim that we do not need to be acquainted with a public language to have vague beliefs. Furthermore, since almost all of our evidence is inexact, it follows that vague propositions occupy a very distinctive evidential role in thought. They are usually the best pieces of information we have when we learn from experience, they are what we usually reason from and to, and they are the objects of our desires upon which we act. If it is the evidential role of a proposition that is partly constitutive of what it is to be that proposition, then our thesis also gives us some insight into what vagueness and vague propositions are.

The claim that inexact evidence is vague evidence requires some refining however. If I have examined a man’s head carefully and have determined that he has no hairs at all, then I have evidence that he is bald. In such a scenario my evidence about the man’s hair number is exact, yet my evidence includes the vague proposition that the man is bald. Having evidence for a vague proposition is thus not sufficient for being in the kind of circumstance characteristic of inexact evidence. However, the proposition that the man is bald is not the strongest proposition I have evidence for – the strongest proposition is the proposition that the man has no hairs. While this proposition entails that the man is bald, it is not equivalent to it: a man with one short hair is bald, so one can be bald without having no hairs at all. The claim I am interested in is rather the claim:

**Vague Evidence:** When your evidence about a subject matter is inexact, your total evidence about that subject matter is a vague proposition.

The basic argument for this claim (very roughly) is that when our evidence about, say, the height of a tree is inexact, the distribution of probabilities over possible tree heights apparently forms a smooth curve. On the other hand, it is not possible to achieve this smooth curve by conditioning on a precise proposition that’s about the tree (and not about, say, our experiences); therefore our total evidence cannot consist only of precise propositions. The primary goal of the next section will be to finesse this argument.

Before I move on, however, it should be noted that there are a couple of questions that I shall not remain neutral on and require some further remarks. The first of these is the thesis of probabilism: the view that ways of measuring the degree to which your evidence supports hypotheses – the credences one epistemically ought to have – are governed by the probability axioms. This is controversial in the present context because some philosophers (see, especially, Field [44] and Schiffer [110]) have argued that to include vague propositions within the remit of such theories would require relaxing the ordinary probabilistic axioms. Although I’ll be assuming probabilism in this chapter, I shall return to that issue and give it a proper defence in the next.

The second point requires a little more discussion. One might object that in the cases described my evidence is not propositional at all, but rather consists of an imprecise visual experience, a hazy memory, or some other non-propositional object. The thought that there is a proposition that summarizes everything that is learnt is what Jeffrey calls the

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2One might argue that certain a priori methods, such as mathematical proof, never leave one with inexact evidence. I would dispute this, but we can set this case aside for the purposes of this chapter.
‘empiricist myth of the sensuously given data proposition’ ([REF] The Art of Probability Judgment, p3.) It is, in fact, possible to make Vague Evidence nominally consistent with this view by replacing talk of evidence with talk of what your evidence fully supports: even if your evidence consists in visual experiences, memories, or what have you, what that evidence supports, presumably, is always a proposition. Thus Vague Evidence can be thought of as just the claim that when your evidence is inexact the strongest proposition fully supported by your evidence is vague.

Although this formulation is neutral concerning whether evidence is propositional, there is a more substantial disagreement around the corner: many philosophers who reject propositional evidence also maintain that very little is maximally supported by your evidence, and is therefore not in a position to accept even the revised formulation (see, for example, Jeffrey [67]).\(^3\) This is related to a second, but distinct objection often leveled at views in which evidence is propositional. According to these views any proposition entailed by your evidence will be fully supported by your evidence and will be such that one ought to assign it maximal credence. This has surprising consequences about betting behaviour, assuming a standard connection between credences and betting: there will be contingent propositions which you should bet your entire life on for a penny. Vague Evidence clearly does not speak to this objection, and it would take us too far afield to respond to it here. However it is important to be clear about this odd consequence of accepting the view that evidence is propositional, and that it is separate from the other reasons that motivate one to deny that evidence is propositional.

At any rate, Vague Evidence is supposed to be understood as an answer to Jeffrey’s first challenge to produce a proposition that summarizes all and only the things learnt after obtaining inexact evidence. It is, however, important to separate this challenge from the more demanding one of producing a sentence that summarises all and only the facts learnt. It is rarely ever possible to articulate what you’ve learnt. Belief, on one natural picture, is a process by which we rule out certain possibilities, and this ruling out process is characterised by its connection with rational action: it might not be a conscious thought, or the explicit articulation of a sentence in a mental language. It might be that a belief that \(P\) coincides with the tokening of some sentence like entity in some animals, such as humans; but the case of belief in animals surely demonstrates that the requirement that we be able to articulate our beliefs in some language (mental or otherwise) is too strong.\(^4\)

There are a great many questions about the nature of evidence that I have left open: How exactly do we obtain evidence? What kinds of propositions are part of our evidence? and so on. For the most part, my discussion can remain neutral on these questions. However to make the discussion more concrete it will help to have some particular answers to these questions on the table. Following Williamson, [133], there are a number of natural propositional attitudes that seem to always confer the status of evidence to things they are held toward. Here are a few:

1. \(S\) saw that \(p\)
2. \(S\) remembered that \(p\)
3. \(S\) could hear that \(p\)
4. \(S\) could feel that \(p\)
5. \(S\) could see that \(p\)

\(^3\)Note, however, that the revised formulation of Vague Evidence is compatible with a Jeffrey style picture about how our credences in the precise are distributed: it could be that one can become fully confident in a vague proposition whilst remaining uncertain about almost all precise matters.

\(^4\)Some authors conflate these challenges – Richard Bradley, for example, makes this stronger demand in [REF] ‘Radical Probabilism and...’'. One only needs to meet the weaker demand to respond to Jeffrey’s objection.
These are all paradigm cases of what I shall call ‘evidential attitudes’: an attitude one cannot have towards \( p \) without thereby having evidence that \( p \).\(^5\) In cases 3., 4. and 5. the auxiliary ‘could’ usually marks that the verb is being understood perceptually. Furthermore, evidential attitudes are characteristically factive, whereas ‘heard that’ and ‘felt that’, without the modal, are not factive or evidential. ‘Jane heard that Hector is getting fired, although she didn’t have any reason to believe that he was getting fired’ seems to be a fine thing to say if, for example, Jane’s source was known to be completely unreliable, whereas ‘Jane could hear that Hector was getting fired, although she didn’t have any reason to believe that he was getting fired’ seems to be harder to maintain. The latter sentence implies that the actual firing is directly audible to Jane, whereas the former sentence implies that she has heard second hand that Hector is being fired, but hearing in this way may or may not have any evidential import. On the other hand, both 1. and 5. are factive and evidential. The auxiliary in 5. can usually be inserted to force the reading where the evidence at hand is perceptual. This is not always the case, however, as seen by the sentences ‘Hector saw that the theorem was true’ and ‘Hector could see that the theorem was true’ – both seem to be ok things to say, yet in neither case is the kind of seeing a perceptual one. The fact remains, however, that whether perceptual or not, one cannot see that \( p \) without \( p \) becoming part of your evidence.

It should be pointed out that one does not need to be particularly linguistically competent to have any of these attitudes towards a proposition. One could hear (see, remember, feel) that \( p \) without speaking any particular public language or standing in any relation to a representation – one only needs the relevant auditory (visual, recollection, sensory) capacities. It is perhaps slightly more controversial to say that one does not need a private language, or to stand in any kind of relation to a mental representation, in order to hear (see, remember, feel) that \( p \). Although I would be willing to make this further claim, my discussion will not rely on this assumption.

We do not always have neat ways of expressing evidential attitudes in natural language. We generally have lots of propositional evidence about whether we are cold or thirsty, where our limbs are, whether we are moving, which way is up, and so on and so forth, which we have by our capacity for proprioception. In English, at least, we do not have any simple verbs for describing these states.

It should be clear that most evidence obtained in the ways detailed above is inexact. In the clearest cases the evidence is inexact because the subject’s eyesight is poor, or she only caught a short glimpse of something, or it was in the periphery of her visual field, or her memories had faded, and so on. But even when our sensory organs are all in good condition our evidence will be inexact. The amount of computing power we would need to calculate the exact path a tennis ball would follow on the information that it has been hit a certain way is typically too high for most humans (even more so when the examples become more complicated, such as spilling a bag of rice.) However we are generally able to do rough and ready estimations, without any conscious calculation, which give us a rough idea of where the ball will land. My best evidence is not that the ball will land exactly at location \( l \) or that it will land within the circular region \( r \) of diameter 3.8m or whatever, it will rather be inexact in the way characteristic of the examples we have been discussing. Most of our conscious and unconscious decisions are based on inexact information of this nature and understanding this information is crucial if we are to apply these epistemological considerations beyond the contrived decision problems, typical in formal epistemology, to commonplace decisions such as whether to hit a tennis ball when it looks like it might be going out.

Let’s now focus on a particular example. Suppose that, after seeing Harry for the first

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\(^5\)Here I borrow heavily from the discussion of ‘factive mental state operators’ in Williamson’s [133]. Most of what I say in what follows is compatible with Williamson’s view that evidence is just knowledge; however it is also compatible with the weaker theory that evidence is just the disjunction of a more limited class of non-inferential factive mental state operators. For reasons to prefer this view over Williamson’s see Bacon [6].
time, I gain some evidence about how much hair he has. I can see that he has some hair, but as usual my evidence is inexact. I claim that

\[ \text{For some, but not all } n, \text{ can I see that Harry has less than } n \text{ hairs} \] (4.1)

As with all the the evidential attitudes we have discussed, ‘sees that’ is a propositional attitude: syntactically \( p \) in ‘S sees that \( p \)' can be substituted for any grammatical sentence, and semantically we may view ‘sees’ as expressing a relation between a subject and a proposition. One slightly distracting feature of the above claim is that these perceptual reports often carry certain conversational implicatures: if I say that I can see that Harry has at most a million hairs it suggests that it’s not the case that I can see that he has at most one. It is important, however, to resist the temptation to think the latter claim is entailed by the former.

We may thus ask: what is the strongest proposition I stand in the seeing relation to in this situation?\(^6\) One may similarly ask what the strongest proposition that my evidence fully supports in this situation is. Assuming, for simplicity, that the only evidence I have in this case is my perceptual evidence we should expect the answers to both these questions to be the same.

I shall consider two answers to this question

1. For some \( n \), the strongest proposition about Harry’s head that my evidence fully supports is the proposition that Harry has at most \( n \) hairs.

2. The strongest proposition about Harry’s head that my evidence fully supports is a vague proposition about the number of hairs Harry has.

Before we tackle this question a few clarifications are in order. By the strongest proposition about Harry’s head that my evidence supports I mean a proposition about Harry’s head which my evidence fully supports and which entails all other propositions about Harry’s head that my evidence fully supports. Assuming that one’s evidence is closed under logical consequence, one can show that there always is a strongest proposition that is part of one’s evidence: the conjunction of all the propositions supported by your evidence. Similarly, (assuming that the propositions about Harry’s head are closed under conjunctions) there will always be a strongest proposition about Harry’s head that my evidence supports.

The notion of entailment between propositions cannot be taken to be strict implication if we are to include vague propositions in this algebra. I shall assume, as per usual, that whether someone is bald or not supervenes on the precise facts about hair number, distribution and colour. To spare myself a few words, I shall simplify things further by assuming that it whether someone is bald only depends on hair number. Under our simplification, it follows that for some \( n \) it is necessary that Harry is bald if and only if Harry has less than \( n \) hairs – although it is vague for which \( n \) this holds. This notion of entailment is useless for epistemic matters such as evaluating your evidence, for it is exactly these necessities we cannot know because of vagueness. We shall need to avail ourselves of a broader notion of entailment in which the claim that \( p \) entails \( q \) implies not only that \( p \) strictly implies \( q \), but that it determinately implies \( q \), and moreover, that the result of prefixing any string of \( \Delta \)s and \( \Box \)s to the material conditional \( p \to q \) is true.\(^7\)

It seems clear that the propositions stating that Harry has at most \( n \) hairs, where \( n \) ranges over numbers, are linearly ordered by entailment: that Harry has at most \( n \) hairs entails that he has at most \( m \) hairs whenever \( m \geq n \). Where does the proposition that Harry is bald fall in this ordering? The answer, of course, is that doesn’t. It appears below (i.e. it entails) some of the precise propositions, including the proposition that Harry has at

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\(^6\)In general it is odd to say that we see, if we are rational, every logical consequence of what we see. But at least in this case it is natural to think that I’ve seen most of the relevant propositions about Harry’s hair number that are logical consequences of the propositions I’ve seen.

\(^7\)This notion of entailment was spelt out in chapter 3.
Figure 4.1: The proposition that Harry is bald neither entails nor is entailed by the proposition that Harry has $N$ hairs

most $10^{10}$ hairs, and appears above (is entailed by) others, including the proposition that Harry has at most 0 hairs. But if it’s borderline whether someone with $N$ hairs is bald, and we know that Harry has exactly $N$ hairs we cannot know whether Harry is bald. It’s not determinately false (and thus epistemically possible) that Harry is not bald and has $N$ hairs, so the proposition that Harry has at most $N$ hairs does not entail the proposition that Harry is bald. Conversely, if I know Harry has $N + 1$ hairs I don’t know whether he’s bald for that would also be to know something borderline. So it would be not determinately false (and hence epistemically possible) that Harry is bald and has $N + 1$ hairs, establishing that the proposition that Harry is bald does not entail that Harry has at most $N$ hairs.

Despite the fact that certain vague propositions are independent of the precise propositions entailment-wise, they are not probabilistically independent of one another. Suppose that the probabilities representing the degree to which my evidence supports various hypotheses about hair number are initially uniformly distributed over the possible numbers
of hair Harry might have.\footnote{Technically there are infinitely many possible hair numbers Harry could have. So we don’t have to worry about infinities I shall just stipulate that we know that he has less than a billion hairs, but our credences are otherwise uniformly distributed over the remaining possibilities. This is an idealisation – presumably the probabilities will trail off slowly – however the substance of my discussion won’t be affected by removing this idealisation.} What are the probabilities of these heights given that Harry is bald? The precise propositions entailed by the proposition that Harry is bald now have probability 1 given that he’s bald. However, the probabilities conditional on Harry’s baldness will presumably drop smoothly: for each $n$, if the probability that Harry has exactly $n$ hairs is $x$ then the probability that Harry has exactly $n + 1$ hairs will be just below (or the same as $x$ once it levels out). Thus if $N$ is the largest number such that the proposition that Harry is bald entails that Harry has at most $N$ hairs then, although the proposition that Harry has at most $N + 1$ hairs neither entails nor is entailed by the proposition that Harry is bald, it is probabilistically supported by it, in the sense that its relatively low initial probability will increase so that it is almost 1 on the supposition that Harry is bald. If I learn that Harry is bald the probability in some of the propositions about hair number not entailed by the proposition that Harry is bald will increase, and the probability of others will decrease.

Formally, $P$ is probabilistically independent of $Q$ for $S$ if $Pr(P | Q \land E) = Pr(P | E)$ where $Pr$ is a rational ur-prior\footnote{A credence it would be rational to have if you had no evidence at all.} and $Pr(\cdot | E)$ represents the degree to which something is supported by $S$’s total evidence $E$ – her credence’s if she is rational. Moreover, $Q$ provides evidential support for $P$ relative to background evidence $E$ iff $Pr(P | Q \land E) > Pr(P | E)$. The foregoing demonstrates that the vague and precise propositions are not probabilistically independent for a rational agent with my evidence, and that being bald provides evidential support for certain hypotheses about hair number. In my view this kind of probabilistic dependence is no accident: if you were to take any rational ur-prior and condition it on a maximally strong precise proposition there is a particular credence it should assign to the proposition that Harry is bald – this will be a small number if you condition on a proposition that entails that Harry has a large number of hairs, and a higher number if you pick maximally strong precise propositions entailing he has lower numbers of hairs. To adopt a prior probability function that violates this constraint is to exhibit a kind of conceptual incoherence akin to believing that there are married bachelors or, perhaps more pertinently, bald people with millions of hairs. This fact explains why your credences about Harry’s hair number and baldness are not probabilistically independent; if the only evidence you have is that described above, every rational ur-prior should agree that on that evidence hypotheses about baldness provide certain levels of evidential support for hypotheses about hair number and vice versa. This can be contrasted with a certain kind of Bayesian permissivism in which all probability functions represent a possible coherent assignment of prior degrees of belief. For this Bayesian there will be rational ur-priors that violate the probabilistic dependence constraints between the vague and the precise. So the thesis I have suggested is inconsistent with this kind of permissivism.

If one were to begin with evenly distributed credences and were to update on the proposition that Harry has less than $n$ hairs, one would have a sharp probability function: $C(\text{H}k) = \frac{1}{n}$ for $k \leq n$ and $C(\text{H}k) = 0$ otherwise, where $\text{H}k$ is the proposition that Harry has $k$ hairs. It would look something like the probability function depicted on the right in figure 4.2, rather than the smooth one on the left. The smooth curve is intuitively what you’d expect to be the right one, and, I would conjecture, is closer to the curve that corresponds to the credences people usually adopt after having learnt from inexact experience.

Sometimes we can tell whether or not someone has some hair from a cursory glance. In such a case the above graphs would need to be modified. Let us assume, however, that Harry is very bald, so that I can’t rule out that he has no hairs at all and that my prior
Figure 4.2: A smooth curve and a sharp curve of \( n \) against credence that Harry has \( n \) hairs.

credences are uniform.\(^\text{10}\) In such a case, one would expect one’s posterior credences, after undergoing the inexact experience, to be closer to the smooth graph in figure 4.2, as opposed to the sharp one.

One way to see whether someone’s credences correspond to the blue or red curve is to look at that person’s betting behaviour. If, for each \( k \leq n \), she would be willing to accept a bet that cost £1 and payed out £\( n \) if Harry had at most \( k \) hairs, and she would not accept any bets of this form if \( k > n \), then the agent’s credences are best described as conforming to the red graph in figure 4.2. The agents betting dispositions change suddenly; she is willing to pay anything up to a dollar for a bet that Harry has exactly \( n \) hairs and pays out \( n \) dollars if you win, but is unwilling to pay anything for a bet that Harry has at most \( n + 1 \) hairs. On the other hand, if her credence corresponded to the blue curve in figure 4.2, then the amount she would be willing to pay for a bet that payed out a constant amount if Harry had at most \( k \) hairs would decrease continuously as \( k \) increases. If my visual experience of Harry’s head is inexact then it should clear that I would become less and less certain that Harry had at most \( k \) hairs as \( k \) increases, which is observably born out in the kinds of bets I would accept.

In these kinds of situations described it is natural to think that betting behaviour conforms to the latter kind of credence distribution. Whether, in ordinary situations, the agents credence is identical to the the credence your evidence recommends (i.e. the evidential probability) is another matter altogether. However, the fact that we do appear to have smooth credences instead of sharp credences, and we are not criticised for doing so, is quite suggestive.

If this is correct there is a striking resemblance between the effect of undergoing an inexact experience and the effect of updating on a vague proposition. Recall that the hypotheses concerning Harry’s hair number and baldness are probabilistically dependent on one another in a systematic way. Presumably the picture will look something like the following. Conditional on each precise hypothesis about Harry’s hair number, the proposition that Harry is bald will vary: conditional on Harry having small numbers of hair the probability of Harry being bald peaks; conditional on having numbers of hair in the borderline region, the probability will be middling between the peak and 0, getting smaller as the numbers increase; and finally conditional on Harry having very large amounts of hair, the probability that he’s bald eventually reaches 0. Since the probability that Harry has various hair numbers conditional on being bald is proportional to the probability of his being bald conditional on those numbers, the graph after conditioning on the proposition that Harry is bald will look like the curve graphed in figure 4.2.

\subsection*{4.2 Updating on vague evidence}

How is the evidential probability that Harry has \( n \) hairs determined from my evidence when I am in such and such a state? In this section I shall argue that extant approaches to updating

\(^{10}\)Have uniform priors is also an unrealistic assumption – I presumably assign lower credences to extremely high numbers. It would be possible, but more complicated, to run the example without the assumption of uniform priors.
do not adequately answer this question. A now standard approach to this question is that one should do something called ‘Jeffrey conditioning’ when one receives inexact evidence. This approach is actually significantly underspecified, although it is often understood as a way of changing credences in response to some non-propositional evidence, such as a visual experience. I shall argue that on this strong interpretation Jeffrey conditioning does not provide a full answer to this question. I shall also argue that the ordinary account of conditioning on a precise piece of propositional evidence does not provide a plausible answer to this question.

### 4.2.1 Jeffrey Conditioning

Let us begin with the first approach. A purportedly distinct way of revising your credences in response to inexact evidence was proposed by Richard Jeffrey in [67]. We shall see, however, that Jeffrey’s generalisation of conditionalisation does not tell us how we ought to respond to inexact evidence. We shall see, in fact, that our own theory of inexact evidence is not an alternative to Jeffrey’s theory but a supplementation of it.

In the simplest case in which one’s credence in a particular proposition is raised by some experience, Jeffrey’s theory states that your posterior credence, $C r'\left(p\right)$, and your prior credence $C r\left(p\right)$ must be related in the following way:

$$C r'\left(p\right) = C r'\left(e\right)C r\left(p \mid e\right) + C r'\left(\neg e\right)C r\left(p \mid \neg e\right)$$  \hspace{1cm} (4.2)

We can consider $e$ to be some piece of ‘evidence’ that you have become less than certain in. Jeffrey conditionalisation (henceforth, JC) thus generalises the relation that holds between your prior and posterior credences that obtains when one updates by conditionalising on $e$ (one can see this by setting $C r'\left(e\right)$ to 1.) This generalisation allows one to account for changes in credences arising from what Jeffrey calls ‘uncertain evidence’: changes in an agent’s evidential state which, according to Jeffrey, does not involve the agent becoming certain in any proposition. Such cases might perhaps include learning from inexact evidence.

In the simplest case, $e$ may be some proposition whose rational credence is determined by your new experience and background beliefs. However, it might be that your credences over a larger partition, $\{e_1, \ldots, e_n\}$, are determined in this way. In which case we may generalise 4.2 to:

$$C r'\left(p\right) = \sum_{i \leq n} C r'\left(e_i\right)C r\left(p \mid e_i\right)$$  \hspace{1cm} (4.3)

If $e_i$ is the proposition that Harry has $i$ hairs then it is trivial to find an instance of Jeffrey’s equation 4.3 which matches the smooth curve in figure 4.2, provided none of the $e_i$ have a prior credence of 0 (one simply sets the coefficients $C r'\left(e_i\right)$ to the values of the graph.) We must be clear from the start which problem Jeffrey’s extension of conditionalisation is supposed to solve. In Jeffrey’s own words

The problem is this. Given that a passage of experience has led the agent to change his degrees of beliefs in certain propositions $B_1, B_2, \ldots, B_n$ from their original values, $C r\left(e_1\right), C r\left(e_2\right), \ldots, C r\left(e_n\right)$ to new values, $C r'\left(e_1\right), C r'\left(e_2\right), \ldots, C r'\left(e_n\right)$ how should these changes be propagated over the rest of the structure of his beliefs? If the original probability measure was $C r$, and the new one is $C r'$, and if $p$ is a proposition in the agents preference ranking but is not one of the $n$ propositions whose probabilities were directly affected by the passage of experience, how should $C r'\left(p\right)$ be determined? [Notation has been modified to fit current conventions.] [67]

It should be noted that formally any proposition can be substituted for $e$ in the first equation, and any partition in the second. For Jeffrey propositions $e_1, \ldots, e_n$ are those
propositions ‘whose probabilities [are] directly affected by the passage of experience.’ What might this mean? A natural thought is that the propositions, \( e_1, \ldots, e_n \), are those propositions whose credences are rationally determined by your background credences and the fact that you’ve had a certain experience. However, if any partition of propositions posterior credence is rationally determined by your new experience and your prior credences, all partitions of propositions are. This is, after all, what JC itself says: once your posteriors over a given partition are fixed, the rest of your credence ought to be determined from this by Jeffrey conditionalisation.

If we assume that evidence is propositional, and that we update by conditioning on our evidence then it is possible to state what contribution a piece of evidence should make to two peoples doxastic state in a way that depends only on the kinds of prior beliefs they have in propositions that entail their evidence. Within Jeffrey’s framework, in which it is often assumed that evidence is non-propositional, it is not so easy to separate out the contribution a piece of evidence makes from the contribution your prior credences make. Given a non-propositional piece of evidence and some prior credences, it would be nice to know what your posterior credences should look like. If two people with identical priors receive the same evidence intuitively they should have the same resultant credences, and it would be nice to know how that resultant credence is determined from the evidence and prior.\(^{11}\) Unfortunately, for all Jeffrey conditioning says, two people with identical priors and identical evidence could respond in radically different ways. Jeffrey conditioning is in fact extremely permissive. To illustrate: suppose that prior to receiving visual experience \( v \), resulting from observing a green cloth by candlelight, the proposition that the cloth is green and the proposition that the number of stars is odd are probabilistically independent. Suppose also that after experiencing \( v \), I become certain that the number of stars is odd. It seems that this kind of transition of credences simply isn’t supported by the kind of evidence that I’ve received.\(^{12}\) Yet according to Jeffrey conditioning this is a perfectly fine change of credences, since I can get it by choosing the right partition and coefficients. I shall refer to a transition between two credences that is permitted by some possible evidence a ‘realizable’ Jeffrey conditioning. The case just discussed is an unrealizable Jeffrey transition.

What one would want instead is some correlation between experiences and propositions telling you how much those experiences support those propositions, from which one can determine, given someone’s background credences, how much posterior credence they should assign to that proposition — JC alone does not do this. This is called the ‘input problem’, and as yet there is no particularly satisfactory answer to it (see the exchange between Field \([42]\) and Garber \([56]\) for a failed solution to the input problem.) Without a solution to the input problem, JC has little content. It just states a relation that must hold between your prior and posterior credences. Furthermore, that relation is very liberal: under simplifying assumptions, if we choose a fine enough partition it can be shown that any transition between two probabilities that preserves certainty is permitted by the constraints of JC. This is perhaps too liberal; one might think that there are some changes of credential state consistent with the JC constraint that would not be permitted by any possible evidence you could obtain.\(^{13}\) Unlike conditionalisation, JC should not be read as telling you how to respond to your evidence, but should rather be read as telling you how, once you have changed your credence in \( e \) to some degree, the rest of your credences redistribute to acco-

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\(^{11}\) If one is taking evidence to be non-propositional then there is an issue as to what counts as two people having the ‘same’ evidence. A common, albeit deniable picture is that people receiving phenomenally identical visual experiences should respond in the same way.

\(^{12}\) It should be noted that as far as Jeffrey himself was concerned, this is a perfectly rational transition: how your credences respond to your experiences is a purely causal involuntary matter, about which rationality has nothing to say.

\(^{13}\) One reason to think this is that that the order in which you receive the evidence between two times shouldn’t matter to your credence at the latter time. But if any Jeffrey conditioning counts as a rational transition then there are pairs of Jeffrey conditions which result in different posterior depending on the order in which you update on them.
moderate this change.\textsuperscript{14} As a theory of inexact evidence, \textit{JC} remains silent. I may increase my credence in \( p \) when I have no new evidence for \( p \), or even if all the evidence is against \( p \), and still comply with \textit{JC} by redistributing my credences correctly. Similarly, it is in full compliance with \textit{JC} to make no change at all to my credences in \( p \) even when there is strong evidence for \( p \). We want to know: when \textit{should} I change my credences and how. When I see a tree in the distance, for all \textit{JC} says, I may just keep my credences about its height the same even though I have new information.

Jeffrey’s theory is thus not really a theory about how you ought to revise your credences in response to inexact evidence after all, nor is it a theory of what inexact evidence is. In order to have a satisfactory theory of inexact evidence, \textit{JC} must be supplemented. In §4.1 I defended the claim that when your evidence is inexact, your evidence consists of a vague proposition, and that your credences ought to be the result of conditioning your priors on this proposition. This, I claim, is just the sort of supplementation \textit{JC} requires. The following is thus another feature of the role in thought that vague propositions have:

With respect to the precise propositions, conditioning on a vague proposition has the effect of a realizable Jeffrey conditioning on a partition of maximally strong precise propositions.\textsuperscript{15}

Those who like the ideology of precisifications could state this principle in terms of Jeffrey conditioning relative to each of a number of precisifications. However, for reasons that will become clear later I prefer this more general formulation. Above I speak of the the partition of maximally specific precise propositions, however in practical contexts, we may simply use a coarser partition; for example, if we are talking about conditioning on the proposition that Harry is bald, we can think of that as Jeffrey conditioning on the partition consisting of each of the propositions that Harry has exactly \( n \) hairs, for each \( n \). To determine the coefficients of this Jeffrey update we look at the conditional probabilities we appealed to earlier: the conditional probability of Harry being bald conditional on each proposition about hair number. (Indeed, the conditional probability of a vague proposition on each maximally strong precise proposition represents a theoretically important aspect of that vague propositions role in thought, and will be a central concept in what follows.) So our principle states that conditioning on the proposition that Harry is bald is equivalent to Jeffrey conditioning over the partition of propositions containing, for each \( n \), the proposition that Harry has exactly \( n \) hairs.

\subsection*{4.2.2 Conditioning on a precise proposition}

In section 4.1 we argued that the smooth blue graph in figure 4.2 could be obtained by conditioning on a vague proposition. The ideas in this section can now be applied to demonstrate that conditioning on a vague proposition would have this effect. To set up an example, suppose that I have looked out of the window and have seen a tree at a distance. According to the current proposal the strongest proposition that I have seen, and that is therefore part of my evidence, is a vague proposition. For the sake of argument, let us suppose it is the proposition that the tree is about 200cm. In order to represent this situation we shall model propositions as sets of epistemic possibilities (or (im)possible worlds.) These can thought of as certain kinds of maximally consistent propositions; all that really matters is that since we do not know whether a tree that is 220cm is about 200cm

\textsuperscript{14}Conditionalisation can be read as telling you how to redistribute your credences once you have become certain in \( e \), but it is most naturally read as telling you to become certain in \( e \) (and redistribute accordingly) when \( e \) is part of your evidence. There is no simple connection between your evidence and Jeffrey conditioning in the same way.

\textsuperscript{15}It should be noted that the converse to our claim – that every realizable Jeffrey conditioning over the space of precise propositions is the result of conditioning on a vague proposition – is not being considered here. Formally one can Jeffrey condition over a set of maximally strong precise propositions without becoming certain in a vague proposition, however one could in principle make the case that such Jeffrey conditionings would never count as realizable in the sense outlined earlier.
(because, let’s suppose, it is vague) we require there to be an epistemic state in which the tree is 220cm and about 200cm, and a state in which the tree is 220cm and it’s not about 200cm. For every possible height of the tree, \( h \), and for every epistemically possible cut off point for ‘being about 200cm’, \( \pm c \) cm, there will be an epistemic state where the trees’ height is \( h \), and the cut off point for being about 200cm is being within \( c \) cm of 200cm. The relevant epistemic states can thus be represented by conjunctions of propositions taken from the following two sets: \( \{ \text{the proposition that the tree is } h \text{ cm } | \ h \in \mathbb{R} \} \) and \( \{ \text{the proposition that, necessarily, a tree is about 200cm iff it’s between } 200-c \text{ cm and } 200+c \text{ cm} | \ c \in [0, 200] \} \). There are therefore many more states than there would be if we had only countenanced precise possible worlds. Suppose, for mathematical simplicity, that we restrict the possible values \( c \) and \( h \) may take to some large but finite set, and that my credences are uniform over this set. After updating on the proposition that the tree is about 200cm the possible heights of the tree graphed against my posterior credences over the possible precise heights of the tree would form a triangular shape with a point at 200cm. In reality, however, my credences would not be uniform: I take it that the cut off point \( c = 1 \) cm is very unlikely, as is the cut off point \( c = 200 \). In reality we should therefore expect my credences to conform roughly to figure 4.3.

Could the graph in figure 4.3 be obtained by conditioning if we had not assumed this theory of vague propositions? If the set of states were simply the set of propositions saying that the tree is exactly \( x \) cm tall, then all the propositions would be precise facts about which sets of numbers the trees height falls in. Presumably the strongest proposition in this algebra that you learn will be of the form: the tree is between \( x \) cm and \( y \) cm tall. If your priors about the trees height are uniform, the result of conditioning on a proposition like this will be a rectangle instead of a smooth graph. In order to get a non-rectangular graph, as in figure 4.3, by conditioning on a proposition we need to add extra states to the model. But this proposal on its own seems mysterious; the crucial aspect of this proposal is that the extra states arise when we can draw distinctions within a single possible world about what vague matters are like there.

This fact marks a significant benefit of the theory of vague propositions I have been espousing. On a possible worlds theory, or any theory which postulates only precise propositions, it is hard to see how there could be a proposition which can take you, as a result of conditioning on it, from having evenly distributed credences to the smooth credences represented in figure 4.3.\(^{16}\)

\(^{16}\)Although it is certainly not impossible to model this without invoking vague propositions. A particularly natural way to do this in a framework of precise propositions is to go for a view in which seeing that \( p \), remembering that \( p \), and the other attitudes listed earlier, do not automatically confer the status of being evidence to \( p \). Instead the evidence one usually acquires is that one has had a certain type of experience, and the priors determine how probable the various heights of the tree are conditional on having that experience.
Neither can a possible worlds theory straightforwardly account for the following scenario. Suppose that Alice and Bob initially have identical evidential probabilities and that they both are looking at a tree in the distance, which, let us suppose, is 500cm high. As expected, their evidence is characteristically inexact. Suppose furthermore that in this situation they both have exactly the same precise evidence; the strongest precise proposition that is part of Alice and Bob’s evidence is the proposition that the tree is between 400cm and 600cm (say.) However there is a difference: Alice’s eyesight is better than Bob’s. While neither Alice nor Bob can rule out that the tree is 401cm or 599cm, Alice’s probability distribution forms a high peak at 500cm which quickly subsides remaining quite low at the edges (400cm and 600cm.) Bob’s probabilities are much more evenly distributed, they increase steadily from 400cm, peaking at 500cm, and decreasing steadily until 600cm. This difference is easily explained by a difference in their propositional evidence on my view, but it cannot be straightforwardly explained by a difference in their precise evidence (although see footnote 16).

What this example shows is that one’s evidential probability distribution does not supervene on one’s precise evidence. Both Alice and Bob have the same precise evidence, yet they have different evidential probability distributions. In particular, this shows that evidential probability is not determined by conditioning on your precise evidence; thus either the agents evidential probability is not determined by conditioning on her evidence, or her evidence does not consist only of precise propositions. Since, as I have argued in this section, the view that we update on non-propositional evidence via Jeffrey conditioning does not solve our problem it seems that the best explanation relies on the existence vague propositional evidence.

It should be noted that the picture described here does not sit well with certain internalist conceptions of evidence. According to the view I’ve defended one cannot have evidence that Harry is bald (or evidence that Harry is not bald) if it’s borderline whether Harry is bald. But, one might think, surely I could have misleading evidence that Harry has exactly no hairs when in fact he has a borderline number. For example, if I initially have no idea how many hairs Harry has and a normally reliable person tells me that Harry has no hairs when in fact he has a borderline number. For example, if I initially have no idea how many hairs Harry has and a normally reliable person tells me that Harry has no hairs when in fact he has a borderline number, some philosophers would say I have evidence that Harry is bald even though this is a borderline proposition. According to other philosophers, however, this is not so: evidence must be factive in the sense that your evidence consists only in true propositions. In the above situation, for example, your evidence would include the proposition that someone has told you that Harry has no hairs, but would not include the proposition that Harry has no hairs since it is false. Your belief might be justified by your evidence in the sense that it is a belief with a high evidential probability, but it would not be evidence in the strictest sense.

I suggest that according to this conception one’s evidence consists only of determinately true propositions. Imagine two parallel cases. In one case the reliable person tells you that Harry is bald instead of telling you he has no hairs, in the other she tells you he’s not bald. Either Harry is bald or he isn’t, so in one of the cases you have been told a true proposition. Yet on the externalist conception of evidence it is natural to think that, even though in one case the proposition you’re told is true, in neither case is the proposition you are told evidence.

4.2.3 Evidence for the whereabouts of cut-off points

We observed in chapter 2 that considerations of vagueness related ignorance indicate that vagueness is a distinctive source of fineness of grain. Thus many theorists are committed to

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The smooth curve pictured is perfectly consistent with this view, although the view does deviate from the attractive picture of evidence that I have been recommending: since, for example, the ur-prior should allow for the possibility of having the experience of a tree without a tree being there, our evidence would never rule out various kinds of skeptical hypotheses and small amounts of credence ought to be reserved for these possibilities.
the existence of ‘vague propositions’ in this very minimal sense. Once we have accepted the existence of these propositions it is also extremely natural to think that we can sometimes learn them. A standard thought about testimony, for example, entails that when you hear a trustworthy person assertively utter a sentence you often get to know and have as part of your evidence the proposition that sentence semantically expresses. Thus if a trustworthy person utters the sentence ‘that tree is around 200cm’ in normal cases we get to add a vague proposition to our evidence.\footnote{Of course when we hear someone utter the sentence ‘that tree is around 200cm’ we learn lots of things: implicatures, facts about their accent, the pitch of their voice, and so on. We are looking for an example in which the strongest thing learnt is vague, but for all I’ve said the conjunction of things I learn is precise. This worry is easily set to rest: since in normal cases where the proposition semantically expressed is vague and not about the pitch of your voice or other such things, it will be epistemically independent of the other things I learn. Thus their conjunction will also be vague even if the other things are all precise.}

The view that we sometimes update on vague propositions therefore seems quite plausible independently of the theory of inexact evidence defended here. However there is a somewhat surprising consequence for any view of this kind.\footnote{Thanks to Cian Dorr here for pointing this consequence out to me.} According to any view that maintains both that we are ignorant about the vague and the thesis of probabilism, vague propositions can provide one with evidence for the whereabouts of cutoff points. Suppose that I already know that the tree is between 180cm and 220cm, but I am uniformly distributed over those possible heights. If I then learn that the tree is around 200cm in height, then I rule out states both where the cutoff point is \textit{being within a margin of} $c$ cm of 200cm and in which the tree is not within that margin of $c$ cm of 200. Let us suppose the largest candidate for $c$ is 20. It follows that out of the states in which the cutoff is a margin of 20cm I do not rule out any of the possible heights of the tree: I already knew the tree was within 20cm of 200cm. Out of the states where the cutoff is 10cm I rule out the states in which the tree is less than 190cm and over 210cm – i.e. roughly half of those states. Finally, out the states where the cutoff is 0cm or close to 0cm I rule out basically all of the heights except for 200cm. In summary, since we are initially uniformly distributed over the possible heights of the tree, this means we end up ruling out far more states in which the cutoff points are low, e.g. the $\pm 1$cm cutoff points, than states in which its high. We should therefore become more confident that the cutoff for being about 200cm is a large margin around 200cm than we were before we had the experience.

The thought behind this argument can be illustrated quite simply using an analogy. Suppose that, instead of being ignorant about cutoff points, someone has rolled a d20 (a 20 sided die) which has landed on some number, $X$, whose value you are ignorant about. Suppose also that as before you know the tree is between 180cm and 220cm and each of the 40 possible heights is supported equally by your evidence. If someone tells you that the tree is within $X$ cm of 200cm it is clear that you should become both more confident that the tree is closer to 200cm and that the die landed on a higher number. You can make the point even more vivid if you imagine that there are 100 trees you know to have a height randomly between 180cm and 220cm and such that the height of each is independent of the height of any other. If someone told you that all of the trees were within the unknown $X$ cm of 200cm you can become pretty confident that $X$ is large, since it is antecedently very unlikely that all the trees would be bunched tightly around the 200cm mark.

Now anyone who thinks that we are straightforwardly ignorant about the vague and accepts probabilism must accept the analogous argument that shows that vague propositions provide evidential support for certain hypotheses about the locations of cutoff points.\footnote{Whether one can get around these issues by rejecting ignorance about the vague, or rejecting probabilism, bears further investigation, although I shall argue that both of these routes have problems of their own in the next chapter.} Although there are disanalogies between ignorance about cutoff points and ignorance about dice rolls, the only analogies we need to run this argument are being granted.

It should be stressed that although these experiences provide confirmation for the hypothesis that the cutoff for being around 200cm is on the wider side of things, the change
of credence being recommended is not necessarily bringing our credences closer to the truth about the location of the cutoff point. Indeed this partial confirmation that the cutoff is a large margin either side of 200cm is only a temporary one, and further precise evidence about the exact height of the tree will bring my credences closer to my prior beliefs about the cutoff points. Indeed, since ones evidence can never be borderline, once I have determined all the precise facts I will be as uncertain about the locations of the cutoff points as I was initially, and there will be no way to improve my epistemic position since I already know all the non-borderline facts.

4.3 A principle of plenitude for vague propositions

According to linguistic theories of vagueness vague propositions have a derivative status. Either there are no vague propositions or there are and they are parasitic on the vocabulary of the language. In either case there are at most countably many vague propositions entailing any particular maximally consistent precise proposition corresponding to the distinctions one can make in the language. This dependence on the language seems to be particularly odd, and in chapter 2 we suggested that there were vague propositions not expressed by any sentence.

An alternative to the modal way of individuating propositions, in which they are identified with sets of possible worlds, would be to individuate them by their attitudinal roles and to identify them with sets of maximally specific epistemic (bouletic/doxastic/etc) possibilities. How should we express the idea that propositions exist independently of language and thought?

I propose the following principle of plenitude for propositions:

For every possible evidential role in thought there is some proposition with that evidential role.

Intuitively an evidential role is a profile of the kind evidential support that proposition receives from each maximally precise description of the world, relative to a rational ur-prior. We may state the principle of plenitude rigorously as follows. Let \( P \) denote the set of maximally specific precise propositions (i.e. those precise propositions entailing all precise propositions which entail them.) Then an evidential role will be represented by a function \( E: P \rightarrow [0,1] \), telling you what your ur-priors should look like conditional on each maximally strong precise proposition:

**The Principle of Plenitude**

Let \( E \) be an evidential role. Then there is some proposition, \( p \), such that for every coherent ur-prior \( Pr \) and maximally strong precise proposition \( w \in P \), \( Pr(p \mid w) = E(w) \).

As an example, the principle of plenitude entails that there is a proposition \( p \) such that conditional on the proposition that Harry has \( N \) hairs, everyone’s ur-prior in \( p \) ought to be 0.798 (set \( E(w) = 0.798 \) whenever \( w \) entails that Harry has \( N \) hairs.) The principle of plenitude entails the existence of an abundance of such propositions. Note that while the evidential roles generate propositions, they do not individuate them. For example, by Plenitude there is a proposition, \( p \), such that one is required to have credence \( \frac{1}{2} \) in it conditional on any maximally specific precise proposition. It follows by probability theory that \( \neg p \) has the same evidential role as \( p \), yet these propositions are always distinct in a Boolean algebra.

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20In principle there could be \( 2^{\aleph_0} \) maximally logically consistent sets of sentences above a given set of precise sentences, however it is not clear that they all correspond to conceptually consistent propositions. (For example, there are \( 2^{\aleph_0} \) maximally consistent sets extending Peano arithmetic, but most of these are not genuinely conceptually consistent because they are \( \omega \)-inconsistent.)
Roughly, the principle of plenitude guarantees that we have the following kind of picture: the space of epistemic possibilities will be divided up into non-overlapping cells, representing the maximally strong precise propositions. Furthermore if for each cell I pick a proportion of that cell that I want to ‘fill’, plenitude guarantees there will be some vague proposition that fills each cell to that proportion. The notion of filling a cell by some proportion simply means having that probability conditional on the cell according to every ur-prior. (Notice that a proposition generated by the principle of plenitude is such that every ur-prior agrees about what proportion that proposition takes up of each cell, even if the ur-priors disagree about the size of the cells themselves. This is an important feature, although I shall defer its discussion until the next chapter.)

Although plenitude will be enough for most purposes, an even stronger principle is motivated by the picture I have been sketching. Roughly, if we cut up each cell into smaller non-overlapping cells, and assign proportions of each of these to be filled, then there is some vague proposition that fills each of the smaller cells to those proportions:

Let $P'$ be any subpartition of $P$ and let $E$ a function from $P'$ into $[0, 1]$. Then there is some proposition, $p$, such that for every coherent ur-prior $Pr$ and every proposition $x \in P'$, $Pr(p | x) = E(x)$.

This principle allows us to generate vague propositions from evidential relationships to other vague propositions that have already been generated. In the appendix it is shown that both principles are consistent.

It is tempting to ask why one ought to have a particular credence, 0.798 say, in the proposition that Harry is bald given one knows the relevant facts about his head. This is simply not a question with a reasonable answer on this way of individuating propositions. Part of what it is to be the proposition that Harry is bald is to have the specific epistemic profile it in fact has, which includes this conceptual requirement. It is like asking, on the modal way of individuating propositions, why the proposition that Harry is bald corresponds to one set of worlds and not another.

However, the thought which I take it this question is trying to latch on to might be better expressed by the question: why does the sentence ‘Harry is bald’ denote a proposition with one set of conceptual requirements rather than another? This is presumably a question of metasemantics and a question whose answer is vague. The appearance of precision is really an illusion. A similar problem arises for the modal way of individuating propositions. One might ask why the proposition that Harry is bald contains worlds where Harry has 784 hairs but not worlds where he has 785 hairs. On the modal way of individuating propositions this is just a bad question: it amounts to asking why a given set has the members it in fact has as members. The sensible question in the vicinity is why does the sentence ‘Harry is bald’ express this proposition and not another, or why are we justified in calling this set of worlds ‘the proposition that Harry is bald’. Both are metasemantic questions, and one should expect there to be vagueness concerning which propositions are picked out by which sentences.

Before we move one, let us apply these ideas to the example of the tree seen from a distance. Suppose that initially my credences are uniform over the possible heights less than 1000cm. After the evidence is in my credences form the curve displayed in figure 4.3. What proposition would have the effect that updating this way would cause this change in credence? Here the principle of plenitude kicks in. According to the principle there will be a proposition whose evidential profile ensures that learning it will change my uniform starting credences to the curve. Let that curve be represented by the function $Pr$. For simplicity

\[21\text{For example, suppose that we know which exact shade a ball is, and that that shade is such that it's borderline whether it's turquoise and blue, just turquoise or just blue. Although having further evidence that it was turquoise would be impossible, we can still ask if the proposition that the ball is turquoise is evidentially relevant by considering the conditional probabilities. Intuitively the probability that it's blue decreases on the hypothesis that the ball is turquoise since we have to rule out the blue and not turquoise possibilities, and there are more just turquoise than turquoise and blue possibilities.}\]
lets say the partition of precisifications is finite, so lets pretend the maximally specific 
precise propositions are the propositions saying that the tree is $n$ cm tall (this simplification 
is harmless.) Let $E$ be the evidential role given by $E(p_n) = Pr(p_n)$. Then the principle of 
plenitude entails there is a proposition, $p$, with evidential role $E$. In particular, if $Cr$ is a 
coherent prior and $e$ is the agents total evidence, so that $Cr(\cdot \mid e)$ is my credence before 
the observing and is thus uniform over the partition $p_n$ for $n \leq 1000$, then conditioning on $p$ 
will result in a curve like that in figure 3: $Cr(p_n \mid e \wedge p) = E(p_n)$. For proof the appendix.

4.4 Evidential roles and degrees of truth

In in many ways, maximally strong precise propositions in this framework are approximations 
of possible worlds in an intensional framework. Assuming this analogy, vague 
propositions then determine a function from ‘possible worlds’ to real values in $[0,1]$: the 
credence any conceptually coherent agent ought to have if she learnt that she was in that 
possible world.

Another approach to vagueness also identifies with each vague proposition a function 
from possible worlds to real numbers in $[0,1]$. According to this view, the number a proposition is 
assigned at such a world is its truth value there. These theorists often espouse 
a non-classical fuzzy logic, due to the way in which the truth values at a world interact 
with the connectives. However other theorists keep classical logic and are close in spirit to 
the current view. Edgington [34], for example, espouses a view in which the truth values 
behave like probability functions (see also Kamp [69], Lewis [80] and Williams [129], for 
versions and developments of the view in a supervaluationist setting.) One might wonder 
how fuzzy views, especially Edgington’s view, differ from the view defended here. Edgington, 
for one, thinks that there is a difference between truth values (which she calls ‘verities’) 
and credences. In [34] she writes:

"Some philosophers (e.g. Williamson 1994) hold that vagueness is a species of 
epistemic uncertainty: there is a precise line which divides the red from the non-
red, etc., but it is epistemically inaccessible to us. Were this true, verity would 
be a kind of credence: the credence that a person with no relevant ignorance 
other than about the precise line, would give to a statement like ‘that’s red’. If a 
is redder than b, but neither is clearly (certainly) red, then one must, if rational, 
be more confident that a is red than that b is: a is more likely to be above the 
mystery line than b is. The nearer to clearly red is the nearer to certainly red. 
Credence and verity, I argued, have the same logical structure. This could be 
interpreted as grist for the epistemicist’s mill. What better explanation of the 
alogy I have developed, than that verity is credence, and so vagueness a kind 
of epistemic uncertainty?" 

The view I am defending is not epistemicism, but it agrees with the epistemicist in respects 
important to this discussion. According to Edgington verities and credences play different 
roles. Credences inform a rational person’s actions, whereas verities do not. In support of 
this she notes the distinction between the following kinds of justifications:

I prefer $A$ to $B$ by a long way. Therefore I prefer $A$ with certainty to $A$ or $B$ happening 
with equal uncertainty, i.e. probability $\frac{1}{2}$, and I should prefer $A$ or $B$ happening with 
equal uncertainty to $B$ with certainty.

I prefer $A$ to $B$ by a long way. Therefore I prefer $A$ with verity $1$ to $A$ or $B$ each with 
equal intermediate verity, $\frac{1}{2}$, and I should prefer $A$ or $B$ with equal verity to $B$ with 
verity $1$. 

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The former principle is a valid principle in most decision theories, yet Edgington gives an example, in which whether \( A \) or \( B \) is borderline figures in my preferences, to demonstrate that the second is not a universally correct principle of decision making.

One might have the converse worry: that the credence a coherent person should assign to a proposition conditional on being in a world plays exactly the same role as a truth value.\(^{22}\) Consider the following ‘truth norm’: if \( p \) is true then one should believe that \( p \) (and one should not otherwise.) One can formulate similar truth norms for partial beliefs, in which credences closer to the truth are better (see [68].) Let \( E(w) \) represent the credence you should have in the proposition that Harry is bald conditional on knowing that you’re in \( w \). Then \( E(w) \), qua the thing that your credences ought to match when \( w \) obtains, seems to play the very same role the truth value of the proposition that Harry is bald plays as described above. Is \( E(w) \), the evidential role of the proposition that Harry is bald, not just the truth value of the proposition that Harry is bald had \( w \) obtained?

Other than Edgington’s argument above (which I find convincing) I think there are two further points to be made. The first has to do with the truth norm. I think that in the sense in which we use the word ‘should’ in epistemology it’s simply not the case that Copernicus should have believed that space-time is non-Euclidean; all his evidence pointed against this so there’s a clear sense in which he shouldn’t have believed this, even though it’s true. However, the word ‘should’ is clearly context sensitive, and with a bit of scene setting it may well be possible to find contexts in which the truth norm holds. However it is far from clear that in the contexts in which the truth norm holds my credence that Harry is bald conditional on \( w \), should-in-this-context be \( E(w) \), and not 1 or 0 (depending on whether Harry is bald at \( w \).) After all, the view that’s being espoused is a bivalent one, and if it’s possible to read ‘should’ in this unintuitive ultra-objective way it is natural to think that all my credences should be 1 or 0. Thus, on this reading of ‘should’, the disquotational (i.e. bivalent) truth values play the truth rule.

The second point I want to make involves an analogy with chances. Note the similarity between the following two claims

For each proposition, \( p \) there is a function, \( E \), such that for each maximally strong precise proposition \( w \), and rational prior credence function \( Cr \), \( Cr(p \mid w) = E(w) \).

For each proposition, \( p \) there is a function, \( F \), such that for each maximally strong piece of admissible evidence at \( t, w \), and rational prior credence function \( Cr \), \( Cr(p \mid w) = F(w) \).

In the both cases there is a special partition of the space of propositions such that every rational prior agrees with every other prior conditional on any element of that partition. The former principle is a consequence of our theory of propositions, the latter a consequence of the principal principle (see [81].) In the latter case \( F(w) \) simply represents the chance that \( p \) at time \( t \). In whatever sense \( E \) plays the truth role for the proposition \( p \) (relative to the partition of maximally specific precise propositions) then note that so do chances (relative to the partition of worlds with the same admissible evidence at \( t \).) Since we are not at all tempted to call chances verities on the basis of truth norms alone, I think that we should not be tempted to call credences verities.

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\(^{22}\)I am indebted to Robbie Williams for pressing me on this objection.
Chapter 5

Vagueness and Uncertainty

Epistemicism is notorious for making a pair of counterintuitive claims. According to epistemicism there was a nanosecond during which I stopped being a child: I was a child at the beginning of it, but not at the end. Moreover, according to the epistemist it is not possible to know when that critical nanosecond occurred. It is these consequences of epistemicism that regularly invite the incredulous stare and which have lead people to try and find alternative theories that respect classical logic; the most famous alternative being supervaluationism.

Unfortunately for this project, it is possible to prove that there’s a nanosecond at which I stopped being a child, and that it is unknown which it is, from a couple of eminently plausible premises. As we noted in the first chapter, apart from classical logic all we need to derive the existence of a nanosecond at which I stopped being a child are the premises that I was a child after one nanosecond of my life had passed, but not after several billion had passed. But once we have accepted the existence of such a nanosecond it would be madness to suppose that we know which one it is. If you are not immediately convinced by this latter claim ask yourself which number it is: if you are unable to produce a satisfactory answer, I would suggest that this is because you do not know which number it is.

Thus the project of finding a more palatable alternative to epistemicism that accepts classical logic is pretty much a no-go if one takes these above consequences of epistemicism to be the source of the unpalatability. However some theorists have suggested that what is so radical about epistemicism is not that there is an unknown nanosecond at which I stopped being a child, but the claim that vagueness amounts to nothing more than this special kind of ignorance. What explains these astonishing consequences is the thought that when there is vagueness about where a boundary lies, there is also no fact of the matter about where it lies.

Unfortunately the locution ‘there’s no fact of the matter’ is mysterious as it is used by these theorists. Since they accept classical logic they think that either Harry is bald or he isn’t even when there is no fact of the matter about whether Harry is bald. Thus one of the following must hold: either (i) Harry is bald but there’s no fact of the matter about whether Harry is bald, or (ii) he isn’t bald but there’s no fact of the matter about whether he’s bald. Both disjuncts are equivalent to something of the form ‘$P$ but there’s no fact of the matter about whether $P$’ and so this use does not seem to be consistent with our pretheoretic understanding of the locution ‘there’s no fact of the matter’: how can there be no fact of the matter whether Harry is bald if it turns out that Harry is bald (or, if it turns

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1 By ‘epistemicism’ I just mean the cluster of views often associated with Tim Williamson, Roy Sorenson, Paul Horwich and a few others; a stricter definition would be unhelpful at this juncture.

2 I personally find this response quite surprising – it is the former, not the latter claim that invites the incredulous stare. Once we have fully absorbed that there is an unknown nanosecond during which I stopped being a child the correct response to these observations should be that epistemicism isn’t quite as radical as it first seems.
out that he’s not). At this juncture the epistemicist could triumphantly point out that ‘$P$’ but it’s impossible to know whether $P$ is perfectly consistent, and seems to play the right role in explanations as well – perhaps the semi-technical talk of there being no fact of the matter is just code for talk about things being unknowable for certain kinds of reasons. This would be an outright defeat for the classical theorist I am describing: the challenge, then, is to find some more technical notion that is consistent with this consequence but excludes an epistemicist reading of it.

In recent years a number of authors have attempted to meet this challenge by concentrating on the epistemists claim that we are ignorant and uncertain about the locations of cutoff points.\(^3\) The operative thought in all these cases is that when one is uncertain about something because they believe it to be borderline they are in a very different kind of state than when they are uncertain about some ordinary matter such as where they left their keys. Uncertainty about my keys seems to be importantly different from uncertainty about where, say, the boundary for baldness lies. One might naïvely put the difference by saying that the former kind of uncertainty is about uncertainty about the way the world really is – about how the facts are – whereas the latter is not uncertainty about a matter of fact. However, in the absence of an antecedent understanding of this talk of ‘facts’, one might hope to elucidate talk of there being ‘no fact of the matter’ by providing an independent characterisation of the two different types of doxastic state.

This has been pursued in a number of different ways. Some have argued that we are simply not ignorant or uncertain about the vague or have tried to make the case that the doxastic attitude we have to vague claims isn’t actually ordinary uncertainty, but is a different doxastic state altogether. Others have argued that rational degrees of belief do not obey the standard probability axioms, and the distinction between ignorance about the whereabouts of my keys and the whereabouts of cutoff points can be made purely in terms of ordinary degrees of belief that don’t conform to the probability axioms.

In what follows I shall defend the view that there is one attitude of uncertainty which we hold to both the vague and precise alike, and that when understood as coming in degrees it ought to conform to the probability calculus. If this is right then we need another explanation of the difference between a purely epistemic ‘factualist’, understanding of the borderline operator and a non-factualist one which I shall explore in section 5.4.1.

### 5.1 Are we genuinely uncertain about the vague?

If we are to accept classical logic then the natural place to resist the claim that there are sharp cutoff points whose locations are unknown to us, is to resist the claim that they are unknown to us. After all, classical logic alone commits us to sharp cutoff points (the nanosecond during which I stopped being a child, for example) so the only thing left to resist is that we are ignorant about these locations. This is Dorr’s strategy in [31]. Dorr argues that when we know the relevant facts about how much of a particular glass is filled with water, say, we typically know whether the glass is pretty full or not. This applies even if it’s a borderline case. According to Dorr, vagueness is a matter of semantic indecision. The relevant fact about vagueness in the vicinity is the fact that the sentence ‘the glass is pretty full’ is vague in English (at context $c$, time $t$, and so on.) This, Dorr contends, and I agree, is a fact about English and ‘the claim that human beings are doomed to ignorance as regards whether the glass is pretty full […] has nothing at all to do with language: if it is true, it would have been true no matter what we had meant by the words ‘the glass is pretty full’.’ If the no-ignorance view was plausible then it would render most of the arguments of chapter 2 moot.

For those who balk at epistemicists accounts of vagueness this alternative is hardly better. According to this view, not only is there an exact percentage, $\%x$ say, at which a particular

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\(^3\)See Field [44], Schiffer [110] Barnett, [9], Dorr [31], Smith [115], MacFarlane[85], Williams [128].
glass is pretty full if filled to be \( >\%x \) full, and is not pretty full otherwise, but we are furthermore supposed to know what that critical percentage \( x \) is. If we are watching a glass being filled, drop by drop, then provided we are keeping count, not only will there be a drop which makes the glass pretty full, but we’ll know which that drop is.\(^4\) The claim that we do not know at what point the glass becomes pretty full is surely not the unintuitive claim that causes people to balk at epistemicism.\(^5\)

Another version of the no-ignorance view is defended in Barnett (see [9], and especially [7]).\(^6\) Unlike Dorr, however, Barnett is not a linguistic theorist. He treats indeterminacy as an operator and presumably therefore admits vague propositions. Since, at least for these purposes, it is open to Dorr to adopt a coarse grained theory of propositions, it is easy on that view to communicate whether Harry is bald in English; one of the sentences of the form ‘Harry has less than \( n \) hairs’ will do. If \( N \) is the critical number then the proposition that Harry is bald simply is the proposition that Harry has less than \( N \) hairs and both are expressed by the sentence ‘Harry has less than \( N \) hairs.’ On the other hand we cannot, according to Dorr, communicate this fact uniquely using the sentence ‘Harry is bald’, and this is what explains our reluctance to assert this sentence when Harry is a borderline case. For Barnett on the other hand, the proposition that Harry is bald is distinct from the proposition that Harry has less than \( N \) hairs, and the former is presumably expressed by the sentence ‘Harry is bald’, and not by ‘Harry has less than \( N \) hairs’ (which expresses the latter.) The belief that Harry is bald can be communicated uniquely with the sentence ‘Harry is bald’.

It seems, therefore, as if Barnett has a special problem of explaining why we are reluctant, and indeed should be reluctant, to assert ‘Harry is bald’ or ‘Harry isn’t bald’, when it’s borderline whether he’s bald. Barnett appeals to the principle:

One should aim to clearly satisfy the rule: (M) assert \( p \) only if \( p \).

according to Barnett there will be cases where you know whether \( p \), but you are not allowed to assert \( p \) (or \( \neg p \)) because it would result in its being unclear whether you’ve satisfied (M). That is, even though you know you’ve satisfied (M), i.e., even though you know you’ve asserted \( p \) only if \( p \), you would not have clearly satisfied (M) and this in itself is supposed to be a bad thing. No explanation for this rule is given; it is just a primitive norm of assertion.

There’s a natural analogy to be drawn here between Barnett’s rule and the claim that one shouldn’t assert things that are rude or would cause embarrassment. Even when you know that \( p \), it can bad to assert \( p \) if it would violate these principles. But even so, one might think these types of norms can sometimes be trumped; maybe by other norms of communication, or by moral norms or by something else. If, for example, someone told me they’d start shooting innocent people unless I asserted all the embarrassing truths I know about certain people it should be obvious what I should do, all things considered.

Now, if we are to believe the kinds of claims made in [7] and [9], Barnett knows, or at

\(^4\)I take it that the ‘knowing which’ facts supervene on the ‘knowing that’ facts. For example, if you know that after drops 0-\( N \) the glass is not pretty full, and you also know that the glass is pretty full at the \( N \)th drop and onwards, then by some closure principles it follows that you know that \( N + 1 \) is the first drop at which the glass is pretty full, and therefore, I claim, you know which the first drop the glass is pretty full at is.

\(^5\)In chapter 2 I discussed a less radical version of the no-ignorance view in which we can be ignorant of vague truths relative to certain modes of presentation, even if we are knowledgeable relative to others. This view could also be invoked as a way of distinguishing epistemicism from other classical views. It would, however, be surprising if it succeeded since this view is pretty much a version of Williamson’s own view.

\(^6\)In [9] Barnett argues for the mere possibility of knowing borderline truths, although doesn’t conclude that we actually know any borderline truths. In [7] he argues for the more radical view that there are in fact borderline truths that we should believe, although he doesn’t talk about whether we would know them. It would, however, be puzzling to think that we’re not in a position to know what we should believe in these cases. In what follows I shall continue to talk about knowledge although I think a similar discussion could be run if I replaced ‘knows’ with ‘ought to believe’.
least is in a position to know, which the last small number is. Yet due to this primitive norm of assertion, he’s not allowed to tell us which it is. In the particular case at hand, however, I think the rule (M) is trumped by other considerations. If Barnett really does know which the last small number is it would be of great value to the rest of the philosophical community if he were to tell us. After all, there has been a long-standing debate amongst classical and non-classical logicians about whether there even is one; if Barnett could settle the matter constructively that would be significant progress. Therefore, given the great benefits, it seems like it would easily be worth the cost of flouting the rule and telling us which the last small number is. Yet he has not told us. I think it would be not unfair to conclude that this is not because he is afraid of violating the rule (M), but because he simply doesn’t know which number it is.

The upshot of this discussion, I think, is that even if we bracket its initial implausibility the no-ignorance view is especially problematic unless it is combined with Dorr’s fairly specific metasemantics.

Another strategy in the vicinity is to concede that our doxastic attitudes towards the vague encode, in some sense, a kind of ambivalence, but to maintain that these attitudes don’t constitute genuine uncertainty. Call this other kind of attitude ‘ersatz uncertainty’. According to this view when one is in the kind of doxastic state characteristic of being uncertain in a proposition you know to be borderline, you are in fact not uncertain in the ordinary sense, but in some other sui generis mental state. Talk of ‘uncertainty’ in cases of vagueness relies on an equivocation between two propositional attitudes. For example Schiffer – a prominent defender of this kind of view – claims that vagueness-related uncertainty is ‘a new kind of propositional attitude, one that comes in degrees and that precludes standard partial beliefs,’ that it is ‘not a measure of uncertainty’, and similar such things. If such uncertainty comes in degrees, we shall also talk about ‘ersatz credences’.

The view helps with the problem we opened the chapter with. According to this view if I can see how much hair Harry has, and I can see that it’s in the borderline region, then there simply isn’t a fact out there that I’m uncertain about, or that I ought to be uncertain about. My beliefs and ersatz beliefs are, in some sense, complete – I believe and disbelieve various determinate things, and I hold a different kind of attitude altogether to the remaining propositions. The appropriate attitude to have towards the proposition that Harry is bald isn’t one of uncertainty at all.

To get clear on what this view means we need to say something about how we individuate doxastic states. One could in principle understand the thesis as an empirical one about the existence of two fundamentally different brain processes. Perhaps genuine credences come from the right side of the brain and ersatz from the left, or something like that. On this hypothesis it is clear why we need the distinction between genuine and ersatz credences to describe what is fundamentally going on – they play distinctive physiological roles. But this is obviously not how we are supposed to be understanding the thesis that there are two fundamentally different kinds of credences. Not only is there no empirical evidence for this hypothesis, but it would anchor the questions we are interested in to the accidental features of human physiology. An intelligent life-form with a different physiology to us presumably isn’t any better placed to find out whether Harry is bald. Thus these lifeforms would also

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7Providing, of course, that his cognitive conditions are reasonably ideal, and so on and so forth. See, for example, p28 of [9].

8Barnett might perhaps play down the value of knowledge to the philosophical community when it is not clear knowledge. Yet I for one would value knowledge of the last small number, clear or not, so there is certainly reason to break the norm (M) if only to satisfy my own idiosyncratic preferences.

9In what follows I will not be focusing on Schiffer’s theory in particular, but rather the specific claim that vagueness related uncertainty and ordinary uncertainty are fundamentally different doxastic states.

10See Schiffer [110], Smith [115] and MacFarlane [85] for different ways of formulating the view. Note that not all of these elaborations on Schiffer’s idea count as genuine ersatz theories on my account. For example Smith, in arguing that degrees of belief are expected truth values, accepts a view in which there is really only one kind of doxastic state, albeit one that does not obey the classical probability calculus. We consider views like this in the next section.
have the puzzling form of uncertainty we have been trying to explain – we ought to be able to apply the concept of an ersatz credence to this kind of creature too.

In light of this, it is better to individuate the attitudes of genuine and ersatz uncertainty by something like their causal functional role and to interpret the claim that ersatz credences are needed to correctly describe an agent’s psychology accordingly. Interpreted this way, however, it is much harder to make precise the idea that the two kinds of uncertainty correspond to radically different attitudes.

One of the reasons for this, to put it roughly, is that the two kinds of attitudes blur seamlessly into one another. Imagine, for example, that you have lined up in front you a sequence of balls starting with a clearly red ball and ending with a clearly orange ball, constituting a sorites sequence for redness. Presumably according to the ersatz theorist we should have high genuine credences that the balls at the beginning of the sequence are red, intermediate ersatz credences that the balls in the middle are red, and low genuine credences that the balls at the end are red.\(^\text{11}\) It would, however, be incredibly surprising if the doxastic attitude that I hold to the claim that the \(n\)th ball is red had radically different causal functional role from the attitude I hold towards the claim that the \((n+1)\)th ball is red for any \(n\). It just doesn’t seem particularly plausible that one of these attitudes combines with my desires to play a certain role in informing my actions, when the adjacent attitude plays no such role.

Another issue is that any plausible theory like this ought to be able to make sense of what I’ll call ‘mixture’ uncertainties. I give two examples:

You are genuinely uncertain whether \(p\) is borderline or not. Example: you have a bag containing two small coloured balls. The first is clearly red, the other bordering on red/orange. You are about to take a ball out of the bag: what should your credence that you will pick a red ball be?

It is borderline whether \(p\) is borderline. Example: you have the sorites sequence of balls described above. You should have genuine credences about whether the initial balls are red or not and ersatz credences about whether the balls in the middle are red or not, but between these it is borderline whether you should have genuine or ersatz credence, and so you should be ersatz uncertain which of the two to adopt.

In both cases you are uncertain about something in some sense or other. However it’s unclear how to accommodate this within the framework that posits two kinds of uncertainty.

Despite the fact that neither state straightforwardly corresponds to being genuinely uncertain or ersatz uncertain, you can still assign numerical values to these mixture uncertainties determined by your ersatz and genuine attitudes of uncertainty in propositions you do have these attitudes towards. These can be used to indirectly assign values to this case. The most straightforward way to do this in the first case would be to work out the probability that the ball is red given that it’s the first ball and the probability that it’s red given it’s the second ball, then times each of these by the probability that it is in fact the first/second ball respectively, and add these together: \(Cr(\text{red}) = Cr(\text{red} | \text{ball1}) \times Cr(\text{ball1}) + Cr(\text{red} | \text{ball2}) \times Cr(\text{ball2})\). In this sum \(Cr(\text{red} | \text{ball1})\), \(Cr(\text{ball1})\) and \(Cr(\text{ball2})\) plausibly represent the numerical values of genuine credences and \(Cr(\text{red} | \text{ball2})\) the numerical value of an ersatz credence, so each of these numbers are grounded in real psychological states.\(^\text{12}\) Thus the number \(Cr(\text{red})\), while not corresponding to a state of ersatz or genuine uncertainty, can still be made sense of in terms of your ersatz and genuine credences and

\(^{11}\)It is interesting to note that if this is the picture there is no such thing as ersatz certainty; whenever you have a numerical credence corresponding to a ersatz state of uncertainty, it will have an intermediate value.

\(^{12}\)Thus to make sense of mixed uncertainty we had to assume that in addition to genuine and ersatz states of uncertainty there are genuine and ersatz states of conditional uncertainty. This assumption seems very much within the spirit of this view.
conditional credences towards other propositions. Similar things can be said about the other kind of ersatz uncertainty.\footnote{The theory of mixture credences (not under this name) has been developed in MacFarlane \cite{macfarlane2012}, MacFarlane \cite{macfarlane2013} and Smith \cite{smith2015}.}

If we are to take the view in question seriously, we should not think of this mixture credence as having any psychological reality of its own. For if mixture credences were attitudes in their own right then ersatz and genuine credences would just be special cases of that attitude. If we write a mixture credence as a weighted sum of an ersatz credence and a genuine credence, we get ersatz and genuine credences out in the special cases where one of the weights is 0. The view that we have two radically different types of attitude is being replaced with the view that we have one kind of attitude which comes in a kind of spectrum of different flavours, with ersatz and genuine uncertainty at opposing ends of the spectrum. (This feels more like a view, defended in Field \cite{field2013}, which I’ll discuss later.)

It might be possible to maintain the view that mixture credences aren’t genuine doxastic states if we individuate doxastic states by physiological processes: mixture credences aren’t real psychological attitudes because there aren’t brain processes dedicated to producing mixture credences. However, if we individuate attitudes by causal functional role then it is much harder to maintain that such states do not have any psychological reality of their own. When we ordinarily talk about how confident someone is about something we are rarely ever talking about their genuine credences or their ersatz credences – we are usually talking about a mixture credence. It rarely, if ever, happens that an agent is maximally confident that a proposition is borderline or maximally confident that it is determinate.\footnote{It is important to distinguish between the credences that agents typically have from the credences they ought to have. On the view I suggested in chapter 4 ones evidence will typically rule this kind of thing out. However the view under scrutiny, and defended by Schiffer, is one about the doxastic attitudes that people actually adopt.}

My uncertainty about the existence of intelligent life elsewhere in the galaxy is not genuine uncertainty: many people reserve some credence for the hypothesis that the only life elsewhere in the galaxy is borderline intelligent. Thus I am not genuinely uncertain here either, I have a mixture credence. A similar thing can be said about the other kind of mixture uncertainty: we often reserve small credences that things that appear to be determinate are in fact borderline.

It thus follows that it is mixture credences that are the things we usually appeal to when we do decision theory: the numbers that we use to calculate expected utilities and so on. It is also, presumably, mixture credences we attribute to people with our everyday talk of confidence for as we noted, most people don’t have many pure ersatz or genuine credences. Although it’s undeniable that there’s a difference between uncertainty about where I left my keys and uncertainty about where the cutoff for being red is, these observations strongly suggest that the difference is not that there are two distinct propositional attitudes being had in each case. The different kinds of uncertainty recognised above merely represent opposite ends of a spectrum of ways you could have that single attitude. So although there is a difference there to be recognised, there is little mileage to be gained from the claim that vagueness-related uncertainty is a fundamentally different doxastic state from ordinary uncertainty.

\section{Should our credences in the vague obey the probability calculus?}

The considerations of the last section suggest that the attitudes we have about borderline matters form a single psychological kind, and that attitude shouldn’t be that of being certain. It is natural to wonder if these points commit us to the epistemicist idea that when it is borderline whether \( p \) we should be genuinely uncertain about whether or not \( p \).
An extremely natural way to resist this epistemicist conclusion, whilst conceding both the idea that there is a single kind of degree of belief held in all cases and that it shouldn’t be 1 in the propositions we know to be borderline, would be to find some distinctive feature of the degrees of belief that are rational in those situations. A few theorists have suggested that we can draw the relevant distinctions if we relax the standard probabilistic axioms governing rational degrees of belief. Roughly the thought is that when we are certain that p is not borderline our credences will obey the standard probabilistic axioms, and the violations of the axioms become greater as ones credence that p is borderline increases. The difference between credences in the determinate and the borderline are not differences of kind, but merely differences of degree; the view is more like the view we mentioned in passing in the last section in which we just have the notion of a mixture credence, and the things being called ersatz and genuine credences are just special cases of those notions.

5.2.1 Alternative probability theories

In [110] Schiffer suggests that ersatz credences, rather than behaving like classical probabilities, are governed by the rules for calculating the truth values of propositions according to the Łukasiewicz connectives. Of course, we’ve already rejected Schiffer’s view on other grounds, however one might think that the insight that probabilism fails for these types of reasons can be separated from the ‘two attitude’ account of vagueness related uncertainty. Perhaps by taking the notion of a mixture credence as primitive and letting credences behave classically when the credence is certain the propositions in question aren’t borderline, and letting them be governed by the Łukasiewicz rules when you’re certain the credences are borderline, and some kind of mixture when your uncertain.

The obvious way to achieve this is to identify one’s degree of belief in a proposition with its expected truth value valuated according to the Łukasiewicz connectives; it is exactly this idea that is explored by Nicholas Smith in [115]. According to this formalism one starts off with a classical probability function over a set of possible worlds, which intuitively represent your ignorance about the precise – ignorance about the facts that are ‘out there in the world’. Relative to each world each proposition gets assigned a truth value in the interval [0,1], with the values of conjunctions, negations and conditionals being determined by the Łukasiewicz connectives. Thus when you are certain that we are in one of the worlds where p only takes values of 0 or 1, our credences in p behave like a classical probability function, and when we are sure that p has an intermediate value our credences behave more like the Łukasiewicz connectives. However the theory also accounts for cases where we are unsure whether p is borderline or not.

Within this framework there is a straightforward way to characterise the difference between ordinary and vagueness related uncertainty. Every classical probability function assigns contradictions like p ∧ ¬p a probability of 0. So if our uncertainty about whether p is ordinary uncertainty our credence in p∧¬p will be 0. However when we are uncertain about p for vagueness related reasons our credence in p ∧ ¬p will be greater than 0: if we assign some credence to a world where p has a truth value of neither 1 nor 0, we are assigning some credence to a world where p ∧ ¬p has a truth value greater than 0, since according to Łukasiewicz logic the truth value of p ∧ ¬p is either the truth value of p, or the result of deducting it from 1, depending on which is largest. Indeed this gives us a measure of how ordinary our beliefs are – if our credence in p ∧ ¬p is 1 2 then our uncertainty is completely vagueness induced, if it is 0 it is ordinary uncertainty, and then there are various other options in between.

This fact, however, highlights how radically strange this proposal is: if you’re certain that p has degree of truth ½ then Cr(p ∧ ¬p) = 1 2. To make this strangeness vivid, note that on this view modus ponens is not a reliable way of reasoning with uncertain premises: I can be 50/50 about the conjunction of A with ‘if A then B’, but completely certain that
$B$ is not the case and so have 0 credence in $B$. But if modus ponens really was a good inference it shouldn’t allow a drop of rational credence from the conjunction of its premises to its conclusion. This non-classical feature isn’t simply inherited from the fact that we are using Łukasiewicz logic, since this logic validates modus ponens – the problem is coming from the theory of credences.

This approach is clearly not helpful for the classical logician who is attempting to avoid the epistemicists striking conclusions. To accept this proposal one already has to weaken ones logic to Łukasiewicz logic, as well as accepting the strange consequences noted above. For this reason I shall focus instead on what is probably the most influential non-probabilistic account of vague credences: the proposal described by Field in [44]. One reason to focus on Field’s account rather than the accounts based on the Łukasiewicz valuations is that Field’s account is completely compatible with classical logic.

Field’s theory is rich and technically involved – I shall just focus on a simplified presentation of the theory, which consists of the following two axioms:

A1. $Q(\Delta A \lor \Delta B) = Q(A) + Q(B) - Q(A \land B)$

A2. $Q$ respects the modal logic KT. This just means that $Q$ assigns 1 to its theorems, 0 to the negations of its theorems and assigns $A$ no more probability than $B$ when $B$ is provable from $A$.

The above theory is in fact strictly weaker than Field’s theory. According to Field $Q$ must additionally respect the logic of determinacy S4, which controversially states that if something is determinate, it’s determinately determinate. The problems I shall raise for Field’s theory can be derived using only the two principles listed above.

Two important consequences of these axioms are the following:

**Iteration:** For any rational credence function $Q$, $Q(A) = Q(\Delta A)$

**Rejection:** For any rational credence function $Q$, if $Q(\nabla A) = 1$ then $Q(A) = Q(\neg A) = 0$.

Iteration follows from the first axiom by setting $A$ equal $B$ and then applying some logic using A2. Rejection then follows straightforwardly from Iteration given the definition of $\nabla$ from $\Delta$. It is worth noting, however, that the principle Rejection, even in conjunction with the axiom A2, does not imply Iteration.

Rejection brings out clearly the incompatibility of this theory with finite additivity, which requires the sum of the probabilities of a proposition and its negation to add up to 1. This allows for a particularly striking characterisation of vagueness related uncertainty. The standard probability calculus only allows you to be in one of two states regarding $p$: (i) be certain whether $p$: either having a credence of 1 in $p$ and 0 in its negation, or vice versa, or (ii) be uncertain whether $p$: both $p$ and it’s negation get a credence strictly between 1 and 0. However in Field’s calculus there is a third option: one assigns both $p$ and its negation a credence of 0. Your credences are neither intermediate, as they would be if you were uncertain, nor do they assign 1 to $p$ or its negation, as they would if you were certain whether $p$. Whereas the epistemicist would treat uncertainty about whether Harry is bald just as it would treat uncertainty about where I left my keys, someone adopting Field’s calculus can say something distinctive about the former case: vagueness does not merely amount to a kind of uncertainty, for one’s credences in $p$ aren’t intermediate between 1 and

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15Suppose that I’m certain that $A$ has truth value of $\frac{1}{2}$ and $B$ has a truth value of 0. Then the expected truth value of $A \land (A \rightarrow B)$ will be $\frac{1}{2}$ and the expected truth value of $B$ will be 0.

16In later work, Field develops the view in a setting suitable for a non-classical logic; I shall focus exclusively on the classical version of the proposal here.

17Adding S4 suffices to derive the crucial formula that relates Field’s theory to Dempster-Shafer theory: $Q(\bigvee_{i=1}^{n} \Delta A_i) = \sum_{j} (-1)^{\lfloor i/2 \rfloor} Q(\bigwedge_{j \in I} A_j)$ where $I$ ranges over subsets of $\{1, \ldots, n\}$. To prove this it helps to note that $\Delta A \lor \Delta B$ is logically equivalent to $\Delta(A \lor B)$ in S4.
0. Yet unlike the Barnett-Dorr view, we are not certain about $p$ either, for we do not assign 1 to $p$ or its negation.

Despite these nice features, I think that Field’s theory is ultimately untenable because it conflicts with a central principle that Delia Graff Fara calls the ‘gap principle’. Gap principles effectively state that sorites sequences don’t have sharp cutoff points at any orders. For example if you had a sorites sequence of people starting with people with no hairs who are clearly bald and ending with people with many hairs who are clearly not bald then the most basic gap principle amounts to the claim that there isn’t a sharp cutoff between the bald and non bald: if the $n$th guy is determinately bald, the $n + 1$th guy isn’t determinately non-bald. In symbolism $\Delta A_i \rightarrow \neg \Delta \neg A_{i+1}$ where $A_i$ represents the claim that the $i$th person in bald. To deny this would be to countenance a determinately bald person adjacent in the sequence to a determinately non-bald person, and this is just to say that it’s not vague where the cutoff point for being bald is.

A sorites for the property of being bald is also a sorites for the property of being determinately bald. Although classical logic guarantees that there’s a last determinately bald person in the sequence, it ought be borderline which that last person is: so if it’s determinate that a given person is determinately bald, it shouldn’t be determinate that the adjacent person isn’t determinately bald. This would be problematic for the same reasons that determinacy at the first order is problematic: if there were a single hair that could make the difference between being determinately bald and not determinately bald, and it weren’t borderline where that cutoff is, why is it we can’t determine where the cutoff point is? This kind of inability to say where the cutoff points are is exactly the kind of phenomena that vagueness was supposed to explain, yet here we would have the phenomenon without the vagueness. Similar observations apply for the cutoff point for being determinately non-bald, and this is just to say that it’s not vague where the cutoff point for being bald is.

The informal considerations above, I hope, give us reason to accept the gap principles. Thus, writing $\Delta^n$ for a sequence of $n$ $\Delta$s, the general gap principle is:\footnote{Note that if we assumed S4, as Field does, the gap principles are all equivalent to the $n = 1$ instance.}

\[ \text{GAP PRINCIPLE: } \Delta^n A_i \rightarrow \neg \Delta^n \neg A_{i+1}. \]

The informal considerations above, I hope, give us reason to accept the gap principles. Thus there ought to be rational credence functions, $Q$, that accept the gap principles.

Moreover since it is practically a conceptual truth that someone with no hairs is bald
and that someone with a million hairs isn’t, we should be able to find a rational credence function that also accepts $A_0$, the claim that the first guy is bald, and accepts $\neg A_{1,000,000}$, the claim that the guy with a million hairs is not bald (aside: the argument below works even if we weaken these two assumptions to: $Q(A_0) > \frac{1}{2}$ and $Q(\neg A_{1,000,000}) > \frac{1}{2}$). But now we may reason as follows, using $\mathcal{M}$ to abbreviate $\neg \Delta^n$:\footnote{This argument is an adaptation of the argument presented in Fara [41].}

1. $Q$ assigns the gap principle probability 1, so $Q(\Delta^m(\Delta^{n+1} A_i \rightarrow \mathcal{M} \Delta^n A_{i+1})) = 1$ applying Iteration $m$ times.

2. $Q(\mathcal{M}^m \Delta^{n+1} A_i \rightarrow \mathcal{M}^{m+1} \Delta^n A_{i+1})) = 1$ from 1, by the fact that $\Delta^m (A \rightarrow B)$ entails $(\mathcal{M}^m A \rightarrow \mathcal{M}^m B)$ in KT.

3. $Q(A_0) = 1$, so $Q(\Delta^{1,000,000} A_0) = 1$ by applying Iteration a million times.

4. Then $Q(\mathcal{M} \Delta^{999,999} A_1) = 1$ from 3 by 2 with $n = 999,999$, $m = 0$ and $i = 0$.

5. Then $Q(\mathcal{M} \mathcal{M} \Delta^{999,998} A_2) = 1$ from 4 by 2 with $n = 999,998$, $m = 1$ and $i = 1$.

... \[
6. \text{So } Q(\mathcal{M}^{999,999} \Delta A_{1,000,000}) = 1
\]

7. But also $Q(\neg A_{1,000,000}) = 1$ so $Q(\neg \Delta A_{1,000,000}) = 1$ (an entailment in KT.)
8. Thus $Q(\Delta^{999,999} \land A_{1,000,000}) = 1$. Thus $Q$ assigns 1 to two claims that are inconsistent in $KT$ (see 8 and 6), which contradicts the second axiom. Although I think this shows that ultimately Field's theory is not suitable for dealing with vagueness, it is the most well developed non-probabilistic theory on the market, and will provide a useful thing to compare probabilism against in what follows.20

5.2.2 Dutch book arguments

Historically, arguments for probabilism have often taken the form of a Dutch book argument. Dutch book arguments start by assuming that your credence in a proposition, $p$, can accurately be revealed by your dispositions to accept certain kinds of bets. Thus, for example, the Dutchbooker will assume that if an agent has a credence of $x$ in a proposition $p$, then they would accept any bet that costs less than $x\cdot$ to buy and pays out $1$ if $p$ and nothing otherwise. Given this assumption we can then argue that anyone who has probabilistically incoherent credences will behave incoherently: they will buy bets that are guaranteed to lose them money no matter what happens.

Of course, the assumption that an agents credences are revealed by their betting behaviour is not always reasonable. For example, you could imagine an extreme philanthropist who cares only about giving others, but whose only means of giving is through losing bets. The philanthropist is not necessarily being probabilistically incoherent when she accepts bets that are guaranteed to lose, nor is she even being irrational: she is achieving what she desires the most.

Another type of situation where the assumption isn’t plausible: suppose your friend offers you a bet, to be paid out immediately, that we will have created strong artificial intelligence by the year 2214. Here I take it that your betting behaviour does not reveal your credences regarding the existence of A.I. by the year 2214, because you can be pretty sure that your friend doesn’t know the answer and therefore you cannot be certain that your friend will pay you if there will be A.I. by 2214.

Bearing these these exceptions in mind, is it possible to run a Dutch book argument for probabilism over the domain of vague propositions? Richard Dietz argues that the Dutch book arguments actually establish the opposite. According to Dietz, probabilism is false, and in fact Field's theory of probability is warranted (see Dietz [29].21) In order to proceed to argue for probabilism I therefore firstly need to diffuse the Dutch book argument against probabilism.

The actual Dutch book argument Dietz employs establishes Field’s theory of probability, however the basic conflict between probabilism and vagueness related betting can be described without getting into this more general argument. The motivating thought is that when people are offered bets on a proposition they know to be borderline, it seems as though they shouldn’t accept favourable bets on that proposition nor should they accept favourable bets on the negation of that proposition. If, for example, I know that it’s borderline whether Harry is bald and someone offers me a bet that costs a cent and pays out $1$ if Harry is bald it seems as though I should reject it, which by the betting-credence connection suggests my credence should be less that 0.01. Similarly I should reject the symmetrical bet that costs a cent and pays out $1$ if Harry is not bald. Thus my credence in the proposition that Harry is bald and in its negation is 0.01 in both cases, and this is incompatible with probabilism, as stated within classical logic, because my credence in the disjunction is 1 and this is not the sum of my credence in the disjuncts as it ought to be.

However, on closer inspection it seems as though this argument involves exactly the kind of situation that we already set aside as being the kind of case where your betting

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20 One might be able to alleviate some of these issues by weakening the theory by replacing axiom A1 with REJECTION. The resulting theory doesn’t entail ITERATION and so the above argument will not go through, although it is unclear if there are variant arguments that will cause trouble – this is not something I shall explore here.

21 Smith [116] also uses Dutch book considerations to argue for a non-standard theory of probability.
behaviour does not reveal your credences. A bet on whether Harry is bald is like a bet on whether there will be A.I. by 2214 in the sense that in both cases we can be pretty confident that the bookie will never know the answer to these questions, and therefore that the bookie won't be paying you back in a way that corresponds with the bet you accepted. Your dispositions regarding bets on the proposition that $p$ only reveal your credence that $p$ if you are certain that you'll get the payoff if and only if $p$.

(Note, of course, that if Field is right then we are not uncertain about whether the payout will correspond with the proposition we are betting on: we assign a credence of 0 both to the claim that it will correspond and to the claim that it won't. However we are still ignorant in the sense that we know neither that the payout will correspond to $p$ nor that it won't, and so it is like the kind of case we set aside in the respect that matters.)

There are ways to define what it means for something to count as a 'bet on $p$' in which it is plausible that you know the payoffs of a bet on a borderline proposition. For example, Dietz defines a bet on $p$ to be something which is cancelled if $p$ turns out to be neither true nor false – the bet pays out if $p$ is true, you lose if $p$ is false, and you get refunded otherwise. This manoeuvre merely disguises the fallacy and moves it elsewhere. When Dietz talks about truth, he is not using it in the deflationary way we have been using it – he rather means something like supertruth. The proposition that $p$ and the proposition that it’s supertrue that $p$ are not identical – the latter entails the former, but not conversely. Consequently a bet that pays out if and only if $p$ is not the same as a bet that pays out if and only if it’s supertrue that $p$. It’s natural to think that the agents betting behaviour with respect to bets of the latter sort might reveal their credence about whether it’s supertrue that $p$. Thus bets are cancelled when the proposition being betted on turns out to be borderline. However, why think that such bets will reveal an agent’s credences in $p$? If you thought that the agent’s credence in the proposition that $p$ and her credence in the proposition that it’s supertrue that $p$ had to be the same then you could make a case that these two things amount to the same thing. But this is exactly Field’s theory of probability, and so a Dutch book that assumes this theory from the get go cannot provide an independent argument for Field’s theory, or an independent way of undermining probabilism.

Note that a similar move could be applied completely generally to bets made when the bookie is ignorant: bets on $p$ could simply be cancelled when the bookie isn’t in a position to verify whether $p$. Perhaps if this were the agreement I’d be indifferent about making a bet with my friend about A.I. in 2214, but clearly the amount I’d be willing to spend on this bets wouldn’t say anything how likely I take it for there to be artificial intelligence by 2214. If I were confident that the bet would be cancelled whatever happens, I could bet as I pleased confident that I’ll just get my money back with no loss or gain.

To actually have a Dutch book establish anything about credences in a borderline proposition, $p$, we’d have to somehow set things up so that the agent could be sure that they will get the payoff if $p$. This might sound impossible when $p$ is borderline, however I think it is at least theoretically possible, if not a little contrived, to construct scenarios where this holds. Consider the following example. Over a period of 4 years Fred revises his will 17 times at regular intervals. At the beginning of the 4 years he is perfectly sane, and by the end of the 4 years he is clearly insane. At the time of Fred’s death, the most recent valid will determines the transfer of money and property to any beneficiaries. Let us suppose that a will written by a mad man does not legally confer ownership, and we may assume this is the only reason any of the wills might fail to be valid. Now suppose at some point at which he is bordering on madness this man offers you a bet over whether he is currently mad. The bet costs $50 to participate in: if he is mad you’ll get $100 and if not you’ll get nothing. How can you possibly be certain that you’ll receive the $100 just in case he’s mad? It is simple - he writes it into his will. We may assume that when he dies it is indeterminate whether the $100 belongs to you, or to whomsoever it is promised in the other versions of his will. However, due to the nature of the situation, it is determinately true that you will
legally own the $100 if and only if he was mad. Thus you may be certain that the bet is not defective; you are certain that you'll legally own the money if and only if he's mad.

It's natural to think that this type of example involves the other kind scenario where betting behaviour and credence come apart, discussed in relation to the philanthropist example. If the agent does not care about owning money then we cannot conclude anything about her credences from her dispositions to accept bets with monetary payoffs. Now you might think that normal people only care about legally owning money because of the things they can buy with it, and thus would have little use for owning money that is only borderline owned. Since in the above example it will be at best borderline whether you own the money if you accept the bet, then it looks as though the assumption that our preferences correspond linearly with money breaks down.

One could respond by pointing out that even if normal people don’t care intrinsically about owning money, you could imagine somebody who did – surely it’s possible to care intrinsically about anything. At any rate, it should hopefully be clear here that to get a Dutch book for probabilism up and running it would have to be quite contrived. Better would be an argument that skipped the connection between degrees of belief and physical behaviour, and worked directly with the connection between degrees of belief and desire. Arguments that start with assumptions about what desires and preferences are rational, and from this determine that credences ought to be probabilistically coherent, are corollaries of the representation theorems for decision theory, and we shall turn to these in the next chapter.

These considerations also suggest a lacuna in the Fieldian account of probability. For although we can argue, from an *ad hoc* principle, that Field’s theory predicts that someone may refuse bets with excellent odds on a proposition and with the same odds on its negation, if they are certain the proposition is borderline, it would be nice to derive this principle from a more general decision theory.

However it is far from clear how to develop such a theory. For example, standard decision theory works with the notion of the ‘expected utility’ of an action, which has the property that for any \( p \), the expected utility of the action is identical to the expected utility of the action given \( p \) times the probability that \( p \) plus the expected utility of the action given that not \( p \) times the probability that not \( p \). However according to Field this result would yield an expected utility of 0 whatever the action was, if we were to choose a \( p \) that we were certain was borderline: we would be multiplying by each summand by 0. This result would be devastating for Field, so whatever theory we use it cannot be founded on the notion of expected utility as standardly conceived.

A natural way around this would be to look at classical probability functions related to Fieldian probability functions in suitable ways and use these probability functions to calculate the expected utilities. Since the probability of \( p \) and \( \neg p \) can add up to less than 1 for Fieldian functions, Fieldian functions systematically underestimate probabilities relative to classical probabilities. However for each Fieldian function there are probability functions that agree or assign higher probability than the Fieldian function for every proposition – indeed, there will typically be many such functions. For each choice of one of these functions there will be a range of expected utilities we can assign to an action: there are thus a number of possibilities for defining preferences from these utilities. Perhaps the agent should prefer \( p \) to \( q \) if the lowest expected value \( p \) can take relative to one of these functions is greater than the greatest value \( q \) can take. This is a strong notion of preference: the range spanned by the possible utilities of \( p \) has to be completely above the range spanned by \( q \). Other proposals in the ballpark are also possible; perhaps we should compare the lowest values of both \( p \) and \( q \), or the two highest values, or the highest of \( p \) against the lowest of \( q \).

I think none of these ideas accord with our intuitions particularly well. For example, let us suppose that I’m presented with a bag containing 4 balls. I know that 3 of them are clearly red, but one of them is a borderline red/orange ball. A ball has been selected at random; I’m pretty, but not totally, sure that the ball is red. Letting my Fieldian
credence be $Q$ and letting $R$ be the proposition that the ball is red then $Q(R) = \frac{3}{4}$ and $Q(\neg R) = Q(\Delta \neg R) = 0$. Now I’m offered two bets: one pays out a dollar if the ball is red, the other pays out a dollar and twenty cents. Now of course, to get this to work we’d have to make the payouts penumbrally connected with the redness of the balls in contrived ways, but assuming that this can be done it seems obvious that I should prefer the second bet to the first, and that this should be born out in my preferences over the proposition that I accept the first bet over the second. However if we look at the first proposal we don’t get this: the expected utilities of the former bet spans $[0.75, 1]$ while the latter spans $[0.9, 1.2]$, and the lowest value of the first bet is not greater than the highest value of the second. (Similar problems can be constructed if we instead compare the highest to the lowest value.)

Comparing the two lowest values doesn’t do any better either. If I’m certain that Harry is bald then my Fieldian credence in that and in its negation is 0. One can then construct classical probability functions related to your credences in the appropriate way that assigns the conjunctive proposition that Harry is bald and I win the lottery the probability 0. Thus the proposal predicts that I should prefer it to be the case that I have won ten pounds than for it to be the case that I have won a million dollars and that Harry is bald. A similar problem can be constructed if we compare the two highest values (just replace winning the lottery with something very bad, and winning ten pounds with something bad but not very bad.)

Lastly, rather than looking at the highest and lowest values that action can take, you might compare, for each choice of admissible probability function, the expected value of $p$ to that of $q$. If they all agree about which is better, then that one is better simpliciter. Unfortunately, if they don’t all agree about which is better we get incomparable preferences: two options that aren’t just as good as one another, but neither is one better than the other. There has been a long standing puzzle concerning how we should relate such preferences to our decisions. When we’re faced with a decision we have to do something, so people typically just behave as though they did have a preference one way or another, so its hard to say what the meaning of these incomparable preferences amount to. (See Elga [37] for further discussion of these problems.)

The brief considerations above are quite clearly not supposed to be an exhaustive discussion of all the possible ways of developing a decision theory within Field’s framework. It is merely a survey of the approaches that strike me as most promising; their shortcomings give us at least a good reason to suspect that an adequate theory will not be forthcoming.

### 5.2.3 Comparative probability judgments

As we noted in the last section, to run a Dutch book argument for probabilism you would need someone who cared intrinsically about the vague, and it is natural to think that there is something inherently irrational about that. Thus there is something a bit awkward about this argument: to show that someones beliefs are irrational we have to consider a situation in which their desires are also irrational to demonstrate that they’re dutchbookable. Although it is surely correct to say that credences that recommend accepting bets that are guaranteed to cost you the things that you care about are bad credences to adopt, this premise loses some of its intuitive force if it was irrational for you to care about those things in the first place.

It would be nice to have another argument for probabilism that doesn’t have this feature. A natural place to start looking for this is our pretheoretic intuitive judgments about probabilities. Needless to say, some of these judgments should be taken with a grain of salt. However I think they do provide us with some kind of defeasible support for probabilism. I’ll start by looking at a couple of intuitive probability judgments that cast doubt on Field’s

\[ \text{114} \]
probability theory, and then I’ll attempt to generalise this thought to give an argument for
probabilism.
Consider the following example: imagine I’m about to roll a hundred-sided die, whose
sides are labelled 0 to 99. The probability that the die will land on any particular number
is 1%. So intuitively I should be fairly sure that the number it’s going to land on won’t be
the last two digits of the age in nano-seconds at which I stopped being a child: which is,
after all, a particular number. There’s only one of this number, and 99 of the others, and
whichever one it is, there’s a 99% chance it won’t land on that one due to the fact that
the die is fair. So I should be 99% certain that it’s not going to land on the last two digits
of time in nanoseconds at which I stopped being a child. But I’m absolutely certain that
whichever number it does land on, it’ll be borderline whether it’s the last two digits of the
critical time at which I stopped being a child, because I’m certain there are at least 100
borderline cases (I was a borderline child for well over a second, so there will be thousands
of borderline cases.) Prima facie, this seems to be a case in which
\[ Cr(\nabla p) = 1 \] (or at least,
high) and \( Cr(p) = .99 \), contradicting Field’s prediction that \( Cr(p) = 0 \) (or low).
Field’s proposal also runs afoul of our intuitions about comparisons of probabilities of
borderline propositions. According to Field if you are certain that \( p \) and \( q \) are borderline,
you cannot think that \( p \) is more likely than \( q \), or that \( q \) is more likely than \( p \). They both
receive credence 0. But this, once again, does not accord with our intuitive judgements.
Suppose you are looking a two glasses, one of which is 65% full the other 67% full. Now
you might be certain that it’s borderline, in both cases, whether the glass is pretty full,
but you should be more confident that the second is pretty full, since it is clearly more full
than the former.
Let’s see if we can generalise this thought. The framework that I shall adopt is Bruno
de Finetti’s theory of comparative probability whose only primitive is the notion of one
proposition being at most as probable than another, written \( A \leq B \). We can state what it
means for \( A \) to be strictly less probable than \( B \), \( A < B \), with the definition \( B \not\leq A \).
Given this interpretation the following principles (adapted from de Finetti’s axiomatisation) seem
very natural, at least when restricted to the domain of precise propositions:

1. \( \leq \) is a total ordering (it is reflexive, transitive and connected.)
2. \( \bot \leq A \leq \top, \bot < \top \).
3. \( A \leq B \) if and only if \( A \lor C \leq B \lor C \), whenever \( AC \leq \bot \) and \( BC \leq \bot \).

One thing to observe about this axiomatisation is that it makes very few logical assumptions.
I have assumed that there is an inconsistent proposition, \( \bot \), intuitively to be thought of
as the conjunction of all propositions, and a tautologous proposition, \( \top \), to be thought
of as the disjunction of all propositions. The existence of these propositions are rarely if
ever contested by deviant logicians and there is no straightforward reason why a deviant
logician couldn’t accept these principles. In de Finetti’s original axiomatisation the claims
‘\( AC \leq \bot \)’ and ‘\( BC \leq \bot \)’ in the third principle – intuitively stating that \( AB \) and \( BC \) have
no probability – are replaced by the statements that ‘\( A \) and \( B \) are incompatible’ and ‘\( B \)
and \( C \) are incompatible’, where inconsistency in classical logic is taken to be sufficient for
incompatibility. For this reason de Finetti’s axiomatisation is not logic-neutral.
Are these principles any less attractive once we allow the domain of \( \leq \) to include vague
propositions? I take it that axiom 2 is practically definitional of \( \bot \) and \( \top \), and that the
transitivity and reflexivity of \( \leq \) are unassailable. This leaves us with axiom 3, which
corresponds intuitively to the finite additivity, and the connectedness of \( \leq \).
To say that comparative likelihood is connected is just to say that for any pair of
propositions one is at least as probable as the other. There are certainly reasons to worry
about this principle that have nothing to do with vagueness. Here is an example from

23The adequacy of the definition depends on the axioms governing \( \leq \), in particular, on the axiom that \( \leq \)
is connected.
Elga [37]: ‘A stranger approaches you on the street and starts pulling out objects from a bag. The first three objects he pulls out are a regular-sized tube of toothpaste, a live jellyfish, and a travel-sized tube of toothpaste.’ It is very hard to imagine, in this scenario, that the proposition that next object will be a jellyfish is either at least as probable or more probable than the proposition that it’ll be a tube of toothpaste. You might feel that intuitively these propositions are incomparable because your evidence is just too unspecific to allow a determinate comparison of probability.

Elga goes on to describe a number of compelling arguments for the claim that, despite appearances, these propositions are comparable. However the important point, as far as we’re concerned, is that this phenomenon has nothing specifically to do with vagueness. Thus, for example, while it might be very hard to say whether or not it’s more probable that Harry is bald than that Sally is old when they are both borderline, matters are no better in the precise case either. It is just as hard to imagine that it’s more, less or just as probable that I’ll live until I’m eighty as that it’ll rain tomorrow. Notice, however, that when the propositions are about a similar subject matters it becomes much easier to make comparative judgments in both the vague and in the precise cases. For example, it is much easier to say which of two people is more likely to live until they’re eighty, assuming you know a little bit about their lifestyles. Similarly you can quite easily make judgments about which of two people is more probably bald than the other, given you know roughly what their hairline looks like even when you know both people fall within the borderline region.

It seems, then, that worries to do with comparability have nothing specifically to do with vagueness; it is more likely that they stem from the general difficulty of making probabilistic comparisons between different subject matters.

It may be, then, that probabilism is false for reasons having nothing to do with vagueness. Let us set those skeptics aside for the time being. For the rest of us – who are fine with probabilism in the general case – the connectedness of comparative likelihood should be no more controversial in the presence of vagueness as elsewhere.

Axiom 3 states that if \( AC \) and \( BC \) are maximally improbable (are no more probable than the conjunction of every proposition) then \( A \) is no more probable than \( B \) if and only if \( A \lor C \) is no more probable than \( B \lor C \). In Field’s theory this principle fails. For someone who knows that Harry is borderline bald the proposition that Harry is bald is no more probable than the inconsistent proposition \( \bot \), yet the proposition that either Harry is bald or he isn’t is strictly more probable than the proposition that either the inconsistent proposition is true or Harry is not bald.

Field’s theory, however, collapses all distinctions of probability between propositions we know to be borderline and, as noted above, this has some counterintuitive consequences. It’s natural to want to explore the alternatives: is 3 plausible, for example, if we acknowledge that we can make comparisons of probability between certainly borderline propositions? It seems to me like a fairly small step between acknowledging that non-trivial comparisons can be made between vague propositions to saying that these comparisons are preserved under disjunctions. If you judge \( A \) to be more probable that \( B \), whether or not they are borderline, then you should judge \( A \lor C \) to be more probable than \( B \lor C \) whenever \( C \) is incompatible with both \( A \) and \( B \).

Let us demonstrate the axiom with an example explicitly involving vagueness. Consider two patches, \( a \) and \( b \) that are on the borderline between green and blue, except that one, patch \( a \), is slightly on the greener end of the colour scale than the other, patch \( b \). Consider the following propositions: \( A \), that \( a \) is green, and \( B \), that \( b \) is green. Presumably, even though we are certain that \( A \) and \( B \) are borderline we should be more confident in \( A \) than \( B \). Now consider the proposition \( C \), that neither of the patches are green and that I’ll get struck by lightning tomorrow. Surely the fact that I am more confident in \( A \) than \( B \) is enough to ensure that I should be more confident in \( A \lor C \) than \( B \lor C \). This is just say, making some logical simplifications, that I should be more confident that either \( a \) is green or I’ll get struck by lightning tomorrow, than that \( b \) is green or I’ll get struck by lightning.
tomorrow and a is not green.

If we add to these principles the constraint that the algebra of propositions forms a Boolean algebra it is natural to wonder, as de Finetti conjectured, if it is possible to construct a classical finitely additive probability function which agrees with comparative ordering about the relative probabilities of each pair of propositions. In fact, as Kraft et al [74] showed the answer is ‘no’: there are some orderings over small finite algebras that satisfy de Finetti’s axioms (and our variants) that are not representable by probability functions. However the axioms do suffice for representability provided the structure of propositions is sufficiently rich. To get around this one can restrict attention to structures that are sufficiently rich. Here is one condition that suffices for the structure to count as sufficiently rich:24

Suppes’ condition If \( A \leq B \) then there is a \( C \) such that \( A \lor C \approx B \).

Here \( A \approx B \) simply means that \( A \leq B \) and \( B \leq A \). This principle isn’t obviously purely about probability, but ensures that there are enough propositions, and thus comparison facts, floating about to guarantee the existence of a probabilistic representation of the ordering facts.25

The upshot of all this, I think, is that unlike the Dutch book arguments, the argument from comparative probability judgments provides an argument for probabilism that has purchase even in the presence of vagueness.

5.3 Logic and Probability

Let’s summarize where we’ve got to. According to epistemicism questions about borderline matters are simply questions whose answers will be forever unattainable – we won’t ever be able find out their answers. In order to distance themselves from this surprising view competing classical accounts of vagueness have attempted to press back against the idea that we really are uncertain about the answers to these questions. However once we have conceded that there is a single attitude that abides by the probability calculus and assigns the various answers to these questions intermediate degrees of belief, is there anything left to distinguish ourselves from the purely epistemic account of vagueness?

Supervaluationists often emphasize the fact that while epistemicism asserts the existence of a unique interpretation of a language determined by the linguistic practices of its users and other relevant factors, the supervaluationist admits many. However this distinction depends crucially on what we mean by ‘determined by the linguistic practices and other facts’. Presumably it doesn’t mean ‘supervenes on’: supervaluationists have no special reason to think that the meanings of our words don’t supervene on facts about how we use those words (and maybe a few other factors). In fact, there is a sense in which a unique interpretation of the language is determined for a supervaluationist – there is exactly one interpretation which, for every \( p \), makes \( p \) true if and only if \( p \) – even if it is indeterminate which this accurate interpretation is.26

On a more epistemic understanding of ‘determines’ the epistemicist agrees with the supervaluationist – Williamson, for example, thinks even if we know exactly how English is

\footnote{Suppes’ condition entails that all atoms are equiprobable, which you might think is too strong a structure condition. Lier’s condition doesn’t have this consequence and also suffices for representability: if \( A \) and \( B \) are atoms and \( A < B \) then there’s a \( C \) such that \( A \lor C \approx B \). See Lier [125].}

\footnote{Note that I have not included the Archimedian axiom which is often included to ensure that the entities that the probability function outputs will have the structure of the real numbers. Thus some of these comparative orderings won’t literally be represented probability functions, they will rather be represented by something pretty close to a probability function: either a function into the non-standard reals, or by Popper function. However these are close enough to probability functions to count the resultant ordering as probabilistic for my purposes. See Regoli [105].}

\footnote{Intuitively, relative to each interpretation \( i \), \( i \) itself counts as the accurate one: so it’s true relative to each interpretation that there’s an accurate one, although there’s no interpretation that’s accurate according to all interpretations.}
used there will be many interpretations of the language that for all we know are the accurate one. Even though the correct interpretation supervenes on use, we might be ignorant about how it supervenes on use because it is vague how it supervenes on use.

These ideas, therefore, look fairly unpromising. More recently Hartry Field has suggested ([45] p164) that the classical logician can escape the epistemic theory by endorsing inferences that the epistemicist ought to find unacceptable (and denying metainferences they will find acceptable), whilst simultaneously accepting all the classical theorems the epistemicist accepts. Let us explore this idea some more, and see how it relates to our discussion of uncertainty.

5.3.1 Global and Local Consequence

Even among people who believe all the theorems of classical logic there is disagreement concerning which inferences are valid; some endorse more inferences and consequently relinquish metainferences that are typically associated with classical logic. That is, some endorse a ‘global’ account of logical consequence, which, roughly, is intended to capture general inferences that preserve determinate truth, and others endorse a ‘local’ account which is intended to capture the general inferences that preserve truth in the disquotational sense.

The global account of consequence is typically associated with supervaluationism, where the notion of ‘supertruth’ is intended to replace the usual role of truth in reasoning and in other domains. However there is an ambiguity in the literature – such supervaluationists often understand global consequence as not merely preserving determinate truth (i.e. supertruth), but as preserving determinately determinate truth, determinately determinately determinate truth, and so on. Thus the inference from \( A \) to ‘it’s determinate that \( A \)’ is taken to be a globally acceptable inference even though it does not preserve determinate truth. The inference does not preserve determinate truth because \( A \) can be determinate without it being determinate that \( A \) is determinate – to say otherwise would be to endorse \( S4 \), an extremely contentious logic for determinacy.\(^{27}\) On the other hand, it is quite natural to think that if \( A \) is determinate at all orders then it’s determinate that \( A \) is determinate at all orders so the inference does preserve the property of being determinate at all orders.\(^{28}\)

The inference from \( A \) to ‘it’s determinate that \( A \)’ is globally valid on one understanding of ‘globally valid’ but not the other. However there are inferences which are uncontroversially globally valid – globally valid on both disambiguations – that are not locally valid. Thus I can, for the most part, draw the conclusions I need by focussing on these inferences. Here is one example of an inference that is globally valid on either understanding:

\[(\Delta ECQ) \quad A, \neg \Delta A \vdash B\]

This is effectively a strengthening of the rule of explosion. If both \( A \) and \( \neg \Delta A \) were determinate then \( A \) would be determinate, and by factivity \( A \) would not be determinate. Thus it is simply impossible for the premises of this argument to be determinate, and ipso facto, impossible for the premises of this argument to be determinate at all orders. Thus in either sense of global validity these premises validly entail everything. This is not a local validity, however, since it is perfectly possible that both \( A \) and \( \neg \Delta A \) be true in the disquotational sense. If it’s borderline whether Harry is bald then by classical logic either Harry is bald and it’s borderline whether he’s bald or Harry is not bald and it’s borderline whether he’s not bald – either way we have a truth of the form ‘\( A \)’ but it’s borderline whether \( A \) from which ‘\( A \)’ but it’s not determinate that ‘\( A \)’ follows. So we shouldn’t be able to move from the truth of \( A \) and \( \neg \Delta A \) to anything whatsoever on the local understanding.

Accounts of consequence that accept the above inference cannot be closed under certain metainference rules commonly associated with classical logic. For example the inference rule

\(^{27}\)A variant of the argument in section [REF] shows why this logic is unsuitable.

\(^{28}\)Indeed, this fact is even provable given the natural assumption that a conjunction of determinate truths is determinate. See the discussion of this in chapter 8.
of reductio would allow one to infer from the validity of the above inference and conjunction elimination that \( \neg(A \land \neg\Delta A) \). This is equivalent to the absurd principle \((A \rightarrow \Delta A)\), which entails there is no vagueness. This same conclusion could also be inferred using conditional proof with some other uncontroversial logic. Thus conditional proof must be relinquished as well.\(^{29}\) For this reason people have called global accounts of consequence ‘semi-classical’.

The difference between the two types of consequence relation is that one represents the preservation of (disquotational) truth, and the other represents the preservation of determinacy, or perhaps determinacy at all orders. When the number of premises of an argument, \( P_1 \ldots P_n \vdash C \), is finite we capture both notions of consequence in terms of the validity of a single sentence (i.e. the validity of a zero premise argument): the argument is a local consequence if the sentence \((P_1 \land \ldots \land P_n) \rightarrow C\) is valid, and a global consequence if the sentence \((\Delta P_1 \land \ldots \land \Delta P_n) \rightarrow \Delta C\) is valid, understanding ‘globally valid’ as preservation of determinate truth. (One could similarly formalise the other understanding of global validity if one had an operator for expressing determinacy at all orders; I’ll just focus on the simpler understanding of global validity.)

Note, however, that once we’ve clarified what the two notions amount to, the disputants agree with one another on the status of \( \Delta \text{ECQ} \). Both disputants agree about the validity of \((\Delta A \land \Delta \neg\Delta A) \rightarrow \Delta B\) and agree about the invalidity of \(((\Delta A \land \neg\Delta A) \rightarrow B)\). Indeed, this just follows from the fact that they agree about the validity of all zero premise arguments. Thus on the face of it this distinction looks as though it is merely a verbal disagreement about how the technical word ‘logical consequence’ is to be used. It seems as though the only sentences the two kinds of theorist disagree about are sentences involving the word ‘logical consequence’ where it is being used without the qualification of ‘local’ or ‘global’.

But this is not a pretheoretic notion, it is a technical notion, and disagreements about how to use technical terms rarely ever turn on interesting questions.

Perhaps the debate would be more substantial if we could somehow anchor the technical notion of logical consequence to questions about how we should reason. If someone endorsing a local account of validity thought that certain patterns of reasoning were good, while at the same time the global theorist thought they were not we would have a substantial dispute on our hands for they would have a concrete disagreement about which kinds of beliefs it is permissible to form on the basis of beliefs in a given set of premises. If we wanted to distinguish ourselves from the epistemicist, we could then phrase it as a disagreement about which kinds of beliefs it is permissible to form on the basis of beliefs in a given set of premises. If we wanted to distinguish ourselves from the epistemicist, we could then phrase it as a disagreement about how we should reason.

It is here that that our theory of rational credences can be put to good use, for we can formulate probabilistic relations between the premises and conclusions of inferences and ask whether the different consequence relations correspond to good probabilistic reasoning. If, for example, the relevant probabilistic relation holds between the premises \( A \) and \( \neg\Delta A \) and \( B \) for arbitrary \( B \), then this provides vindication of a global account of validity, and if not it provides vindication of a local account of validity.

A natural connection to place between entailment and rational credence is that your credence in the conjunction of the premises of a valid argument must never exceed ones credence in the conclusion. Thus, for example, if \( A \) entails \( B \) then \( Cr(A) \leq Cr(B) \) for every rational credence function \( Cr \). What does this mean for the inference \( \Delta \text{ECQ}? \) On the picture I have been sketching ones credence in \( A \land \neg\Delta A \) can be greater than 0: if you are certain that it is borderline whether \( A \) you should be uncertain whether \( A \) and thus your credence in \( A \land \neg\Delta A \) is greater than 0. However we can clearly always find a \( B \), a contradiction for example, which we assign credence 0. Since our credence in the conclusion of an instance of \( \Delta \text{ECQ} \) can be lower than our credence in the conjunction of the premises, this appears to point against the validity \( \Delta \text{ECQ} \) and thus the global account of consequence.

In fact, assuming probabilism, it follows that if your credence in each instance of \( A \land \neg\Delta A \) is required to be 0 – as \( \Delta \text{ECQ} \) seems to dictate – then you are required to be certain in each

\(^{29}\)See Williamson [131], chapter 5, for a more comprehensive discussion of the metainferences that fail.
instance of its negation, which is equivalent to being certain in each instance of \( A \rightarrow \Delta A \); this is absurd for it entails that nothing is borderline. Non probabilistic theories are better off in this regard. For example according to Field’s account you are required to always have a credence of 0 in \( A \land \neg \Delta A \), and indeed all inferences that are globally valid (in the strong, preservation of determinacy at all orders, sense) are inferences in which it is not rational to be more confident in the conjunction of the premises than the conclusion according to Field’s theory.\(^{30}\)

At any rate, this seems like a substantial disagreement: someone who thinks that global consequence forbids a drop in credence from premise to conclusion cannot be a probabilist, and the debate about probabilism, at least, seems to be substantial. However this is not a particularly new way of distinguishing ourselves from the epistemicist; we could have done this just by adopting Field’s probability calculus, and we have already given reasons to prefer probability theory over this theory.

5.3.2 Two Kinds of Probabilistic Consequence

Does it follow, then, that probabilism favours the local account of validity and that a non-probabilistic theory, such a Field’s, favours the global account of validity? More importantly, given our arguments for probabilism and arguments against non-probabilistic theories like Field’s, do we now have an argument that local validity is the correct account of logical consequence?

I think that even here we have to be quite careful, because there are actually lots of different relations that can hold between premises and a conclusion, each of which makes for some precisification of ‘good reasoning’. For example, another question we might be interested in is whether knowing the premises puts one in a good position to know the conclusion. Or, in terms of credences, an analogous question might be put in terms of preservation of full confidence:

**Preservation of Certainty:** The inference from \( \Gamma \) to \( A \) preserves certainty iff
\[
Cr(A) = 1 \text{ whenever } Cr(B) = 1 \text{ for every } B \in \Gamma \text{ for every rational credence function } Cr.
\]

An extremely natural question to ask is, which inferences preserve certainty? Inferences that preserve certainty are, in one well defined sense, good inferences. This is, of course, a different question from asking for which inferences a drop in credence from premise to conclusion is forbidden and so it need not have the same answer.

Indeed, assuming the theory I have been sketching thus far, the answers to these questions are different. For example, while one’s credence can drop from the premises of \( \Delta ECQ \) to the conclusion, \( \Delta ECQ \) is still an inference that preserves certainty. Anyone who is certain that it’s not determinate that \( A \) cannot be at the same time certain that \( A \) – either you assign some credence to \( \Delta \neg A \) in which case you are not certain that \( A \), or you are certain that it’s borderline whether \( A \) which again, means that you are not certain that \( A \). Since it is impossible to be rationally certain in both of the premises of \( \Delta ECQ \), it is vacuously true that all rational credence functions that assign the premises credence 1 assign the conclusion credence 1.\(^{31}\)

According to the theory of rational credences I have been sketching it looks as though the inferences that preserve certainty are governed by some kind of global consequence relation. Whether it should be governed by the relation that corresponds to preservation of

\(^{30}\)See Williams [129] §3 for further discussion of these and related points.

\(^{31}\)Let me highlight a potential caveat to what I’ve been saying. Recall that in section ?? I gave an example in which you were certain that a proposition was borderline but 99% sure that it was true. It is natural to think that one could construct infinitary variants of that example in which you are both certain in a proposition and certain that it is borderline. If one takes conditional probability as primitive, something I’ll more rigorously in later chapters, one can define a stronger notion of certainty: you are supercertain in \( A \) if your credence in \( A \) is 1 conditional on any proposition – i.e. if \( Cr(A \mid B) = 1 \) for every \( B \). To account for these examples one could simply replace ‘certainty’ for ‘supercertainty’ in the above discussion.
determinacy, or preservation of determinacy at all orders will depend on further questions which I haven’t addressed yet. The question at stake is whether higher order vagueness requires one to be uncertain, or whether one can be certain in a higher order vague proposition provided it is not first order vague. I shall return to this questions in chapter 8.

Is there a similar characterisation of local consequence in terms of a notion of probabilistic inference? In the last subsection we considered inferences that didn’t allow a drop of credence from premise to conclusion. Unfortunately this notion of probabilistic inference is hard to characterise without a more informative account of which posterior credences are possible, which in part requires knowing which kinds of propositions could be ones total evidence. However in the present theory there is a notion of probabilistic inference that does correspond straightforwardly to local consequence. At a first parse, premises $\Gamma$ entails conclusion $A$ if the conditional probability of $A$ on the conjunction of the premises $\Gamma$ is 1 for every rational credence – i.e. if $Cr(A | \Gamma) = 1$ for every rational $Cr$. Actually there is a slightly more general formulation of this idea which will be more useful:

**Preservation of Conditional Certainty**: The inference from $\Gamma$ to $A$ preserves conditional certainty iff $Cr(A | C) = 1$ whenever $Cr(B | C) = 1$ for every $B \in \Gamma$ for every rational credence function $Cr$ and proposition $C$.

Note that if the inference from $\Gamma$ to $A$ preserves conditional certainty then for any rational credence function, $Cr(A | \Gamma) = 1$ since clearly $Cr(B | \Gamma) = 1$ for every $B \in \Gamma$.\(^3^2\)

What happens to $\Delta ECQ$ on this notion of probabilistic entailment? We argued that this inference vacuously preserved certainty because no rational credence function assigns a credence of 1 to both $A$ and $\neg \Delta A$. However, it is quite easy to construct a rational conditional credence function that assigns credence 1 to both $A$ and $\neg \Delta A$. Suppose that I am certain that it’s borderline whether Harry is bald, and that my credence that Harry is bald is consequently middling – for the sake of argument let’s say that it is exactly $\frac{1}{2}$. If my credences can be written $Cr$, and the proposition that Harry is bald is $A$, then the conditional credence function $Cr(X \mid A) = Cr(X \wedge A)/Cr(A)$ is a perfectly well defined probability function because $Cr(A) = 1/2 > 0$. Moreover $Cr(\neg \Delta A \mid A) = 1$ since $Cr(\neg \Delta A) = 1$ and clearly $Cr(A \mid A) = 1$. Letting $B$ be any proposition such that $Cr(B \mid A) < 1$ (e.g. letting $B = C \wedge \neg C$) we see that the inference from $A$ and $\neg \Delta A$ to $B$ does not preserve conditional certainty.

This highlights an important feature of rational credences on this view – rational credences are not closed under conditioning. If $Cr$ is a rational credence function and $A$ is a proposition such that $Cr(A) > 0$ it does not follow that $Cr(\cdot \mid A)$ is also a rational credence function. For note that in the above example $Cr$ was, by assumption, a rational credence function yet $Cr(\cdot \mid A)$ was not, where $A$ is the borderline proposition that Harry was bald. This observation also explains why preservation of conditional certainty is a stronger condition than preservation of certainty – if every rational conditional credence was a rational credence they would amount to the same thing.

This might initially strike one as puzzling, for there is a well known argument that the set of rational credences is closed under conditioning that starts from the assumption that conditioning is the correct way to update your credences in response to evidence. The argument goes as follows: if $Cr$ is the credence function of a rational agent and $Cr(A) > 0$ then that agent has not ruled $A$ out. Thus it is surely possible that the agent learns $A$, updates by conditionalisation, and remains rational in doing so – in which case her posterior credence function, $Cr(\cdot \mid A)$ is a rational credence function.

The mistake in this argument is to assume that every proposition that the agent is uncertain about could be evidence that they could recieve. For example, if I know that it’s borderline whether Harry is bald, then I should be uncertain whether Harry is bald, certain that he’s not determinately bald, and therefore uncertain in the proposition that Harry is bald.

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\(^3^2\)If we take conditional probabilities as primitive then preservation of conditional certainty is actually stronger than having a conditional certainty of 1 in the conclusion on the premises.
bald but not determinately bald. In other words uncertain about a proposition of the form $A \land \neg \Delta A$. However no proposition of this form could be someone’s evidence, assuming that one’s evidence is always determinate, for $\Delta (A \land \neg \Delta A)$ is inconsistent.

If there are epistemically possible worlds where $A$ is true while your present evidence is $E$ then there are worlds at which your present evidence is $E$ and $A$ is part of your future evidence. This is false: if $E$ entails that $A$ is borderline then $E$’s being your present evidence is consistent with $A$ and with $\neg A$, but not with $A$ being part of your evidence. For if your evidence is true, then $A$ is borderline and thus $A$ cannot be part of your evidence.

Of course I argued in chapter 4 that it was possible for your total evidence to be vague, but this doesn’t mean that it’s possible for your total evidence to be borderline. Your total evidence can be vague so long as it is in fact determinately true.

So what should we make of the distinction between local and global consequence on this view? If you want to know which inferences preserve certainty then you should be looking at a global consequence relation. If you want to know which inferences preserve conditional certainty you should be looking at a local consequence relation. However these do not correspond to different ways of reasoning — there is no disagreement about which beliefs or credences are rational to have, we are simply asking two distinct questions about the relation between the premises of an inference and its conclusion. There is no question of which relation represents the correct way to reason. They both state correct ways to organize your beliefs and conditional beliefs respectively, and these are not in conflict with one another. Which of the two probabilistic relations we prefer to honour with the word ‘logical consequence’ is merely a verbal issue.

5.4 Do all rational disagreements about the vague boil down to disagreements about the precise?

Is there anything distinctive to be said about vagueness related uncertainty? I have considered and rejected three theses that one might hope to use to elucidate what is distinctive about this state: (i) that vagueness related uncertainty is a fundamentally different psychological attitude, (ii) that one is permitted to violate the classical laws of probability when one’s uncertainty is due to vagueness and (iii) that there is no vagueness related uncertainty — that when one is certain in the precise facts one is certain about everything.

Of course, an important fact about borderline propositions is that we are ignorant about their truth value, and this ignorance isn’t eliminable — it is hard to see what evidence one could acquire, even in principle, that would put you in a better epistemic position with regard to borderline cases. However many people have the intuition that there is some important difference between our ignorance about whether Harry is bald, say, and our ignorance about precise facts for which it is also hard or impossible to improve our epistemic position. One might worry that, without anything distinctive to say about our state of ignorance in cases that arise from vagueness, the view collapses into a form of factualism about vagueness in which the only distinctive thing about vague truths is that they are very hard to verify. This does very little to distinguish them from other facts that are hard to verify, and one might think this is tantamount to accepting some form of epistemicism.

Another strategy would be to ground the difference in a metaphysical distinction between two types of true proposition — the true propositions that correspond to facts about reality, and those that don’t. We don’t know whether Harry is bald because there simply is no fact out there in the world to be known. However even if we can articulate that distinction in a satisfactory way, it is not clear that precision and vagueness have much to do with corresponding to reality in the metaphysical sense. The boundary delineating the vague from the precise propositions is itself vague, whereas it cannot be vague which propositions ‘correspond to reality’ in the metaphysical sense (granting the coherence of
It is worth comparing these problems with a number of related issues most commonly associated with expressivism about moral language. These problems generalise straightforwardly to similar views about conditionals and epistemic modals (and indeed many other expressions, although I shall limit my attention to these three.) What is common to these views is that they all take some portion of language, whether moral, hypothetical or epistemic, to fail to carve out real distinctions in the world.

Early formulations of moral expressivism maintained, variously, that there are no moral facts, that moral sentences do not express propositions and that speech acts involving moral vocabulary that appeared to assertions were really a different kind of speech act altogether. However moral sentences freely occur as the arguments of complementizers (‘that’, ‘whether’) and compose freely with verbs like ‘asserts’ and ‘believes’. Thus if we are understanding propositions as we have been, as the denotations of that-clauses, then there are moral propositions and we do assert and believe them. What is being denied is not the existence of the things that attitude verbs take as arguments: denying that would result in rendering many everyday attitude reports false or meaningless. What is being denied is the existence of an entity that corresponds to ‘the way the world is’ – some propositions carve out substantial distinctions whereas others, such as moral propositions, do not.

There is a real challenge in spelling out exactly what this distinction amounts to (see Field [49], Dreier [33], Dorr [30].) A trend among some more recent expressivists has been to cash out traditional expressivist slogans in terms of theses identifying doxastic attitudes towards moral propositions with attitudes (possibly not doxastic) towards non-moral propositions (see, for example, Gibbard [57]). While the early views denied that there were beliefs in moral propositions, and proffered other non-doxastic attitudes towards non-moral propositions as a replacement, these views freely acknowledge that we believe moral propositions, but instead identify these states of belief in moral propositions with other kinds of attitudes toward non-moral propositions.

In the three cases we brought up, we have the following theses in which beliefs about certain suspect kinds of propositions are identified with an attitude had towards an ordinary non-suspicious proposition:

**Conditional:** To have a certain credence in a conditional proposition, that if \( p \) then \( q \), is just to have certain credences in non-conditional propositions. In particular it is just for you to have credences in \( p \land q \) and \( p \) that have that ratio (Adams [1], Edgington [35], and others.)

**Epistemic Modals:** To have a certain credence in the proposition that it might be the case that \( p \) is just to have a certain credence in \( p \). In particular, your credence in the proposition that might \( p \) is one if your credence in \( p \) is non-zero, and your credence is zero otherwise. (Schulz [112].)

**Moral Propositions:** To have a certain credence in a moral proposition, that it is good that \( p \), is just to have a certain non-doxastic attitude towards the non-moral proposition that \( p \). For example, it is for your credence that it is good that \( p \) to be identical to the degree to which you desire that \( p \). (See the ‘Desire as Belief’ view criticized in Lewis [83]. A similar thesis identifying moral attitudes with non-moral attitudes is endorsed in Gibbard [57].)

In each case we have a thesis about the role a certain type of proposition plays in thought replacing a metaphysical claim about the status of those truths. To believe that the cradle

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33We will expand on this further in chapter 3.

34It is strictly speaking incorrect to talk of ‘non-doxastic attitude’ since the view is one that sometimes identifies the purportedly non-doxastic attitudes with beliefs. Desire as belief theses aren’t the only way to account for moral propositions – Gibbard maintains that deciding whether you ought to \( \phi \) is just deciding whether to \( \phi \); thus having a certain practical attitude towards a moral proposition is simply having that attitude towards a non-moral proposition.
fell if the bough broke (to some degree, or fully) is to do nothing over and above believing, to certain degrees, non-conditional propositions about the bough breaking and the cradle falling.

At this juncture one might worry that the literal identifications of these attitudes are too strong. Take, for example, Gibbard’s thesis that deciding what one ought to do is deciding what to do. It seems quite clear that one can decide that something is what one ought to do without deciding to do that thing. Perhaps a rational person will always make that last step, but it is a step that it seems as if one could fail to make. Similarly, it is natural to think that one could be fully confident that it’s not raining, while also being confident that it might be raining. Of course, it would be completely irrational to do this, but it is not metaphysically impossible, just as it is irrational, but not metaphysically impossible, to believe that it’s raining and it’s not raining.

No doubt one could tell a special story about attitude reports that accommodates these intuitions. One could also weaken the principles to govern only rational credences. Whichever route one goes, the principle that these identifications hold among rational people is granted by all parties, and I think that these restricted theses do a good job of elucidating some of these expressivist slogans. In each case we have what I’ll call the ‘rational supervenience’ of attitudes in one kind of proposition on another. For example, expressivists endorsing CONDITIONALS, ipso facto, endorse the thesis that, for rational people, your credences in the hypothetical propositions are completely determined by your credences in the categorical. Once you have distributed your credences over the categorical propositions you have no rational leeway regarding how you assign your credences over the remaining hypothetical propositions. That is we have:

**Supervenience of Hypothetical Beliefs on the Categorical Beliefs:** if two rational agents assign the same credence to every categorical proposition, they will assign the same credence to every hypothetical proposition.

This supervenience principle has a tight knit connection to the principle CONDITIONALS. CONDITIONALS tells us what it is to believe a conditional to a certain degree purely in terms of our degrees of belief in non-conditional propositions – so credences in simple conditional propositions are straightforwardly determined by credences in the categorical. A shortcoming of CONDITIONALS, however, is that it only tells us what it is to believe a simple conditional proposition, and not an arbitrary hypothetical proposition. I take it that hypothetical propositions include also arbitrary disjunctions, conjunctions and negations of conditional propositions, as well as conditional propositions with hypothetical antecedents and consequents. To get the full force of the supervenience principle we need principles that tell us what it is to have a credence in one of these extended hypothetical propositions purely in terms of our credences in categorical propositions (this problem is quite technical, but the existence of a solution in the general case is implied by the results in Bacon [6],35)

Regardless of the details, the takeaway message is that these versions of expressivism about each subject matter entail a kind of rational supervenience thesis. In the hypothetical case, for example, these effectively guarantee that all disagreements about hypothetical propositions derive from disagreements about the categorical.

Could these rational supervenience theses give precise cash value to the idea that hypothetical propositions are second rate, or derivative while categorical propositions are not? The driving intuition here seems to be that while you can have genuine disagreements about categorical matters, all disagreements about the hypothetical boil down to disagreements about the first rate, categorical facts. You might therefore think that the supervenience thesis goes a good way to elucidating some forms of non-factualism.

35Using the results in [6] one could in principle state the assertability conditions for an arbitrary sentences purely in terms of complicated equations involving credences only in categorical propositions. Thus one could, in stating the meta-semantic theory, dispense with hypothetical propositions altogether. This idea certainly seems to be more in line with writers working in the tradition of Adams.
It is, however, unclear to me whether this strategy will succeed in completely capturing non-factualism about hypothetical facts. One reason for skepticism is that supervenience is not an asymmetric relation: it could turn out that categorical beliefs also supervene on the hypothetical in the sense that once you know what a rational agent's credences in the hypothetical propositions are you can work out their credences in the categorical propositions. This symmetry problem is also a problem for the stronger thesis that simply identifies a credence in a hypothetical fact with a certain distribution of credences in categorical facts: the assertion of the identity of these states alone can't capture expressivism, because it could equally be taken to be a reduction of conditional categorical beliefs to unconditional hypothetical beliefs.

In light of all this, I'm not quite sure what to say on behalf of the expressivist. However, whether or not the supervenience theses (and the identity theses) fully capture the idea that certain facts are second rate, they do help distinguish these views from a pretty stark kind of realism about conditional facts. According to the stark realist the truth of hypothetical propositions don't depend in any important way on the categorical facts. Even once you have formed opinions about all the categorical facts there may still be a large number of opinions to be had about the hypothetical facts left open to you. If we are merely worried about distinguishing ourselves from the stark realist then the supervenience thesis does a good job at doing that.

5.4.1 The supervenience of vague beliefs on precise beliefs

Our primary concern in this chapter has been whether all classical accounts of vagueness collapse into epistemicism – a stark form of realism about vague matters. So a natural question to ask, given our previous discussion, is whether a vague beliefs rationally supervene precise beliefs in the way articulated above. If the supervenience thesis held it could serve as a partial articulation of the idea that vague truths are not as substantial as precise truths.

Excluding those who think that vagueness is compatible with knowledge, almost everyone agrees that whether it is permissible to be certain or uncertain about a vague proposition is determined once you know the relevant the precise facts. If you know exactly how many hairs Harry has (and any other relevant precise facts), and they put him in the borderline region, then, modulo Dorr and Barnett, everyone in the debate agrees that rational people must be uncertain whether Harry is bald. The issue at stake is whether the degree to which you are uncertain is determined by your credences in the precise facts. Although rational people must assign an intermediate credence, when they know how many hairs Harry has, they might nonetheless disagree about which intermediate credence to assign. The kind of supervenience thesis I shall defend here will rule this kind of disagreement out – once you know all the precise facts, you must agree about the vague.

However there are two different thoughts in the vicinity that are very easy to confuse. The first thought, explained above, is that there cannot be any disagreements – i.e. differences in credence – about vague matters between two rational people who know the relevant precise facts. The second thought is the more general thought that there cannot be any disagreements about vague matters between two rational people who agree about – i.e. assign the same credences to – the precise propositions. The second thesis is stronger because it applies to agents who are ignorant about the relevant precise facts, and it is analogous to the principle connecting hypothetical beliefs to categorical beliefs discussed above.

\[36\text{In evaluating this latter supervenience claim one has to be careful not to confuse a hypothetical proposition with a proposition that is expressed by a sentence containing conditionals: if } A \text{ and } B \text{ are categorical, the conjunction } A \land (A \rightarrow B) \text{ is logically equivalent to the categorical sentence } A \land B, \text{ given some natural assumptions about the logic of conditionals, so the former sentence does not express a hypothetical proposition.}\]
I suggested, in chapter 4, that a person’s rational credences need not supervene on their precise evidence and their prior probabilities. Assuming that credences are determined by conditioning on priors this suggests that some people’s total evidence is vague. For all that’s been said, it might be possible for two people to have different vague evidence, yet have the same credences in every precise proposition. This would be problematic for the stronger supervenience principle, but not the former.

Of course, these kinds of rational disagreements only arise when people have different evidence, and they are therefore not that surprising. It’s natural to think that the real question is whether people with the same evidence can agree about the precise and yet disagree about the vague. In other words, does one’s credences in the precise propositions, provided they are the rational ones to adopt on your evidence, uniquely determine what credences one ought to have in the vague propositions? This kind of supervenience thesis can be formulated as a constraint on which priors are coherent, namely, that all coherent priors agree with one another conditional on any maximally strong precise proposition.

**RATIONAL SUPERVENIENCE**  
Necessarily, for any pair of coherent ur-priors, \( Pr, Pr' \),  
\[ Pr(p \mid w) = Pr'(p \mid w) \] 
for every maximally strong precise proposition \( w \), and proposition \( p \).

Here is an informal description of the supervenience constraint. Think of logical space as being carved up into a number of non-overlapping cells representing the maximally strong precise propositions: states within a given cell will describe states of affairs that agree about all precise matters and differ from one another about things like whether Harry is bald and the like. Different coherent priors will typically disagree with one another about the sizes of these cells – some will regard certain cells as probable and others as less so. (It helps to think in terms of a Venn diagram where the areas of the cells correspond to their probabilities.) However if the supervenience principle is true then all priors must agree with one another about what proportion of each cell, each proposition takes up. If the proposition \( p \) takes up three quarters of a cell according to at least one prior, it must take up three quarters of that cell according to all priors. Intuitively, although you can change the sizes of the cells as you move from prior to prior, these changes must only come about by ‘stretching out’ each cell uniformly, so their contents scale proportionally.

It should be noted that for this principle to have any traction we must make some assumptions about the space of coherent priors. In particular, we must rule out the neo-Carnapian view that there is only one true ur-prior (see Williamson [133]). According to the neo-Carnapian the supervenience principle is vacuously true – since there is only one ur-prior the ur-priors always agree with each other conditional on each maximally strong precise proposition. If, however, the set of ur-priors is assumed to be sufficiently rich then it states a substantial thesis about the nature of vagueness and vagueness related uncertainty.

It is worth comparing the supervenience thesis to an account of inexact evidence we discussed back in chapter 4. The view was one in which all propositions were treated as precise – for the sake of argument, assume they are just represented by sets of possible worlds – but in which the changes in credence obtained from inexact evidence could be modeled by Jeffrey conditioning over a partition of precise propositions. Notice that such a theorist needn’t jettison all talk of belief in vague propositions. They could, if they wanted, say that although we fundamentally only have credences towards the precise, all it means to come to learn that a tree is around 200cm is to have a certain smooth distribution of credences over the precise hypotheses about the height of the tree. Putting it more precisely, suppose that \( E \) is the evidential role for the proposition that the tree is around 200cm: assuming my theory, \( E \) can just be thought of as the function from possible worlds to the interval \([0, 1] \) telling us how confident our priors should be that the tree is around 200cm conditional on that world obtaining. This theorist could then assert all it means to have credence of \( x \) in the proposition that the tree is around 200cm is to distribute your credences over worlds in such a way that \( \sum_w E(w) Cr(w) = x \). More generally:
**Expressivism about Vagueness:** To have a credence of $x$ in $p$ (i.e. for $Cr(p) = x$) is just for $\sum_{w} E(w)Cr(w) = x$ where $E$ is the evidential role of $p$.

This principle has much in common with the identity theses endorsed by other kinds of expressivists discussed above. However, like these expressivists you might think that it’s at least metaphysically possible to irrationally have a credence in a vague proposition without having the right credences in the precise.

As with our earlier discussion, you might think that the supervenience thesis, although weaker, still partially explicates the idea that vague beliefs are epiphenomenal. Our rational beliefs about the vague are completely determined by our evidence and our rational credences about the precise. There is no flexibility about how to distribute your credences once you’ve learnt all the precise facts: rationality dictates how you then distribute your credences in the vague.

It is perhaps worth comparing vague propositions to other propositions that are often alleged to be, in some sense, non-substantive, or at least to have a secondary or derivative epistemological status. Certain propositions – propositions expressed by paradigm examples of ‘analytic’ sentences – are such that your credences in them supervene on your credences in the more substantial propositions; they are dictated, given your other credences, by very general norms of conceptual coherence. An example would be the proposition that all crimson things are red; here the supervenience is trivial since all conceptually coherent credences assign probability 1 in this proposition. Another example would be the proposition that there are eight solar planets and all vixens are female foxes. In this case, your credence is derivative on your credence in the proposition that there are eight solar planets, indeed, it ought to be identical to it. Closely related examples are provided by the case of conditionals and epistemic modals considered already.

The derivative status of our beliefs in these propositions, in the case of conceptual truths and conjunctions with conceptual truths, are due to conceptual entailments between them and the substantive claims. This marks a salient difference between these examples, and the type of supervenience beliefs in vague propositions, conditionals and epistemic modals have. We argued in chapter 4 that there are no conceptual entailments between the proposition that Harry is bald, and the proposition that Harry has $N$ hairs. The relation between one’s epistemic attitudes in the vague and the precise does not consist purely in entailments, but it is still a relation which determines your attitudes towards the vague uniquely, given your attitudes in the precise. Once one is omniscient about all the substantive non epistemologically derivative claims, our credences in the other propositions are fixed by norms of conceptual coherence. In the examples just considered this is just a matter of us having a credence of 1 or 0 in those propositions. For vague propositions, there it may be that some intermediate credence is the only conceptually coherent credence to have.

This explains one puzzling feature of vague propositions. Even if your evidence is as complete as it can be, you may still be rationally uncertain about the vague. I might, for example, know everything there is to know about Harry’s hair number, distribution and colour, but not know whether he’s bald. It is natural to think that if one is rationally uncertain about something, there must be some further evidence that could be obtained that would settle the matter. Explaining why my uncertainty about Harry’s baldness is uneliminable is achievable on the current view: my uncertainty, given my particular evidence, is a requirement of conceptual coherence. It is part of the constitutive role of that proposition in thought, as it were, that I have a particular credence, $x$ say, when I’m rationally certain on my evidence that Harry has this particular hair number and distribution.

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37 The term ‘conceptual’ is close enough to what I mean here, at least for the purposes of this chapter. However, whatever the special status of these entailments is, I’m inclined to think it does not extend to many of the alleged examples of a priori entailments; such as, say, mathematical entailments.

38 There are some analogies to be drawn here between this idea and that of Horwich [65]. Horwich’s...
I shall henceforth use the term ‘conceptual requirement’ to denote both the simple kinds of requirements, such as having credence 1 in the proposition that vixens are female foxes, and the more complicated ones, such as having a certain credence in the proposition that Harry is bald, given you know he has $N$ hairs. As always, I shall understand these requirements as being fairly objective – one may reasonably fail to know what you are required to believe.

5.4.2 Defending the supervenience thesis

Let us now move on to defending the supervenience thesis.

We began this discussion by drawing an analogy between the kinds of requirements characteristic of propositions expressed by conceptual truths and the requirements characteristic of vague propositions. In fact it can be seen that these requirements are of one and the same type. Consider

\begin{align*}
\text{All crimson things are red} \quad (5.1) \\
\text{All auburn things are red} \quad (5.2)
\end{align*}

(5.1) denotes a paradigm example of a conceptual truth. Therefore it is a conceptual requirement that I have full credence in the proposition that all crimson things are red. However (5.2) is not a conceptual truth: there are auburn things such that it’s vague whether they’re red. I therefore am not required to assign full credence to the claim that all auburn things are red.

The supervenience thesis entails that for each shade $F$ there is some specific credence, $x$, that I’m supposed to have in the claim that all $F$ things are red provided I know all the precise facts (let’s say, all the facts about which objects are which precise shades.) When $F$ is ‘crimson’ that number is 1. As $F$ varies over the shades in between crimson and auburn the number cannot remain at 1, since we know that we should not be certain that all auburn things are red. According to the picture I prefer, conceptual requirements such as the requirement to have credence 1 in the proposition that crimson things are red transform seamlessly to the kinds of requirements I postulate – for example the requirement to have a credence of 0.745 in the proposition that auburn things are red. Is it possible that when $F$ is substituted for ‘auburn’ there are multiple credences it is permissible for me to have?

On the simplest picture, the one I am trying to motivate, the answer is ‘no.’ Just as there is one specific credence I’m required to have in the proposition that all crimson things are red, there is one specific credence I’m required to have in the proposition that all auburn things are red.

The simplest picture also provides my answer to input problem for Jeffrey conditioning we mentioned in section 4.2: given a prior credence $Pr$, and evidence $e$ (whether propositional or not), what partition and what coefficients should I Jeffrey condition relative to? In my set up $e$ is a vague proposition and your posterior credence should be the result of conditioning on $e$. We argued that this was equivalent to Jeffrey conditioning on a partition theory provides an explanation of why we are disinclined to apply certain vague predicates in borderline cases. According to this theory it is constitutive of the meaning of a vague word, such as ‘bald’, that to be competent with it you be disinclined to apply the word or its negation in cases were the subject is borderline bald. Of course, we are not disposed to apply the precise word ‘has an even number of hairs’ or its negation to people either. However this pattern of linguistic inclinations obtains because we are ignorant of something whereas the application of ‘bald’ is precluded by our linguistic competence alone. Note, however, that this theory relies on a controversial theory of meaning whereas the present proposal does not. Moreover, it falls afloat of the problems we discussed in chapter 2. The theory can explain why competent English speakers are usually disinclined to utter the sentence ‘Harry is bald’ or ‘Harry is not bald’ when they know that Harry’s hair number is in a certain range, but it doesn’t explain why they don’t know whether he’s bald in those circumstances and this, I take it, is one of the most important jobs of a theory of vagueness.

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of precise propositions—the partition of precisifications of \( e \). The supervenience thesis provides us with an answer to what the coefficients are: the coefficient for the precisification \( p \) is the probability that every conceptually coherent prior assigns to \( e \) conditional on that precisification. This number is independent of the agents priors, as required.

What would it take for the simplest picture to be false? Could there be two rational agents who were certain about all the relevant precise matters, and yet disagreed about a vague proposition? It is very hard to see how this disagreement would manifest itself.

If two agents could disagree about a vague proposition \( p \) whilst having all the precise information available and having the same preferences how would this difference manifest itself. Could it manifest itself in a difference in behaviour? Could it manifest itself in some other aspect of the agents internal mental state?

Under certain assumptions a difference in one’s credences in the vague really could manifest itself in a physical behavioural consequence. The example I have in mind runs as follows.

Alice and Bob are in an auction house and they are both bidding on a relatively inexpensive vase which is a shade of turquoise that is indeterminate between being blue and green.

Now suppose we know the following things about Alice and Bob. Firstly, they have exactly the same telic desires and have the same credences in the precise propositions. We also know the following facts. (1) as it happens they are both very peculiar people and they care intrinsically about owning green things. (2) although they both know exactly what precise shade the vase is, Alice is more confident than Bob that the vase is green.

In this scenario I think it is clear that Alice should be willing to bid more than Bob for the vase. Since they care about the same things its value is the same for both Bob and Alice. However, since they know that it’s vague whether the vase is green they are uncertain whether it’s green and therefore the purchase is a gamble on its being green. However, according to Alice’s subjective credences the odds that it is green are better than they would be according to Bob’s. Therefore Alice ought to outbid Bob.

However there is a feature of this example which is particularly bizarre. In order for it to work both Alice and Bob have to care intrinsically about owning green things. Indeed, any example of a difference in behaviour due purely to differences in beliefs about the vague sees like it will have to involve agents who care intrinsically about the vague. There is something very odd about this, the full discussion of which I must delay to the next chapter.

However, if one thought that it was not possible to rationally care intrinsically about the vague then there is a serious problem for the supervenience deniers. If whenever two people are certain in all the precise facts they are behaviourally equivalent, then what role are the differing credences in vague propositions playing in the agents psychology? Notice that if they really are behaviourally equivalent even their linguistic behaviour will be the same: they will both report the same degree of confidence in \( p \). If no failure of the supervenience thesis could manifest itself in any kind of behavioural difference it seems hard to imagine what theoretical importance differences like this might have.
Chapter 6

Vagueness and Desire

A compelling thought, no doubt inspired by the idea that vagueness is primarily linguistic, has it that beliefs and desires about vague matters are, in some sense, redundant. A naïve version of this idea might maintain that behaviour consists in precise bodily movements and that any explanation of this behaviour in terms of a person's beliefs and desires about vague matters could be equally or better put by appealing only to that person's beliefs and desires about precise matters. If you happened to know the precise propositions whose truth a person values, and to know how confident they are about all the precise eventualities, one would be able to determine how that person will behave assuming they're rational. This is the thesis I shall be examining:

**Practical Irrelevance:** In deciding what to do, you only need to take into account your beliefs and desires (and attitudes more generally) about precise matters; in this sense your attitudes toward the vague are irrelevant.

The thought is best explained by examples. It seems clear that someone can care about being rich and consequently go about doing things that they believe will make them rich; this much is certainly consistent with Practical Irrelevance. Someone subscribing to this thesis, however, will insist that what is really going on is that the person in question distributes their desires in such a way that they care about having certain precise amounts of money over others, with a preference towards larger amounts. This is what ‘caring about being rich’ amounts to, or at least supervenes on, and so our desire to be rich is in some sense derivative. A similar example can be run with belief: suppose that I can be described as believing that a certain glass is pretty full, and that consequently my credences concerning the various the precise percentages that the glass is filled to drop off smoothly. According to the above thesis, the practical significance of this belief is exhausted by the effects it has on my credences about the precise things — when it comes to making decisions it wouldn’t matter whether I had the vague belief so long as my credences in the precise remained the same.

Practical Irrelevance is more of a slogan than a specific thesis, and the different precisifications of it bear no more than a family resemblance. While there is a version of the thesis that I like, I shall be giving reasons to resist several natural precisifications of the basic idea. I will leave it to the reader to judge whether there is still some important true insight in the vicinity at the end of our discussion.

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1One might think that this all follows from a more general thesis that vagueness is causally inert: while one can certainly say that the rat scampered because he noticed the cat, even though talk of noticing cats is clearly vague, one might insist that there must always be some precise fact that was the basic cause of the rat scampering. This relates to a quite general puzzle about connection between physical causation, relating fundamental physical (and presumably precise) propositions, and the causal interactions we observe all the time between everyday macroscopic objects and, in particular, between our thoughts and the physical world. I shall take it for granted here that the everyday notion of causation is perfectly respectable, and that mental to physical causation is both possible and pervasive.
For those who think that vagueness is just a matter of semantic indecision, or some other public language phenomenon, the idea that vagueness is practically irrelevant will seem extremely appealing. Presumably vagueness plays little role in our day-to-day non-verbal decision making: on one view the things we want, believe and make happen are all precise – it is only when we want to latch on to these things using words that vagueness comes into play. Another variant admits the existence of vague propositions, but treats these as mere constructions out of precise things describing the way the world really is along with the ways we use language to talk about the world. Again, it is natural to think that the important psychological explanatory work can be done by appealing only to our beliefs desires about the way the world really is, with the aspects of content that derive from the way we talk playing little or no psychological role.

It is natural for this kind of theorist to endorse a particularly strong precisification of the practical irrelevance thesis: if two people have exactly the same (relevant) attitudes towards the precise propositions, and they have the same actions available to them, they will act in exactly the same way if they are perfectly rational. In what follows I will offer some reasons to reject this strong version of the practical irrelevance thesis and thus, indirectly, some further reasons to reject accounts of vagueness that predict that attitudes towards vague matters have no important psychological role.

On the other hand if you’re an epistemicist who thinks that vagueness is just a matter of ignorance then you might expect Practical Irrelevance to fail in a fairly drastic way. Our beliefs about things we are ignorant about are often extremely relevant to us in practical reasoning. If I’m uncertain whether it will rain then my degree of uncertainty will inform my decision about whether to bring a coat or not; being ignorant of some matter is no indicator of its practical relevance. In particular, ignorance about whether \( p \) doesn’t necessarily prevent you from caring whether \( p \): if vagueness were merely a matter of ignorance then there would be no reason why someone couldn’t care intrinsically about the vague.

Thus another pertinent precisification of the practical irrelevance thesis is the view that it is irrational to care intrinsically about vague matters, which appears to be violated by a purely epistemic account of vagueness. This principle bears directly on the problem we considered in the last chapter: to what extent do all classical accounts of vagueness collapse into some form of epistemicism? The answer I offer here is that it is not rational to care intrinsically about the vague. This, I think, is the precisification of Practical Irrelevance that holds true.

I’ll begin in section 6.1 by evaluating some different ways of understand the practical irrelevance thesis in the context of a standard decision theory that assumes probabilism. Once we’ve cleared up which variants of the practical irrelevance thesis are true, I’ll reconsider the view that vagueness doesn’t involve one being genuinely uncertain about anything, and views that adopt non-standard theories of probability within this framework.

### 6.1 Vagueness and Decision Theory

Let me begin by outlining a standard formalism for representing an agent’s beliefs, desires and decisions – decision theory à la Jeffrey [67]. According to this framework each consistent proposition, \( A \), is assigned a real number, \( V(A) \), that represents the expected value (sometimes called the news value) of \( A \). Roughly, \( V(A) \) measures how positively you regard things to be on the supposition that \( A \). To get a better grip on this function we shall relate it to several other concepts.

Firstly we can state how it interacts with ones credences, another real valued function on propositions which measures how confident your are in each proposition. Call any set

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2Of course psychology, like any other special science, is couched in vague language. This is not to say that the subject matter of psychology must essentially involve vagueness in some way.
of propositions, $E_i$, which are pairwise incompatible but whose disjunction is $A$ a partition of $A$; intuitively a partition of $A$ is an exhaustive list of more specific ways that $A$ could be true. If $Cr$ is your credence function and $V$ your expected value function then the following ought to hold for any partition of $A$:

**Averaging:** $V(A) = \sum_i V(E_i)Cr(E_i | A)$.

That is to say, how positively you regard things to be on the supposition that $A$ is a weighted average of how positively you regard things on the supposition of each of the ways $A$ could be true.\(^3\) You average the different ways $A$ could be true by weighting them by your confidence in $A$ being true in that way assuming $A$ is true. When the partition consists of maximally strong propositions, $i$, we can think of $V(i)$ as determining what we value outright, before we take into account uncertainty. Informally Jeffrey treats the maximally strong propositions as possible worlds, although nothing in his formalism requires this interpretation; I will refer to them neutrally as ‘indices’.

(Aside: the following technical observation helps us get a more concrete mathematical model of $V$ on the table. Note that we can introduce a random variable, $u$, taking value $V(i)$ at the index $i$: $u$ here represents the utility of the actual index, unknown to us, and the expected value of $A$, $V(A)$, can simply be understood as the expectation, in the mathematical sense, of $u$ given $A$, written $E(u | A)$. Thus given a credence function $u$ and $V$ are interdefinable.)

We can secondly relate an agent’s $V$ function to what they are in a position to make true, and the normative notions of what they should and may make true. Each agent will be associated, at a time, with a set of propositions, $\mathcal{A}$, which represent the propositions that they are in a position to make true. Call this the agent’s action set. The second role that $V$ plays is thus:

**Maximise:** If the agent is in a position to make $A$ true, it is permissible to do so if and only if no member of the agent’s action set has a greater value. It is obligatory if every other member has lower value.

In a slogan, one should maximise expected utility. More likely than not there will be many functions which satisfy the roles captured by **Maximise** and **Averaging**, so talk of a single function $V$ is a bit of a fiction. For our purposes, however, this does not matter – the mere existence of a function satisfying these roles is enough to ensure important structural constraints regarding what it is permissible to make true.

At any rate, the terms with which we shall be theorizing include the notion of an agent’s credence, the agent’s expected value, what an agent is in a position to make true, what they should and may make true. I also used logical concepts when I talked of incompatible propositions and disjunctions of propositions. In order to make sense of this formalism – so that one can assume that rational credences are probability functions, that there are partitions, and so on – one must assume that the space of propositions have the logical structure of a complete Boolean algebra.\(^4\) As noted already, however, given this assumptions nothing in this formalism requires that propositions be sets of worlds – they might be sets of world precisification pairs, for instance, or something else.

It is crucial to bear in mind, in this regard, that the objects to which $V$ assigns values are the objects of our desires, suppositions and beliefs – $V$ is not assigning values to physical objects, chunks of reality or ‘facts’ in some inflationary sense. Moreover, if we are to avoid a revisionary story about the relation between vagueness and ignorance (see chapter 2) then the objects of thought cannot be taken to all be precise (some of these objects will have

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\(^3\)When uncountable partitions are in play we must use a generalised kind of integration; I shall set these technicalities aside.

\(^4\)Or, at least, if propositions were not Boolean – if they were structured propositions or sentences for example – then the structure of the probability theory would allow us to partition these propositions into equivalence classes that did have a Boolean structure. Moreover, a strictly Boolean structure is needed to represent the value function in terms of a utility function.
distinctive epistemic profiles, for example.) Thus since it is possible to desire, believe and suppose that I’m rich, the value function will assign a value to the proposition that I’m rich in addition to the various precise propositions about my wealth.

Someone who endorses Practical Irrelevance within this framework, then, should not take exception to the idea that we can assign values to vague hypotheses. The thesis is rather that these values are utterly otiose. While one can sensibly ask how good things seem to be for me on the supposition that I’m rich, this value is redundant for the purposes of decision making since one could as easily explain my actions in terms of my suppositions, beliefs and desires in the precise. There are precise propositions about my wealth whose values are, at a more basic level, guiding my actions. Knowing an agent’s value, and perhaps also their credence, in each precise hypothesis is all that we need to explain their behaviour, and all that they require to reason practically.

6.1.1 Vagueness and Preferences

Maximise and Averaging leave $V$ fairly unconstrained. Indeed, for someone who is not in a position to make very much true, someone in a coma perhaps, Maximise imposes hardly any constraints at all.

It is natural to think that there is another concept we can relate $V$ to that would further constrain an agent’s value function. While it hardly seems plausible to think that there is any psychological reality to the claim that a person is matched with a particular real number, say 4.1954, that measures how good things seem for them on a supposition – after all, what about the agent’s mental state could possibly ground a particular assignment of values and not some scalar transformation of it – one might still think that there is some reality to comparative judgements of value. That is to say, it seems perfectly sensible to ask whether things seem better for a person on the supposition of $A$ than on the supposition of $B$. Here $A$ and $B$ need not be propositions the agents are in a position to make true since one can suppose many things to be true that are beyond your powers to bring about.

**Suppositional Preference:** The agent considers things to be better for her on the supposition of $A$ than on the supposition of $B$ if and only if $V(A) > V(B)$. She considers things to be at least as good iff $V(A) \geq V(B)$.

Indeed, provided your suppositional preferences satisfy some natural structural constraints (to be discussed shortly), one can show that it is always possible to represent your suppositional preferences using a value function $V$, to some degree of uniqueness, relative to some probability function, also determined to some degree of uniqueness. The function $V$ is, strictly speaking, a convenient fiction, but the fact that it’s always possible to represent rational preferences in the above way entitles us to use it.

If we include suppositional preferences among the attitudes used to rationalise action then ones attitudes towards vague propositions are not irrelevant. There are vague propositions whose supposition should cause you to rank precise propositions in a way that the supposition of no precise proposition would cause you to. Learning such a proposition puts you in a practical situation you could not be in without vague information.

Let us begin by noting that the supposition of a vague propositions can clearly change the order of your preferences. There are things that would be great to buy on the supposition that you’re rich, but, since they would bankrupt you otherwise, would not be a good idea to buy. Since the proposition that you’re rich is vague, the truth of this vague proposition is relevant to your decision – if you were to learn it, you should behave differently.

It is far from obvious, however, that this shows that our attitudes towards the vague are not practically indispensable. There are closely related precise propositions that would produce this same reordering on their supposition, such as the proposition that you have more than a certain amount of money. What we want, then, is a vague proposition whose supposition induces a reordering of the precise propositions that could not be obtained by the supposition of any precise proposition.
Such examples are formally quite simple to construct. Imagine that there are two equiprobable maximally strong precise propositions $A$ and $B$ whose values are $V(A) = 2$ and $V(B) = 1$. Conditional on any precise proposition the order of $A$ and $B$ will remain the same, with $A$ beating $B$, provided they are ranked (if the precise proposition being supposed is inconsistent with a proposition that proposition won’t be ranked.) Suppose furthermore that $A$ is partitioned into two equiprobable vague propositions, $A_1$ and $A_2$ with $V(A_1) = 0$ and $V(A_2) = 4$. $B \lor A_1$ is a vague proposition, and on its supposition the ordering of $A$ and $B$ is reversed since clearly $V(A \land (B \lor A_1)) = 0$ whilst $V(B \land (B \lor A_1)) = V(B) = 1$. Thus if I acquire the inexact evidence $B \lor A_1$, via some imprecise perceptual faculty for example then I will rank both $A$ and $B$, and $B$ will outrank $A$.

This is one type of formal counterexample to the thesis that attitudes towards the vague are practically irrelevant. Note, however, that in this type of example one had to care intrinsically about the vague. $A_1$ and $A_2$ had different values, even though they entailed the exact same precise propositions – they both entail $A$, and since $A$ is a maximally strong precise proposition, they have the same precise consequences. The difference in value has nothing to do with the value of some precise matter. Clearly there’s nothing wrong with valuing vague things – one can care about being rich, or bald or whatever – but these cases normally come with valuing some precise underlying parameter such money in dollars, or hair number. The case being described here is one where, even once one has supposed all of the precise facts to be a certain way (i.e. one has supposed $A$), one still has preferences for the vague proposition $A_2$ over $A_1$.

Whether this kind of preference is rational is something we’ll return to later later. For now let me describe another class of examples that do not depend on this feature. In this class consider three equiprobable maximally strong precise propositions $A$, $B$ and $C$ with values $1$, $2$ and $5$ respectively and suppose that these values remain the same for any proposition that entails $A$, $B$ or $C$ (for example if $X$ entails $C$ then $V(X) = V(C) = 5$; this ensures that no-one cares intrinsically about the vague.) Now suppose that the propositions you are in a position to make true are $B$, $A \lor B$ and $A \lor C$.\footnote{Note that there is nothing incoherent about the idea that both $B$ and $A \lor B$ can be part of the action propositions. For example, I can choose to place a coin on the table heads up, or I could flip it ensuring that it would land either heads or tails.}

I claim that there is no precise proposition such that upon learning it you would rank $A \lor B$ below $A \lor C$, and $A \lor C$ below $B$. One can see this by noting that if a precise proposition is consistent with $C$ then $A \lor C$ is ranked above $B$ conditional on it, if they’re ranked at all, and if it is inconsistent with $C$ then, conditional on it, $A \lor C$ is ranked below or on a par with $A \lor B$.

On the other hand, however, there are vague propositions that could induce this ranking: Suppose that $C$ is partitioned into two vague propositions $C_1$ and $C_2$ with $C_1$ taking up a quarter of the total probability of $C$, i.e. $Cr(C_1 \mid C) = 0.25$. Let $V$ represent your values conditional on $A \lor B \lor C_1$ – then $V(A \lor B) = 1.5$, $V(A \lor C) = 1.8$ and $V(B) = 2$, yeilding the conditional ranking $A \lor B < A \lor C < B$.

This is a case in which the supposition of a vague proposition appears to alter the practical situation in a very distinctive way. These are not idle suppositions either – vague suppositions play an important kind of role in practical reasoning. If I’m deciding whether to wear my coat when I leave the house I consider the options of taking or leaving it on the supposition that it is cold, and on the supposition that it is not cold. In some cases it is better to wear the coat both on on the supposition that it is cold and on the supposition that it isn’t; in which case I can straightforwardly conclude that I should wear the coat. This kind of reasoning is an instance of Savage’s ‘Sure Thing Principle’.\footnote{Note that the principle is only good when the relevant suppositions are probabilistically independent of the options. In this example it seems plausible that whether I wear the coat is completely independent of the weather.} The proposition that it is cold outside is vague, so the behaviour of preferences under vague suppositions cannot be ignored. Of course, similar reasoning can be carried out with precise supposi-
tions, but there is nothing wrong with the reasoning described above, and it is moreover psychologically implausible to think that people suppose completely precise things when doing a quick piece of reasoning like that. It is more likely that the relevant supposition is whether it’s sufficiently warm to go out without a coat, and it would be perverse to require a precise account of what being sufficiently warm amounts to.

Another way in which these observations can have real practical import is when we acquire vague information (see chapter 4.) Suppose that you are taking part in a jellybean contest. You get to look at a jar of jelly beans from a distance, and then you must decide how full you think it is, and choose the option from a list of ranges that you think it’s most likely to be in. The ranges are given in terms of what percentage of the jar is full, and the ranges you can choose from are: (1) in the 70’s, (2) between 60% and 90%, but not in the 70’s, or (3) between 70% and 90%. You stand to make $5 if the jar is in the 60’s, $2 if it is in the 70’s and $1 if it is in the 80’s; you get nothing if the percentage is not within the range that you chose.

Let’s suppose that before you look at the jar you find being in the 60’s, 70’s or 80’s equally probable. Since your eyesight isn’t perfect, and the jar is in the distance, you certainly do not learn, after looking at the jar, that the percentage to which the jar is full is in some particular precise range. You’re evidence after looking won’t be anything like, say, the proposition that the jar is between 65% and 75% full, it is more likely that your total evidence regarding the jar will be a vague proposition. Perhaps your evidence after looking is that the jar is pretty full. Being petty full is perfectly compatible with the jar being in the larger of the two ranges, however when it’s in the high 60’s it’s borderline whether the glass is pretty full. After learning that the glass is pretty full I can’t rule out the possibility that the percentage is in the 60’s, but I become much less confident in it.

The case I have described above instantiates the formal example I gave earlier. My preference ranking after seeing that the jar is pretty full will plausible be to rank option (2) below (1) and option (1) below (3) which seems to be distinctive to this piece of vague information.

6.1.2 Vagueness and Action

Emily decides to have a drink of water. The following things happen: Emily’s elbow forms an angle of 116.3 degrees, her shoulder muscle contracts and her hand moves a total of 19.7 centimeters towards the glass of water waiting on the table in front of her. Her thumb and fingers contract by 1.3cm each and her elbow tights until it makes an angle 33.9 degrees, bring the glass of water towards her mouth.

The above is a fairly precise description of the sequence of events that transpired after Emily decided to have a drink of water – it could, in principle, have been described in completely precise terms. It might be tempting, then, to think that actions are completely precise things.

An action certainly consists in a completely precise sequence of events – denying this sounds as though it would involve a commitment to vague objects or vague events of some kind. So there is certainly some sense in which actions are guaranteed to be precise, however there is a way of formulating this question in the theoretical framework developed above where question requires a little thought. The most straightforward way to reflect the idea that actions are precise is to maintain that the agents action set – the set of propositions they are in a position to make true – consists only of precise propositions. Perhaps, for example, prior to deciding to have a drink of water the precise proposition describing the sequence of events outlined in the first paragraph is one of the things Emily was in a position to make true.

The hypothesis that the agents’ action set consists only of precise propositions entails a natural precisification of Practical Irrelevance. If the action propositions are precise then, according to Maximise, the action chosen will just be a function of the V values we
assign to precise propositions (assuming we choose rationally):

**Irrelevance to Action**: The proposition a rational agent makes true is determined solely by their suppositional preferences (or, $V$ values) among precise propositions.

**Maximise** states that the action a rational agent makes true is determined solely by their suppositional preferences among action propositions; if the action propositions are always precise then the above principle follows. In some sense this principle states that some of our attitudes toward the vague are epiphenomenal – you can have suppositional preferences among vague propositions, but they won’t affect how you behave.

It should be noted that this principle is consistent with other attitudes toward the vague being relevant. For example, the value you assign to a precise proposition might be determined by attitudes, such as credences and desires, you have towards the vague. By **Averaging**, the value of a precise proposition can be written as a weighted sum of values of vague propositions. It could turn out that what you really care intrinsically about is vague, and that the values you assign to the precise propositions only represent what you care instrumentally about – things that raise the expected value by making it more likely the vague matters you care about are true. Even though you only need to know how the agent assigns expected values to the precise to know what she’ll do, you still need to know her credences and desires about vague matters to know what those expected values are.

Be that as it may, I want to argue that even this modest form of **Practical Irrelevance** is false. Vagueness seeps even into our actions, and consequently the action a person chooses cannot be determined purely by looking at their suppositional preferences among precise matters. It is important to be clear what this amounts to. It need not amount to denying that actions are constituted by precise events or anything like that, it is rather a claim about the kinds of things we are in a position to do. Indeed, while an aspect of an agent’s behaviour consists of precise bodily movements, the central concept for decision theory is rather the intentional notion of doing, or making true:

**Vague Actions**: The propositions we are in a position to make true (i.e., that are among our action propositions) at any given time are almost always vague propositions.

As we shall see shortly, we must distinguish this thesis sharply from the thesis that the action propositions are sometimes borderline (neither determinately true nor determinately false). Indeed, the action propositions for a determinately rational agent with determinate preferences will usually be determinately true or false, their vagueness notwithstanding.

For **Vague Actions** to be plausible at all we must distinguish sharply between what happened after Emily decided to have a drink of water, and what she did. Some of the things that happened after Emily decided to have a drink are relevant to the goodness of the outcome of her decision – if she had bent her elbow a slightly different angle, for example, she would have spilt her drink. But lots of other things happened too: the crickets in her garden continued chirping, someone somewhere died in a car crash, and so on. This even applies to some of Emily’s bodily movements: her heart continued beating, she continued to digest her last meal, and so forth – these are things that merely happened to Emily, not things she did. The difference is that most of these things are beyond her control, and are neither among nor entailed by the things she was in a position to make true. As a rule of thumb, the things Emily made happen closely reflect the things we hold her responsible for – the kinds of things for which she can be subject to blame or praise.

Another issue in the vicinity is that the notion of making true relevant to decision theory is an intentional notion: for example, to have made it true that I ate the apple I must have intended to eat the apple. There is a purely causal way of understanding ‘makes true’ which therefore needs to be distinguished and ruled out. If I roll a die and it lands on a 6 then, in the purely causal sense, I made it land on a 6; or similarly if I throw a dart at a dart board and it lands on a point, $x$, then I made it land on $x$ in the purely causal sense. But
since I didn’t intend the dart to land on $x$ – the most I intended was that it hit the board – I didn’t intentionally make the dart land on $x$. The distinction applies even to my own bodily movements: if, after bending my elbow, it forms an angle of 116.3 degrees, this is rarely something I intentionally made true but merely something I caused.

So the first distinction we must be clear on is the distinction between the things that happened which Emily (intentionally) made happen, and those which she didn’t (intentionally) make happen. Clearly only the former kind of things are the things Emily gets to choose from before she makes her decision, and are the kinds of things taken into account when we evaluate her actions after the decision.

The second thing we need to get clear on is how to think about the things she did do, and their role in principles like Maximise. The issue is that even if we exclude from our attention the things that happened that were beyond Emily’s control, there are still a multitude of things that Emily did intentionally make happen in the above scenario. Some of the things Emily did in the above example include: picking up the glass, picking something up, moving her arm, and so on. The proposition that Emily picked up the glass and the proposition that Emily picked something up, for example, have different expected values: if the table had both a scalding hot plate and glass of water, for example, then the value of picking something up is worse than the value of picking the glass up, provided she initially knows she’s going to do one of the two things and assigns non-zero credence to picking up the plate. There is thus a puzzle about what one means by the actions Emily has available to her. Which of the things she made happen do we use to calculate the expected utility of her action? Or in other words, what are the things we are trying to maximise when we make decisions?

One awkward feature of the Jeffrey style decision theory we’ve been using thus far is that the things that get assigned values are propositions, whereas phrases that denote actions, like ‘picked up a glass’, are not propositional. That said, actions clearly have some kind of logical structure – for example, picking up a glass entails picking up something but not vice versa, running to catch the bus entails running, and so on. The presence of this logical structure gives us a straightforward answer to the question we just raised. In the above example Emily made a number of things happen: it is surely the most specific thing that she did that we should evaluate her actions by, and the thing whose value she should be attempting to maximise. It doesn’t matter if the expected value of picking something up is low (because there’s a scalding hot plate on the table) if Emily only picked up something in virtue of doing the more specific act of picking up the glass. In this framework this can be represented by the conjunction of all of propositions she made true.

To summarize: in any given scenario we can divide the propositions into those that Emily made true, and those which she didn’t. To calculate the expected value of what would be made true in that scenario we simply conjoin the propositions that Emily made true and calculate its expected value. It is important to note, then, that an action proposition is not merely a proposition which it is possible (i.e. consistent with Emily’s limitations) for Emily to have made true, it must be a proposition such that it is possible for it to be the conjunction of all the things that Emily made true.

With the action propositions thus delineated, Maximise then tells us to make the action proposition with the highest expected value true. With a clearer question in sight, we are now in position to return to our discussion of Vague Actions.

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7 There are many questions that we haven’t addressed here. For example, there is a question of whether to model ‘making true’ as a factive normal modal operator: in which case the propositions that Emily made true are closed under logical entailment and under conjunction introduction. Whether or ‘making true’ behaves like this or not doesn’t really affect our discussion.

8 If ‘made true’ isn’t closed under conjunctions (see footnote 7) then we must reword Maximise as telling us to make things true in such a way that the conjunction of things made true has maximum value.
Let us begin by noting that the precise proposition describing the events following Emily’s decision, presented at the beginning of this section, does not primarily consist of things that Emily intentionally made true. While these are certainly things that happened, it is quite implausible to suppose that Emily made it true that her elbow contracted to an angle of exactly 116.3 degrees; this is simply too specific a thing to have been under her control, and she certainly didn’t intend anything that particular.

On the other hand, she no doubt made some weaker propositions true. For example she made it true that she contracted her elbow by some amount. But recall that an action proposition is not merely a proposition Emily could have made true: it’s a proposition that could have been the strongest thing she made true. Presumably, then, the proposition that Emily contracted her elbow by some amount is not the strongest thing she made true — she has more control over her elbow than that. Nonetheless, might the strongest thing she made true still be a precise proposition? Perhaps the strongest thing she made true is the proposition that she contracted her elbow to an angle between 102.4 and 128.6 degrees? Again it seems as though this is just too precise a thing to have been the strongest thing she made true; what Emily made true is at least partly a matter of her intentions, and she certainly did not have intentions that precise. But more importantly, Emily’s ability to control her movements isn’t perfect. Someone who does not have perfect motor control cannot hope to bend there elbow to exactly 115 degrees, say, or to some precise interval around 115 degrees — the best they can do is aim to bend elbow at roughly 115 degrees.

There is an extremely natural analogy to be drawn here between the examples of imperfect motor control we have been considering here, and the example of imperfect perceptual faculties that we discussed in chapter 4. There it was argued that when our perceptual faculties aren’t completely precise the conjunction of our evidence is typically vague. Similarly for agents without perfect motor control, it is natural to think that the conjunction of the things we make true will also typically be a vague proposition. In the case of evidence we saw this by observing that the evidential probabilities we’d expect to have after an inexact experience conform to a smooth curve that is different from the sharp curve you’d get from conditioning on a precise proposition.

In a similar way, when I decide to extend or flex my elbow, but I don’t have perfect motor control, I am doing something that I think will result in my elbow making certain precise angles, although in a way that will leave me uncertain which precise angle I will end up making. If I’m trying to make a right angle presumably I should be more confident that I will make an angle in the 85-95 range than 95-105 range. Indeed, it is natural to think that the probability of each precise angle, conditional on my chosen course of action, will smoothly drop off either side of 90.

However if the strongest things we make true are always precise then it seems as though we wouldn’t get the smooth curve that seems to be predicted. The difference between the smooth curve and sharp curve isn’t a mere curiosity either — these probabilities are essential to applying decision theory. AVERAGING says that in order to calculate the value of this action we can look at the value of each precise angle and multiply it by the probability of it conditional on that action and sum them; yet these sums can change between the smooth and sharp curves even if our utilities remain constant.

In our discussion of vague evidence, we considered the idea that one’s evidence isn’t propositional at all, and that one should update by Jeffrey conditioning over a collection of precise propositions instead. There is an analogous move to be made here as well. Rather

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9An argument can be given for the vagueness of the strongest thing Emily made true if we assume ‘makes true’ has a modal logic of $S4$. Namely, one can show, using $p$ to denote the strongest thing Emily made true on a given occasion: if it’s possibly borderline whether Emily made $p$ true then it’s possibly borderline whether $p$. (This follows because if $p$ is the strongest proposition Emily made true, then $p$ and the proposition that Emily made $p$ true will be identical, assuming $S4$.) To show that it’s possibly borderline whether $p$ it suffices to show that it’s possibly borderline whether Emily made $p$ true; I leave it as an exercise to construct a sorites starting with a scenario in which Emily clearly made $p$ true and in one in which she clearly didn’t make $p$ true.
than treating actions as a choice between making a single, potentially vague, proposition true, we could imagine an action being a collection of precise propositions paired with probabilities representing the probability that we will make that precise thing true. Thus perhaps choosing to make certain elbow movements is a bit like choosing to enter a kind of lottery: the possible results are that I flex my elbow to an angle of exactly $x$ degrees, and each of these results happen with probability $q$, where the result of graphing $q$ against $x$ conforms to the kind of smooth curve you would expect. Call such things ‘mixed strategies’. Mixed strategies are thus the analogue of Jeffrey conditioning for the case of action: a choice, on this view, is a choice about which mixed strategy over precise eventualities to adopt rather than a choice about which proposition to make true.

Mixed strategies on this account play a fundamental role in thought and action. Indeed, much like the kinds of Jeffrey conditionings that arise through obtaining inexact evidence, there is a straightforward correspondence between mixed strategies and the things I called evidential roles – functions from maximally strong precise propositions to $[0, 1]$. Since on this picture it is mixed strategies/evidential roles that play the proposition role – the things we know, believe, make true, desire and so forth (see chapter 2) – there is little reason not to call these entities propositions. The result is a theory of vague propositions much like that outlined in chapter 4.

Although I maintain that an agent’s action propositions are often vague, it is important to distinguish this claim from the thought that the action propositions are often borderline (i.e. neither determinately true nor determinately false). Indeed, it is natural to think that determinately rational agents with determinate preferences always have determinately true or determinately false action propositions. The thought, to put it roughly, is that if $A$ has the optimal value out of all of my action propositions (and this is a determinate truth) and $A$ is borderline then it’s borderline whether I’ve satisfied MAXIMISE, and therefore it’s at best borderline whether I’m being rational.\footnote{Thanks to Cian Dorr for pointing this argument out to me.} We can turn this into an argument that the action set of any determinately rational agent with determinate preferences consists only of propositions which are determinately true or determinately false on the assumption that it’s determinate that there are no ties among her preferences on actions. Let $S$ be a determinately rational agent and $V$ her value function.

1. For any pair of propositions $A$ and $B$: either it’s determinate that $V(A) < V(B)$ or it’s determinate that $V(B) < V(A)$. (By the assumption that it’s determinate what $S$’s preferences are, and the fact that there are no ties.)

2. For any proposition $A$ either it’s determinate that $A$ is an action proposition or it’s determinate that $A$ is not an action proposition. (It’s determinate what $S$’s action propositions are.)

3. For any action proposition $A$, determinately: $A$ if and only if $V(B) < V(A)$ for every other action proposition $B$. (The agent determinately satisfies MAXIMISE and the action propositions are pairwise incompatible.)

4. Therefore for any action proposition $A$ either it’s determinate that $A$ (if $A$ is optimal) or it’s determinate that $\neg A$ (if $A$ is not optimal).

The first and second premise are debateable, but are probably true in many relevant cases; for example the first premise is entailed by the thought that it is a precise matter what credence and utility the agent assigns to a proposition. However, even though this is usually false – ascriptions of desire and belief are often slightly vague – the vagueness usually isn’t enough to make a difference to the ordering of propositions. (3) follows from the claim that the agent is determinately rational and that it’s determinate that there are no ties in $A$. If the agent is determinately rational and it’s determinate that there are no ties then she determinately makes the best proposition in her action set true: this make the right
left direction true. If we assume that the action propositions are incompatible with one another then the other action propositions must be false giving us the other direction. Finally from the precision of the right hand side of (3) we may infer that for every action proposition \( A \), determinately: \( A \iff \text{[something determinate]} \). This ensures (4).

This argument has a number of caveats, but it is suggestive nonetheless: even if there are exceptions, the action propositions will often consist only of determinately true or false propositions.

Note, however, that this conclusion is completely compatible with Vague Actions. Being vague is not the same as being neither determinately true nor false – the proposition that Ghandi was bald, for example, is a vague proposition despite the fact that it’s determinately true. Vague Actions just says that the action propositions are usually vague, and this is completely compatible with them usually being determinately true or false. Moreover, the mere fact that they consist of determinately true or false propositions does not mean that our attitudes toward the vague are irrelevant, because even if a proposition turns out to be determinately true we won’t typically know which it is, and it’s our ignorance, and in particular our ignorance about the vague, that changes how we act.

Let us summarize the points we have made above and relate them to the practical irrelevance thesis. The thought we started off with was the idea that the behaviour of a rational person will always be a function of their rational desires and beliefs. Moreover, if the behaviour of an agent is always a precise matter then the desires and beliefs which matter for the agents behaviour will ones with precise contents. Certainly there are conceptions of behaviour, popular among early kinds of behaviourists, that identify behaviour fairly narrowly with precise bodily movements. However it is quite clear that this conception of behaviour will neither play the role in decision theory we need it to play nor will it track the kinds of thing we care about when we make evaluative judgments about a persons behaviour. Here I have argued that behaviour, as it figures in decision theory, is best described in terms of a fundamentally intentional notion: that of making something true. Making true is a propositional attitude in the sense that it requires you to be in a certain state of mind, although like other externalist attitudes such as knowledge, it cannot be held unless the world around you complies.

On the alternative account of behaviour our attitudes towards the vague are practically important. For example, it’s possible that on a given occasion the only thing that you’ve made true is a vague proposition. In this case you simply cannot describe your behaviour purely in terms of the precise things you have made true. More importantly, for a rational person, which proposition is made true is determined by which proposition they assign the highest value to so ones beliefs and desires in the vague determine which action they perform.

Thus on this picture it is possible for two people to assign the exact same values to all precise propositions, be in a position to make the same propositions true, while rationally behaving in different ways. If Alice and Bob, say, agree about the precise in the sense that they assign the same values to all the precise propositions, they might still end up behaving differently if they assign different values to the vague. The best proposition in Alice’s action set might be different from the best proposition in Bob’s action set – for Alice’s best proposition might be one of the vague propositions Alice and Bob disagrees about. This can happen even if Alice and Bob have the same action set, since all that agreement about the precise ensures is that they will both make the same proposition true in the special case

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11 The assumption that the action propositions are pairwise incompatible is not obvious, and does not follow from anything I’ve said so far. Although many people do make this assumption, it might be possible to avoid this argument by denying it. For the time being, however, I will grant the assumption.

12 Recall that the supervenience of vague beliefs on precise beliefs discussed in chapter 5 rules out the possibility of two people with the same evidence agreeing in their credences about the precise whilst disagreeing about the vague. However it is still possible for two people with different evidence to agree about the precise whilst disagreeing about the vague.
where their action sets are the same and consist only of precise propositions.\textsuperscript{13} Moreover this will result in a concrete difference in behaviour: since Alice and Bob will choose the highest value proposition to make true, they will end up making different vague propositions true.

### 6.2 Is it irrational to care intrinsically about the vague?

At this juncture I would like to circle back to a question that we wrestled with in the last chapter. What puzzled us there was that it seemed that in accepting classical logic, and in treating ignorance due to vagueness as straightforwardly analogous to ignorance about the world we concede too much to the epistemicist. If we are to truly to think of vagueness as something more than mere ignorance we need to say something to preclude a purely epistemic interpretation of borderlineness.

Could our examination of decision theory in the presence of vagueness shed any light on this question? If we assume an epistemic interpretation of vagueness, might the decision theories look different? I think the answers to both questions are positive. Although the formal theory of preferences might look the same, on a purely epistemic understanding of vagueness practical irrelevance fails in a quite spectacular way: in terms of practical reasoning there is very little to distinguish ordinary ignorance about the things that affect what we desire and ignorance about these things that is due to vagueness. For if vagueness is just a special kind of ignorance then there is nothing to prevent us from finding vague matters themselves to be intrinsically desirable, much like we find other outcomes that we are ignorant about to be desirable. On the other hand, by denying the decision theoretic analogy between vague and precise desires, I shall argue, we open up the possibility for a bouletic interpretation of vagueness that does not identify it as merely a species of ignorance.

Why does decision theory look different on a purely epistemic understanding of vagueness? The thought, to put it in very broad terms, is that according to an interpretation in which vagueness is just a special kind of ignorance there’s really nothing to distinguish decisions made under uncertainty about factual things, and decisions made under uncertainty about borderline things: borderlineness just is a kind of uncertainty about factual things according to that view.

Let us now finally formulate a precisification of the practical irrelevance thesis that, I think, does capture an important truth about the difference between our beliefs and desires about vague outcomes and about precise outcomes: while we can care intrinsically about the latter, it seems as though there’s something incoherent about caring intrinsically about the vague. Yet this assumption does not look particularly well motivated on the epistemic theory. If you think that vagueness is \textit{merely} a matter of ignorance, and that in general nothing precludes you from caring about things you are ignorant about, then there is nothing to preclude you caring about the vague.

To demonstrate what this might amount to we might consider an example. Suppose Harry has the telic desire to be bald, and, for simplicity, let us suppose this is the only thing he cares about. He could have this desire in a number of ways – for example, he might have the desire derivatively in virtue of his having the desire to have no hairs at all. Let us suppose that this desire is not had derivatively, it is the strongest proposition about his head that he desires: Harry wants to be bald and any other epistemic possibility about the state of his head that he finds desirable is already entailed by his being bald. There is therefore no specific number of hairs he wants; if he wants to have 98 hairs or less this is only because he thinks this will make him bald and not because he cares about having smaller number of hairs over larger numbers. He does not care whether he has 98, 99, 100, ..., hairs unless it would make the difference between his being bald and his being not bald.

\textsuperscript{13} Talk of two agents having the very same action set is a bit of a fiction, because actions propositions typically involve the things only one agent can control. The fundamental point could be restated by invoking a suitable notion of two action sets being isomorphic instead.
that is, he cares about his numerical hair number only insofar as it contributes to making
him bald.

Compare this with another case. Alice cares intrinsically about whether there is intelli-
gent life elsewhere in the universe right at this second. Although Alice expects never to be
able to know the answer to this question, I think that this kind of desire is still perfectly
coherent. Perhaps Alice is a science fiction enthusiast, or assigns high value to worlds with
more diversity and culture than worlds with less. Or perhaps she is a fundamentalist chris-
tian, and disvalues possible worlds where humans do not occupy the role of being the sole
bearers of intelligent life in the universe. Whatever the story, it seems like something it
could be perfectly rational to care about.

For simplicity, again, we may assume this is the only thing she cares about. Like Harry,
she doesn’t really care about other features of the universe. Let’s also suppose that she’s
certain that if there is intelligent life it’s well outside the region spanned by her lightcone at
any point in her life. Whether or not there is intelligent life out there is completely causally
independent of her and is something she could never hope to verify. Nonetheless, she
thinks that if there is intelligent life out there that would be a pretty cool thing (or not,
depending on the story).

I think there are two kinds of reactions you could potentially have to these two examples.

Reaction 1. Harry’s desire is just like Alice’s desire. They both desire something
which could never have a causal effect on either of them. The truth values of both
the propositions they desire will never be known. The fact that Harry is bald (is not
bald) is like the fact that there is (is no) intelligent life in the universe. They are
both perfectly coherent desires to have.

Reaction 2. Harry’s desire is not at all like Alice’s. While their desires are both
unknowable and causally independent of their immediate surroundings Harry’s desire
is simply incoherent. Harry cares about whether he is bald or not when he already
knows exactly how many hairs he has. Based on this desire Harry acts as if there is
a fact of the matter out there, whether he’s bald, just as there’s matter of fact about
whether there’s intelligent life in the galaxy; only Alice is entitled to act like this.

It is not unnatural to associate the first kind of reaction with an epistemicist who thinks
that vagueness is merely a matter of ignorance, albeit a special kind of ignorance. For
the epistemicist facts about baldness in borderline cases are not especially different in kind
from facts about life in the far reaches of the galaxy. Since it is perfectly fine to care about
things you are ignorant about – even when that ignorance is impossible to remove – there
is no distinctive reason why we could not care about whether a borderline case of baldness
is bald or not.

The second kind of response is naturally associated with supervaluationists and other
theorists who think that in borderline cases there simply is no fact out there to be known,
and no fact worth caring about. Propositions about life in the galaxy are different since,
while they are like borderline cases in the sense that they are unknowable, there is still a
fact of the matter out there which one could reasonably care about. For this latter kind of
theorist Alice’s desire is perhaps a little eccentric, perhaps even irrational in a loose sense,
but it is not incoherent. Bob’s preferences, on the other hand, seems to be conceptually
confused.

This observation is intended to help us with the problem we raised in chapter 5. If
episcentists and other ‘factualists’ think that vagueness is just a matter of uncertainty,
then what is this extra thing, beyond uncertainty, those who deny the factual nature of
borderline cases think that these cases involve? According to the theory I’m advancing
that difference is articulated by two further theses: (i) there cannot be rational differences
of opinion that are soley differences of opinion about the vague and (ii) you shouldn’t care
intrinsically about the vague. The thought, in both cases, is that if vagueness amounts
to nothing more than uncertainty, then there would be no reason to think there couldn’t
be rational disagreements about purely vague matters, or to think that one couldn’t care intrinsically about vague matters. There is no general reason to think that one cannot disagree or care intrinsically about things we are uncertain about, so these two principles give us two ways in which vagueness amounts to more than the presence of uncertainty.

The first principle was formulated precisely in terms of the supervenience of vague on precise beliefs in the last chapter, and was defended there. We shall spend the rest of this section sharpening and defending the second claim.

Before we do that, it will be worth noting that the conjunction of (i) and (ii) have fairly drastic decision theoretic consequences. For a view in which vagueness is merely the presence of uncertainty – i.e. an account in which neither constraint (i) nor (ii) are accepted – vague beliefs and desires, far from being epiphenomenal, can play a fairly concrete, even surprising, role in informing our actions. The cases that most starkly demonstrate are cases which have the following structure. Suppose there are two rational agents, Alice and Bob, such that:

1. Alice and Bob have exactly the same evidence and assign the same credences to every precise proposition.
2. Alice and Bob have exactly the same desires (their utility functions are identical).
3. Alice and Bob are in a position to make exactly the same propositions true.

If we accept both (i) and (ii) then it is pretty straightforward to show that Alice and Bob must assign the same values to all propositions, and thus make the same proposition true if they are rational (assuming, for simplicity, there are no ties). However a ‘mere uncertainty’ account of vagueness could allow both (i) and (ii) to fail simultaneously. In which case, I claim, it is possible to construct examples where Alice and Bob make completely different choices. For example, recall the decision puzzle we considered in the last chapter:

Alice and Bob are in an auction house and they are both bidding on a relatively inexpensive vase which is a shade of turquoise that is borderline between being blue and green.

Now suppose we know the following things about Alice and Bob. Firstly, they have exactly the same telic desires and have the same credences in the precise propositions. We also know the following facts. (1) as it happens they are both very peculiar people and they care intrinsically about owning green things. (2) although they both know exactly what precise shade the vase is, Alice is more confident than Bob that the vase is green.

The scenario described violates both (i) and (ii): it requires that Alice and Bob care intrinsically about the vague, and that they disagree to some degree about whether the vase is green, even though they know the relevant precise facts such as its exact shade.

In the above scenario it is clear that Alice should bid more than Bob for the vase. Since they care about the same things its value is the same for both Bob and Alice. However, since they know that it is vague whether the vase is green they are uncertain whether it is green and therefore the purchase is a gamble on its being green. However, according to Alices subjective credences the odds that it is green are better than they would be according to Bob’s. Therefore Alice ought to outbid Bob.

On the proposed view the kind of scenario described above requires Alice and Bob to have deeply irrational preferences, and at least one of Alice and Bob to have incorrect values for uncertain propositions.
credences. On the other hand, the scenario seems fine for the kind of pure uncertainty account of vagueness I have been describing. According to this kind of factualism about vagueness one can reasonably disagree about a proposition, even given all the relevant precise evidence, and one can care intrinsically about the vague.

6.2.1 The indifference principle

It’s obvious that you can care about matters which are vague: for example you can rationally care about being rich, being happy, having lots of friend, and so on and so forth. The proposition that you’re rich or that you’re happy, and so on, are all vague propositions. This is all compatible with the claim that one cannot care intrinsically about the vague. However these examples do demonstrate that we need to be careful how to spell out the thesis, as its plausibility is very sensitive to the way it is stated.

In order to do this we shall assume that the algebra of propositions we are working with is atomic. In a supervaluationist framework the atoms would not correspond to possible worlds, but to pairs of possible worlds and precisifications. If we are to work in a neutral setting in which propositions are taken as primitive, however, a state is a proposition such that for any proposition (vague or precise) it entails that proposition or its negation, but doesn’t entail everything (or alternatively: entails everything it’s entailed by.) We can also partition the set of states into maximally strong precise propositions: propositions \( w \) such that for every precise proposition, \( p \), \( w \) either entails \( p \) or \( p \)’s negation. In general the strongest precise proposition entailed by a state (i.e. atom) \( i \) is a maximally strong precise proposition. I shall use the fairly neutral term ‘index’ for the atoms, and ‘maximally specific precise proposition’ for latter thing.

With these notions in place we can state the principle that correctly encodes the thought that one should not care intrinsically about the vague:\(^{15} \)

\[ \text{IP. If, for every precise proposition, either both } A \text{ and } B \text{ entail it, or entail its negation, then one should be indifferent between } A \text{ and } B. \]

Note that this principle does not entail that you couldn’t rationally care about being rich. For simplicity let’s assume you only care about your financial situation and let’s also assume that whether you’re rich supervenes on how much money you have. Given these assumptions \( \text{IP does entail that you should be indifferent between states that agree on how much money you have. If you furthermore know how much money you have, you should be indifferent between all the different maximally specific ways things might, for all you know, be, even if they disagree about whether you’re rich or not.} \]

6.2.2 Caring about the vague

One might take exception to the indifference principle on the grounds that there are phenomena that on the face of it seem closely related to vagueness, yet where it seems as though there’s absolutely nothing wrong with caring in these cases. For example, according to some theories of time most contingent propositions about the future are indeterminate. Yet it seems perfectly rational to care about what will happen to you in the future. Indeed, someone who didn’t care about their future presumably would be indifferent between all options in a decision, and so people wouldn’t ever feel the need to deviate from the status quo.

I think in this case it’s tempting to turn the argument on its head. To my mind this makes for a fairly compelling argument that truths about the future are not indeterminate, they are just epistemically inaccessible. Certainly either there will be a sea battle tomorrow or there won’t, and we do not know which, but that surely does not mean that there is no

\(^{15}\text{It should be noted that, since there is vagueness concerning which propositions are precise, there can be cases where it’s vague whether you may care about } p. \)
The fact of the matter. The fact that it’s perfectly rational to care about which outcome will obtain seems a reason to think that there is a fact of the matter for us to care about. Also, unlike in the case of vagueness, we do know some truths about the future – I know what I’m having for dinner tonight, and lots of other mundane facts about the near future. If we combine this observation with the fact that we can care about the future, the analogy between the future and borderline cases quickly dissipates.

Other objections in a similar vein I think represent a misunderstanding of the indifference principle. One might object that one ought to be able to care intrinsically about being happy, about being hungry or about having pleasurable experiences and so on, yet each of these things are vague. For the sake of concreteness let us focus on happiness. IP is certainly consistent with one caring about being happy – what IP rules out is that you care about being in states which count you as happy over states in which count you as not happy, even if they agree about all the precise facts.

Let us consider a simple sorites for happiness: suppose that you are going to give me a portion of a cake. If you give me a tiny slither I will not be happy, whereas if you give me a large slice a will be happy. Keeping other variables relevant to my happiness fixed there will be some amount of cake in the middle such that it is borderline whether I’d be happy if you gave me that much cake. Since it is borderline whether I’m happy in these scenarios I don’t know whether I’m happy when I get that much cake. Now consider two epistemic possibilities in which I recieve that middling amount of cake and which differ only in that in one I’m counted as happy and in the other I’m not. It seems to me that since I am recieving exactly the same amount of cake in both scenarios, and ex hypothesi the other factors relevant to my happiness are fixed, it would be bizarre to think that I should countenance a difference between the two cases. Happiness is not something you can directly pursue, it is a side effect of pursuing more specific things; it is those more specific things that you care intrinsically about. Similar things can be said about hungeriness, pleasure and so on; you usually care about them because you care about more specific, precise things and this is perfectly compatible with the indifference principle.

Be this as it may, other cases where the indifference principle seems to fail are harder to brush aside. Let me take up an example discussed at length in Williams [127], drawing from the literature on personal identity: survival. I take it that survival is not always a completely precise matter. There are episodes that someone could undergo in which it would be clear that they would survive – perhaps if a few of their memories were erased, or some of their matter were replaced. And there are other episodes after which it would be clear that they do not – perhaps if all of their matter and all of their memories were replaced. It is a matter of routine to construct a sorites sequence connecting the two kinds of cases, so it surely follows that there are episodes someone could undergo in which it would be borderline whether they survive: there is someone before the episode, call her Alpha, and someone after the episode, call her Omega, but it is borderline whether the person before and the person after is the same person or not.

Now Williams argues convincingly that in such cases Alpha should care, at least to some degree, about what happens to Omega. For example, if Alpha had the chance to pay a small amount of money to prevent Omega being punched in the face, she should probably pay that money. We could explain this, as Williams does, by supposing that the person before the episode cares intrinsically about what happens to her, and since she does not know whether she is the person after the episode or not, she should derivatively care about what happens to the person after the episode. This style of explanation would be disastrous for the indifference principle, for it supposes that Alpha cares intrinsically about whether Alpha gets punched in the face, and as we have seen, it is borderline whether Alpha gets punched in the face in the scenarios where Alpha doesn’t pay the money. Thus Alpha cares intrinsically about the vague and seems rational in doing so.

However I think there are a few complications with this argument that make it unclear whether it really conflicts with the indifference principle. The complication is that the usual
way to flesh out the scenario described above allows one to explain these preferences without invoking desires towards the vague, circumventing any violation of IP. The standard way to think to think about the vagueness that arises in this puzzle is that there are (at least) three salient temporally extended entities in the vicinity of Alpha and Omega: one relatively short one ending at the time of the episode which indeterminately disrupts Alpha’s identity, call this A, another short one that begins at the time of that episode, call this thing B, and a long one coinciding with both A and B, call this ‘C’.

The ‘standard’ story locates the vagueness in the names ‘Alpha’ and ‘Omega’ – according to that story ‘Alpha’ is referentially indeterminate between referring to A and to C, and ‘Omega’ is referentially indeterminate between referring to B and to C, thus the sentence ‘Alpha=Omega’ is linguistically borderline. Of course, for a non-linguistic theorist a different story would be told about the source of the vagueness, however they will nonetheless also accept the above thesis about the referential indeterminacy of ‘Alpha’ and ‘Omega’ and the linguistic borderliness of ‘Alpha=Omega’ (recall that a sentence is linguistically borderline if it expresses a borderline proposition). Note also that what I am calling the ‘standard story’ hasn’t taken sides on the mereological situation involved here: we could think of C as the fusion of A and B, or we could think of it as mereologically disjoint but coincident with both, or in some other way – it will not matter for our purposes.

Now since by hypothesis Alpha is a self-interested person, and either Alpha is identical to A or Alpha is identical to C, it follows by logic that either A or C, or maybe even both, are self-interested people. On the assumption that Alpha is identical to A, A is a self-interested person and therefore cares only about what happens to A. However since it is a completely precise matter what happens to A – it’s determinate that A’s life ends at the time of the disruption – A does not care intrinsically about the vague. Secondly, on the assumption that Alpha is identical to C we get a parallel conclusion about C: C is a self-interested person, and so C does not care intrinsically about the vague since C cares only about what happens to herself, and it’s determinate that she survives the disruptive event. Since, determinately, Alpha is identical to either A or C, and determinately A and C only care about the precise, it follows that, determinately, Alpha does not care intrinsically about the vague.

This is, I think, enough to deflate Williams’ argument. Nonetheless, one might still be left with a feeling of puzzlement – why is it that it seems as though A ought to act as though she were uncertain about what will happen to her? Why does A seem to act as though she cares about what happens to the person after the disruption? Fortunately we can explain this without invoking vagueness related uncertainty or desire. At time t before the episode A and C have exactly the same subjective experiences (or at least, things that are not determinately not subjective experiences) so it is natural to think that both A and C have self-locating uncertainty: for all A knows, she’s C, and is going to survive, and for all C knows, she is A and is not going to survive. So neither A nor C know whether they’re the entity that survives, and thus will do things that benefit the entity that survives to the extent that they think that they are the entity that survives.

Thus in the scenario described above the only thing A cares intrinsically about is what happens to A. The problem is A doesn’t know whether she occupies the qualitative role A actually occupies or the qualitative role that C occupies, and therefore doesn’t know whether she will survive. However, it is a perfectly precise matter what happens to A on either hypothesis so she does not care intrinsically about the vague; similarly for C.

6.2.3 **Is it even possible to only care intrinsically about the precise?**

So far we have been only considered specific examples in which it appears as though it is permissible to care about the vague. However one might have a much more general worry about the indifference principle. According to an extremely pervasive view, pretty much the only concepts that can be used to express precise propositions and properties are the
concepts of logic, mathematics and fundamental physics. On this assumption, however, the
indifference principle seems to entail two extremely implausible principles.

Firstly it seems to entail that I can only to care intrinsically about the propositions of
fundamental physics, mathematics and logic, for if these are the only precise propositions
these are the only things I can care intrinsically about. Secondly, it follows that it was only
until quite recently, with the discovery of modern physical concepts, that we were able to
have thoughts about these precise propositions at all. That is to say, up until the birth
of modern physics, pretty much everybody had irrational desires because the only desires
they could have formed would have been formed using vague concepts.

This objection, if it has any force, rests on some presuppositions that are fairly specific
to a linguistic conception of vagueness. The objector assumes, for example, that in order
to have a vague belief or desire, one must first mentally do something with vague concepts,
such as articulate a vague sentence in the language of thought. However this picture is
emphatically denied on the account of vagueness I have been sketching so far.

The idea that we can obtain a useful categorization of precise and vague propositions
by looking at the kinds of sentences or concepts that express them is also a bad starting
point. Indeed, on the view I have been endorsing it is plausibly not true that the sentences
of fundamental physics express precise propositions. For example, in chapter 3 I argued
that the proposition that there are electrons is vague, because there are certain physical
hypotheses such that conditional on them, we should be certain that it’s borderline whether
there are any electrons (hypotheses in which electrons are clouds of smaller particles).

The upshot of this is that the objector, in talking about our concepts, is assuming a
radically different picture of vagueness. The lines between vague and precise propositions,
on my view, simply do not correspond to the lines drawn by sentences (or concepts) that
are used in a certain way, they are drawn by the role that those propositions have in
thought. Indeed, according to my view pretty much the only sentences that pick out precise
propositions are the sentences of mathematics and logic, and thus very few contingent
precise propositions are picked out this way.

Thus, to respond to the objection, while I agree that sentences of a public (or private)
language rarely express precise propositions it does not follow that we do not bear any
interesting relations to precise propositions in the sense articulated by my preferred theory of
propositional attitudes. The notion of a precise proposition can be implicitly introduced by
the role it plays in a theory of rational propositional attitudes, a theory in which knowledge
of physics is not a requirement of rationality.\textsuperscript{16}

### 6.3 Probability in the Absence of Uncertainty

Let us return to one of the themes of the last chapter. There I argued that when we are
confident that it is borderline whether Harry is bald, we ought to have an intermediate
credence regarding whether Harry is bald. This conclusion sounds more puzzling according
some accounts of vagueness than others, and might lead one to resist the view as follows.
If borderline propositions do not have truth values, one might reason, there is no relevant
proposition whose truth we are uncertain about in this scenario so it seems to be incorrect
to describe this as a case where we are uncertain about something. Similarly, if there is no
fact about whether Harry is bald, and I know this, then there’s nothing to be uncertain
about so we shouldn’t be asking how confident one should be about whether Harry is bald.
If I know and am completely confident that it’s borderline whether Harry is bald, one might

\textsuperscript{16}It is also possible that the objection rests upon an equivocation with the word ‘property’. There are
two conceptions of properties: an abundant conception according to which properties are abstract objects,
serve as the semantic values of predicates, feature importantly as the objects of belief and other intentional
attitudes, and a sparse conception according to which properties are physical things whose existence depends
on fundamental features of the world and which are discovered by science. The property of being an electron,
in the latter sense, may in fact never feature in the thoughts of an agent, even an agent who has all the
concepts needed to state a final theory of fundamental physics.
object, then any assignment of confidence to the proposition that Harry is bald would be epiphenomenal. Unlike, say, a degree of confidence about whether it will rain tomorrow – which might inform my decision to bring an umbrella – any confidence about Harry’s baldness would be practically inert. If I did have a credence in that proposition I could have pretty much any credence I like without negative consequences, and there would no way to test what my credences are.

If I’m right then this objection rests on a fallacy: it appears to rest on a strong version of the practical irrelevance thesis. According to the considerations in the previous sections one’s credences in the vague are not epiphenomenal: one can test a persons credences in the vague by observing their behaviour, and indiscriminately changing your credences in vague propositions can result in actions with negative consequences, even if they leave your credences in the precise the same.

Now my opponent in the last chapter could concede this point and still insist that there’s no sense to be made of uncertainty when one considers there to be no fact of the matter. That is to say, the following concession is perfectly compatible with their view:

Although being genuinely uncertain in the face of vagueness is a mistake, vagueness related attitudes still give rise to distinctive practical behaviour.

In this section I want to explore this kind of response. I will argue that, even if this theorist is right about there being no such thing as genuine uncertainty due to vagueness, there will be some probabilistic notion in the vicinity of uncertainty that informs our practical reasoning. In particular, I shall argue that there is some kind of mental state, or mental feature that supervenes on your mental states, than does satisfy the axioms of the probability calculus, and moreover governs rational behaviour in the way we thought credences were supposed to. Whether we want to go one step further and call this thing a measure of ‘genuine uncertainty’ can be left up for debate. For most purposes, however, this conclusion is strong enough.

Before I do that, I first want to draw an analogy with another debate that I think might be helpful here. An extremely natural way to interpret the standard formalism of quantum mechanics (without augmenting it or taking it to be a partial description of the fundamental facts) is to treat the classical macroscopic reality around us – us and the things we observe – as constantly splitting into multiple ‘branches’ (see Everett [39].) So, for example, when I flip a coin which has a genuine chance of landing heads and of landing tails, we can expect that in reality there will be one branch in which the coin lands heads and another in which it lands tails. For the time being, let us set aside discussion of the merits of this idea, and assume that this is a correct description of the fundamental universe. According to this picture there’s straightforward difficulty for making sense of probability: I know exactly what’s going to happen in the coin flipping case – there’s going to be (with certainty) a branch in which the coin lands heads, and another branch in which it lands tails. Since all the potential outcomes actually occur on this picture it’s impossible to make sense of uncertainty about which outcome will occur.

This was a long standing obstacle to this interpretation of quantum mechanics, and for many it was a decisive objection: the power of quantum theory comes from its ability to predict the chances of things happening, after all, so it would be useless if there was no sense to be made of probability. However in [28] David Deutsch showed that, even if there’s no room for uncertainty on this kind of hypothesis, one can still make sense of probability by looking at rational action. More importantly for that project, he was able to show a person who acts rationally (and knows some the relevant quantum theory) will act as

\[15\] Actually I think it rests on a distinct fallacy which is often brushed over. This theorist is using the word ‘true’ in a way that prevents one from substituting \( p \) for ‘\( p \) is true’, whilst also equating uncertainty about \( p \) with uncertainty about whether \( p \) is true. It remains far from clear to me why the lack of a truth value should preclude uncertainty about whether or not \( p \).

\[16\] These ideas are further developed by Wallace [126] and Greaves [60].
though she was uncertain about the outcomes, and moreover, uncertain to the degrees that quantum theory predicts (i.e. as given by the Born rule.)

Note that if there’s no such thing as genuine uncertainty in the many worlds interpretation of quantum mechanics, or indeed the case of vagueness, then there’s a prima facie puzzle about how to interpret the decision theory, involving value functions, that I outlined in this section. For example AVERAGING explicitly appeals to credences, and if there is no real uncertainty in the cases of interest then this appeal is problematic. This is solved in Deutsch’s framework by dispensing with talk of values and credences, and just talking about an agents preferences over various acts – things that have a direct behavioural interpretation. In our framework we can take these to be given by our suppositional preferences over propositions, as defined in section 6.1. While our preferences over action propositions have the clearest behavioural interpretation, even our preferences over other propositions can manifest themselves as dispositional behavioural properties: counterfactuals like ‘if I were in a position to make A or B true, I would make A true’ can ground many of your suppositional preferences.

Just as Deutsch was able to do in the many worlds case, we can reconstruct talk of probability, without taking it for granted that people are uncertain in the face of vagueness. In order to do this one must make a few assumptions about the agents preference relation: they must obey some natural constraints that seem natural for a preference relation to be rational. One’s preferences ought to be irreflexive, asymmetric and transitive, for instance. You cannot prefer A to B and B to C without preferring A to C, you cannot prefer A over itself and if you prefer A to B you shouldn’t prefer B to A. (Similarly obvious things can be said about the relations being ‘preferable to or as preferable as’, ‘as preferable as’.)

Another constraint is that any two propositions be as preferable as one another, or that one be preferred to the other. This principle is perhaps more controversial, although as in our discussion in section [REF], its non-obvious appearance doesn’t seem to be distinctively to do with vagueness. One can also state a version of AVERAGING purely in terms preference, without invoking credences. A straightforward consequence of AVERAGING is that if \( E_1 \) and \( E_2 \) partition A into two, and \( V(E_1) < V(E_2) \) then \( V(E_1) < V(A) < V(E_2) \) – this is is obvious since \( V(A) \) is just defined as a (weighted) average of \( V(E_1) \) and \( V(E_2) \). As a constraint on suppositional preferences it seems intuitively correct even without appealing to the justification of it in terms of AVERAGING: if \( E_1 \prec E_2 \) then \( E_1 \prec A \prec E_2 \).

The final two constraints are perhaps harder to understand, but no less intuitive once they are understood. According to the decision theory we’ve outlined, A and B can have the same value even if they are not equiprobable. However, in such cases, if you were to disjoin both A and B with a more preferable proposition C disjoint from both of them, it would pull their values apart. This is another consequence of AVERAGING, but this time we’re making use of the fact that the average is weighted by probability: if A and B are not equiprobable then their respective disjunctions with C will have different values because the A and B parts will be weighted differently. On the other hand, if A and B are equiprobable then their disjunction with C will have the same value for every C of the kind described because the weightings of the respective disjuncts will be the same. Thus we should have that if \( A \approx B \), then either \( A \lor C \approx B \lor C \) for no proposition C more preferable but disjoint from both A and B (in the case that A and B are not equiprobable), or \( A \lor C \approx B \lor C \) for every proposition C more preferable but disjoint from both A and B (in the case that A and B are equiprobable).

The final axiom effectively states that the preference relation is continuous in the sense that whenever the supremum or infimum of a set of propositions lies in a certain interval (let us say, A is preferred to it and it is preferred to B) then some member of the set lies in that interval (i.e. some member is such that A is preferred to it and it is preferred to B.)

The axioms listed above can be summarized as follows

1. \( \prec \) and \( \preceq \) are transitive relations. \( \prec \) is irreflexive and \( \preceq \) is reflexive.
2. \( \preceq \) is total: for any \( p \) and \( q \) either \( p \prec q \) or \( q \prec p \).

3. If \( A \land B = \bot \) then
   
   (a) If \( A \prec B \) then \( A \prec (A \lor B) \prec B \)
   
   (b) If \( A \approx B \) then \( A \approx (A \lor B) \approx B \)

4. If \( A \land B = \bot \) and \( A \approx B \), then either \((A \lor C) \approx (B \lor C)\) for no \( C \) with \( C \land A = C \land B = \bot \) and \( A \not\approx C \), or \((A \lor C) \approx (B \lor C)\) for every such \( C \).

5. If \( A \prec C \prec B \) and \( C = \bigvee_{\alpha} C_{\alpha} \) then for some \( \beta \), \( A \prec \bigvee_{\alpha>\beta} C_{\alpha} \prec B \). Similarly if \( C \) is the infimum \( \bigwedge_{\alpha} C_{\alpha} \) then for some \( \beta \), \( A \prec \bigwedge_{\alpha>\beta} C_{\alpha} \prec B \).

By only talking about preferences over propositions, then, we can formulate Jeffrey’s decision theory without invoking the notion of a credence, and thus we can apply Jeffrey’s decision theory without assuming that vagueness involves genuine uncertainty.

However, an important result due to Ethan Bolker [15] shows that if your preferences satisfy these constraints, then there is a unique (or uniquish) function \( V \) and probability function \( C \) which satisfy AVERAGING and represent your preferences in the sense that \( V(A) < V(B) \) iff \( A \prec B \) for every \( A \) and \( B \). The above purely preference theoretic axiomatisation of Jeffrey’s decision theory shows that the appeal to the value function \( V \) and the credence function \( C \) are dispensable in favour of talk about preferences. However Bolker’s representation theorem shows that one can also reconstruct these functions from the preferences and use them as though they part of the agents psychology. In other words, even if uncertainty about the vague were problematic we have decision theoretic substitutes for credences – the probability function that is generated by the representation theorem:

**Theorem 6.3.1. Bolker’s Existence Theorem**

Suppose that \( \prec \) is preference relation satisfying the above axioms. Then there exists a countably additive probability function \( Pr \), and a real valued function \( u \) on the maximally strong propositions such that:

\[
A \prec B \iff E(u | A) < E(u | B)
\]

The function \( V(A) = E(u | A) \) is a Jeffrey style value function which satisfies averaging.

**Theorem 6.3.2. Semi-Uniqueness** Suppose that \( V \) represents \( \prec \) and \( V \) satisfies averaging with respect to \( Pr \), and similarly for \( V' \) and \( Pr' \). Then there is a real number, \( \lambda \) with

\[
\frac{1}{\inf_X(V(X))} \leq \lambda \leq \frac{1}{\sup_X(V(X))}
\]

such that:

\[
Pr(A) = Pr'(A)(1 + \lambda V'(A))
\]

\[
V(A) = V'(X) \frac{1+\lambda V(X)}{1+\lambda V'(X)}
\]

Note that in the special case where your values are unbounded then \( \lambda \) must be 0. So in that special case we get genuine uniqueness of \( V \) and \( Pr \).

The representation theorem shows that we have decision theoretic substitutes for particular numerical credences. However, we can demonstrate the general strategy of substituting genuine doxastic attitudes with decision theoretic ones by considering a much simpler example: a decision theoretic substitute for the notion of being uncertain – i.e. having a credence strictly between 0 and 1.

Given that we can make sense of an agents preferences between arbitrary propositions, it is also possible to make sense of what we might call *conditional* preferences. Just as you can ask whether someone prefers \( A \) to \( B \) you can also ask whether they prefer \( A \) to \( B \) on the supposition of \( C \). The former question amounts to asking whether things seem better to the agent on the supposition of \( A \) than on \( B \), whereas the latter can be understood as asking whether things seem better to the agent on the supposition of both \( A \) and \( C \) than
on both $B$ and $C$. If we formalise a preference with the relation $A \prec B$ then conditional preference on $C$, written $A \prec_C B$, can be defined as simply $AC \prec BC$.

If the agents preferences and preferences conditional on $A$ are defined and identical, then we can say she is practically certain of $A$. Not only would her best action be her best action conditional on $A$, but her preferences over all propositions remain the same on the supposition of $A$. Thus for all possible practical reasoning $A$ is taken for granted.$^{19}$ The representing probability function assigns $A$ an intermediate value iff $A$ is practically uncertain (assuming the agent doesn’t have completely flat preferences).

Let us now see how practical uncertainty can explicate uncertainty about the vague. The example will trade crucially on the fact that it is coherent to suppose things that are impossible to know. To see this not that it’s perfectly coherent to suppose that the number of particles in the universe at this moment is even and that nobody will ever know this fact. No-one could ever know this conjunction, since it would involve someone knowing that the number of particles in the universe is even and also knowing that nobody knows this, which is jointly impossible. In the case of vagueness it means that even if we know that it’s borderline whether $A$, one can still ask what one’s preferences are on the supposition that $A$, even if it is impossible to know $A$.

Here is an example in which uncertainty due to vagueness reveals itself in our preferences. Suppose that I know I have between $x$ and $y$, and suppose also that it’s determinate that anyone that has between those two amounts of money is borderline rich. Let us also suppose that my credences are uniformly distributed over the possible amounts of money I could have between $x$ and $y$. Even though I’m in fact uniformly distributed over these values, on the supposition that I’m rich I’m more confident that I have larger amounts of money than the smaller, for this is exactly the kind of effect that conditioning on vague propositions has on my credences over the precise propositions. Thus things are better for me on the supposition that I’m rich than on no suppositions, which means that my unconditional preferences rank the proposition that I’m rich above the tautology. However, trivially, I rank the proposition that I’m rich along with the tautology conditional on my being rich since, for any $A$, $V(A \land A) = V(\top \land A)$. It follows that I’m practically uncertain whether I’m rich. Of course, I introduced the above preferences talking about credences, however one doesn’t need to be able to make sense of credence to ascribe that pattern of preferences.

Note also that since it’s in fact borderline whether I’m rich at $t$ I won’t ever have evidence that I’m rich at $t$. However, as I have argued in section 6.1, this does not make these kinds of preferences epiphenomenal – they often will have real practical import.

Thus it seems that one can make sense of probability talk even if you do not accept the thesis that vagueness involves any real uncertainty. One could, at this juncture call the resultant probabilities ‘credences’ – for after all, they play the right functional role by connecting with our desires and actions in the way decision theory proscribes. This would be my preference, although the conclusions I have drawn about the coherence of probabilistic talk depends on making this identification.

6.4 Alternative Decision Theories

In the last section we effectively gave an argument for probabilism. Provided our suppositional preferences between propositions satisfy a certain list of axioms – axioms which seem plausible even when applied to suppositional preferences between vague propositions – then there is a probability function an utility function that govern those suppositional preferences. Thus, assuming these purely qualitative axioms on preferences, the doxastic state that plays the action guiding role will be one that obeys the probability calculus.

It is still natural to wonder, however, what would happen to decision theory if we adopted a non-standard probability theory such a Field’s. Of course, one cost is that we’d

$^{19}$Related notions plays an important role in some philosophical accounts knowledge; see for example Fantle and McGrath [40].
have to relinquish some of the qualitative constraints on preferences we urged for in the last section. But setting costs these aside, it would be interesting to know what happens to the various sharpenings of Practical Irrelevance in these alternative decision theories. This area is still very much underexplored, and the number of options are presumably vast, so I shall limit myself to a small number of proposals that seem to me to be particularly natural.

In terms of the quantitative decision theory it is either Averaging or Maximise that must go. In section ?? we discussed the view, outlined in Field [44], that being confident that a proposition is borderline mandates having a low confidence in both the proposition and its negation. As we noted there, Averaging is presumably false in this theory. If I am certain that it’s borderline whether Harry is bald then (call this \( B \)), for any proposition \( A \) my conditional credence in \( AB \) and in \( AB \) given \( A \) should be 0. I’m certain \( B \) (and \( \bar{B} \)) is borderline, and thus certain in this conditional on \( A \), so my credence in \( B \) (and \( \bar{B} \)) and my credence in \( B \) (and \( \bar{B} \)) given \( A \) must be both be 0 since these are both credence functions that represent me as being certain that \( B \) is borderline. Averaging entails that \( V(A) = V(AB)Cr(AB \mid A) + V(\bar{AB})Cr(\bar{AB} \mid A) = V(AB).0 + V(AB).0 = 0 \) for all propositions \( A \).

However it is much in the spirit of Field’s view that the standard decision theory can be applied in the limiting case where the relevant propositions are all precise. In fact there are natural decision theories that keep a variant of averaging:

**Restricted Averaging:** \( V(A) = \sum_i V(E_i)Cr(E_i \mid A) \) where the \( E_i \) partition the space and are all precise propositions.

Note that while one cannot calculate the value of a proposition relative to a vague partition, there is no restriction on the kind of proposition you can evaluate using this principle – \( A \) gets a value even if it’s vague.

One might worry that since vagueness is so pervasive this principle is applicable in only a few instances. At the very least, Restricted Averaging fails to track the way we psychologically compute values for propositions. One might informally think about what good things would occur under the supposition of two or more exclusive exhaustive states of affairs. However, mathematical and logical thoughts presumably won’t carve out an interesting partition (i.e. a partition with more than one element) since they are not contingent, and it is implausible to require that these suppositions are suppositions involving the concepts of fundamental physics either. When we do this kind of reasoning we inevitably involve vague partitions, and this seems to be a perfectly acceptable practice. Yet according to the theory I am discussing, this informal reasoning is fallacious: one simply cannot apply Averaging when the relevant possibilities are vague. Only the unrestricted averaging principle can account for psychologically realistic practical reasoning.

Be that as it may, let us ask whether this type of theory lends any support to Practical Irrelevance. There are in fact many ways to develop decision theory in a way that respects Restricted Averaging. Two useful functions can be defined from our original \( V \) function:

- \( V^-(A) = V(A^-) \) where \( A^- \) is the weakest precise proposition that entails \( A \).
- \( V^+(A) = V(A^+) \) where \( A^+ \) is the strongest precise proposition \( A \) entails.

\( V^- \) is the value of the proposition you get by shaving away the ‘vague corners’ of \( A \), and \( V^+ \) is the value of the proposition by adding them in. See diagram [REF].

[[REF] Diagram here (background of squares with shape that cuts diagonally across some squares. Two more diagrams: one with diagonal squares removed, and one with them added representing \( V^- \) and \( V^+ \) respectively.])

\( V^- \) is compatible with Restricted Averaging and a Fieldian account of credence. \( V^-(A) \) is just the weighted sum of the maximally strong precise propositions that entail \( A \).
When a maximally strong precise proposition, $E_i$, does not entail $A$ then either $A$ entails that $E_i$ false or it entails that $E_i$ is borderline, and in either case its probability, conditional on $A$, is 0 according to a Fieldian account. Thus the weighted average will equal $V^− \cdot V^+$, on the other hand, is the weighted sum of the maximally strong precise propositions consistent with $A$. This could be natural on an account of credence that is dual to Field’s — one in which the credence in a proposition and its negation are both 1 when you are certain that the proposition is borderline. (This type of view seems more appropriate for subvaluational accounts of vagueness (see Hyde [66]) than for supervaluational accounts.)

Note that the first option is closely related to another function:

- $V^\Delta (A) = V(\Delta A)$

This seems natural in Field’s framework since it is analogous to his stipulation that $Cr(A) = Cr(\Delta A)$. Since Field accepts an $\mathbf{S4}$ logic of vagueness it is possible to maintain that $V^\Delta$ and $V^-$ are equivalent. Without this assumption, however, differences arise due to higher order vagueness: in general the weakest precise proposition that entails $A$ is not the same as the proposition that $A$ is determinate (although they will always be necessarily equivalent; see model theory from chapter 3.) Most importantly for our current purposes, $V^\Delta$ might not satisfy Restricted Averaging unless we make the further assumptions distinctive to Field’s framework.

One could revise Jeffrey’s decision theory by employing either of the above value functions to determine which action proposition has the greatest value.\footnote{One might wonder what happens to the qualitative decision theory. It is relatively straightforward to see that averaging, continuity and the axiom of testing equiprobable propositions all fail. Thus very little of the standard logic of decision survives in this framework; whether one can find a replacement theory and prove a representation theorem for it bears further investigation.}

An alternative to Field’s theory of credences is a theory in which probability functions, instead of taking real numbers as values, take intervals. Call these ‘fuzzy credences’ (see Levi [79].) A fuzzy credence can equivalently be represented by a set of classical probability functions.\footnote{Formally, a fuzzy credence must contain the function that maps each proposition to a Fieldian probability function while the set of classical probability functions that always assign values greater than a given Fieldian probability function will behave like a fuzzy credence.} In this context it is natural to think that each member of a fuzzy credence will agree about the probabilities assigned to precise propositions, but may diverge on vague propositions. It will be useful to use the notation $[Cr]$ for the set of probability functions that agree with $Cr$ on each precise proposition; $[Cr]$ will always be a fuzzy credence. This formalism is closely related to Field’s theory, since the function that maps each proposition to the infimum of the values the proposition takes relative in a set of probability functions will be a Fieldian probability function, while the set of classical probability functions that always assign values greater than a given Fieldian probability function will behave like a fuzzy credence.

Nonetheless, there’s a natural way to use this formalism to state a decision theory distinct from the one above. Rather than employing a single value function, we should have a set of value functions, one for each probability function in the agents fuzzy credence. Each of these value functions will satisfy the unrestricted version of Averaging for some function in the fuzzy credence, so in some sense averaging is preserved. However the relation between each value function and action will be changed. This time an agent should make an action proposition true only if it’s all of its values are better than any value of any other proposition. Assuming the agent has a fuzzy credence of the form $[Cr]$, we can state this rule just using the functions $V^−$ and $V^+$ above:

**Restricted Maximise** If the agent is in a position to make $A$ true, it is permissible to do so if $V^−(A) \geq V^+(B)$ for every other action proposition $B$. It is obligatory if every $V^−(A) > V^+(B)$ other action proposition $B$.

This is a view which, in a sense, keeps Averaging but rejects Maximise. Unlike the decision theories consider so far, however, it is silent in some cases. If no action proposition

\begin{center}
\begin{tabular}{|c|c|c|}
\hline
Action & Value & Decision \\
\hline
A & $V^−(A)$ & True if $V^−(A) \geq V^+(B)$ \\
B & $V^+(B)$ & False otherwise \\
\hline
\end{tabular}
\end{center}
has no more value than any other action proposition has value then this theory says nothing about what’s permissible or obligatory to do in this situation.

This principle is in fact completely consistent with MAXIMISE, since in the cases where the restricted version delivers verdicts it delivers the same verdict. It is therefore important to have an answer to the question of what one ought or may do in the situations where the restricted principle doesn’t apply. If the answer is that we should chose the option with best value, in the Jeffrey sense, then the theory is not distinct. Indeed, this is one of the main problems for this formalism quite apart from its specific application to the case of vagueness (see Elga [37], and Moss [93] and Williams [127] for positive views.)

At any rate, with these alternatives on the table we can once again ask whether our arguments against PRACTICAL IRRELEVANCE generalise to this setting. Our first point was that the effect of learning a vague proposition can have a distinctive effect on how we rank the various actions, and which action we perform: in some cases, after learning a vague proposition the precise propositions might be ranked in a way that would not have happened if we were to learn any precise proposition.

Note firstly that the notion of conditional preference can be introduced to all of the decision theories introduced above. In Jeffrey’s theory the conditional value of \( B \) on \( A \), written \( V_A(B) \) is defined by first conditioning the probability on \( A \) and then calculating the ordinary expected utility of \( B \); the result in this case is just \( V(AB) \). Thus the values \( V^- \) and \( V^+ \) after learning a proposition \( A \) are given by the following equations: \( V^-_A(B) = V_A(B^-) \) and \( V^+_A(B) = V_A(B^+) \). Given these definitions the discussion of section 6.1 transfers straightforwardly to the present setting, for note that \( V_A, V^-_A \) and \( V^+_A \) agree with one another about the values of all the precise propositions since if \( B \) is precise \( B = B^- = B^+ \). In that section we gave an example of a value function, \( V \), such that conditional on some vague proposition, \( A \), it ranked the precise propositions in a way that couldn’t be achieved by learning any precise proposition. However, by the above observation, the same function shows that the corresponding functions \( V^+ \) and \( V^- \), conditional on \( A \), will rank the precise propositions in way that is cannot be achieved conditional on any precise proposition.

What about our discussion of vague actions? In section 6.1 we showed that there could be two people who (i) were in a position to make the same propositions true, (ii) assigned exactly the same values and credences to every precise proposition (iii) were rational in the sense that they would always make a proposition the the highest value true, but (iv) could nonetheless end up acting differently due to differences in their vague values. In the present context we cannot arrive at such a strong conclusion, for both \( V^+ \) and \( V^- \) satisfy a strong form of value supervenience: if Alice and Bob assign the same value to every precise proposition then they assign the same values to all propositions. This fact is an obvious consequence of our definitions, because the value of any proposition, \( A \), according to either \( V^+ \) or \( V^- \), is always identical to the value of some precise proposition \( A^+ \) or \( A^- \) respectively. (Note that even the supervenience of vague beliefs on precise beliefs defended in chapter 5 doesn’t commit us to this kind of value supervenience – for example, while the supervenience principle I defended there told us that two people with the same evidence couldn’t agree about the precise whilst disagreeing about the vague, it said nothing about two people with different evidence.)

However despite this supervenience thesis, we can still get a weakening of our conclusion in section 6.1, by removing condition (i). For even if Alice and Bob assign the same values to all propositions they can act differently if the actions available to them are different. In some cases the best proposition available to Alice might be a vague proposition that Bob isn’t in a position to make true, and so Alice will make that proposition true, and Bob will not. Which actions are available to Alice and Bob can be inherently vague, and so vagueness cannot be bracketed in practical deliberation, even when we are using one of these alternative decision theories that validate a form of supervenience of vague values on precise values.
Although formalisms invoking the notion of a precisification have their difficulties, it cannot be denied that they have had a positive influence on the philosophy of vagueness. Much like the relational semantics for modal logic or the similarity semantics for counterfactuals, to mention two prominent examples, precisifications have considerable heuristic utility. The abstract semantics provides us with enough structural constraints to get a clear picture of what is going on, as well as providing us with an important tool with which to build models and answer questions about validity and consistency. There is a defeasible onus on any attempt to do away with them to provide an alternative framework with similar benefits.

It is tempting to contrast supervaluationism with accounts that take the notion of determinacy and borderlineness to be primitive, unanalysable operators; according to the latter kind of theory this type of formal model theoretic treatment is typically absent (see, for example, Field [44]). It is, however, important to distinguish the project of giving a heuristically illuminating model theory from the project of giving a reductive analysis of borderlineness in terms of the notions employed by that model theory. This distinction can be seen with the notion of an admissible precisification: informally, a precisification that counts a fully grown elephant as ‘small’ is not admissible because it gets things determinately wrong – it’s not even borderline whether a fully grown elephant is small. This informal explication of admissibility is clearly not going to help someone who does not already know what ‘determinate’ or ‘borderline’ means, and cannot serve as a reductive analysis of borderlineness. One can try to give an alternative explanation of the technical notion of admissibility that doesn’t explicitly invoke borderlineness, but it should be clear that the success of this project is independent of the value of the formalism.

The situation here is similar to other formal analyses that have been successful in philosophy such as the analysis of modality in terms of possible worlds or the analysis of counterfactuals in terms of similarity. It is far from clear that one can really achieve a reductive analysis of modality in terms of the notion of a possible world, or of counterfactuals in terms of similarity – one cannot have a grasp of the notion of a possible world or of the relevant measure of similarity unless one already has the modal or counterfactual concepts at your disposal. Yet the benefits of these abstract analyses of modality and counterfactuality are clear. I suspect that it will similarly be controversial whether my alternative to the supervaluationist model theory provides the basis for a reductive analysis of borderlineness, however whether or not this is true, the model theory has important heuristic value.

In brief, this alternative model theory does not employ the notion of a precisification or a possible world that we found to be objectionable in chapter 3, but instead works in terms of a certain class of functions from propositions to propositions (more precisely, ‘a group of automorphisms’ – a notion we will introduce later) that preserve certain properties. A
propojition, \( p \), is said to be precise if it is fixed (mapped to itself) by all of these automorphisms, and determinate if every proposition \( p \) is mapped to by one of these functions is true. A logic of determinacy can thus be developed in a way completely parallel to the supervaluational framework, although formally the underlying semantics is quite different. Although the formal account is inspired by our philosophical explanation of borderlineness in terms of rational desires and beliefs, the framework is quite general and is in principle compatible with other philosophical interpretations.

Crucial to this project will be a philosophical account of propositions that is capable accommodating the existence of vague propositions. The notion of a precisification and a possible world are completely absent from this formalism; instead propositions are taken as primitive and certain types of maximally strong propositions play the role that world-precisification pairs played in the supervaluationist framework. Vague propositions cannot be sets of possible worlds for these are not fine-grained enough. But they are not linguistic entities either – sentences of a particular language, or world-precisification pairs, where the precisification is understood as an interpretation of that same language, are both more fine-grained but it is not clear that they are kinds of things that can be believed and thought by people who do not speak that language. Thus it is fairly urgent that we have a theory of propositions that does not fall into either of these camps.

7.1 Vague Propositions

So far I have been contrasting views that theorise in terms of worlds and precisifications to a more general view that simply takes the indices at which propositions are evaluated to be primitive. While we have a clear conception of what a precisification is – a bivalent interpretation of a language – and we have a number of competing, but well developed, theories of what possible worlds are, we are completely lacking an account of these more fine-grained entities which I’ve been calling ‘indices’ or an account of fine-grained propositions in general. Calling them ‘epistemically possible worlds’ might seem helpful as a way of fixing ideas, but it serves only to shift the burden of explanation. In this section I will develop a theory of moderately fine-grained propositions according to which indices are identified with a certain kind of proposition.

Before we consider the theory, let me make some general remarks about the scope of this project. Firstly, it will be clear as you read on that the account of fine-grained propositions I propose here does not solve, and is not intended to solve, all of the problematic issues surrounding propositional attitude verbs like ‘knows’, ‘believes’, ‘desires’ and so on. For example, the theory I am going to develop will distinguish between the proposition that Harry is bald and necessarily equivalent propositions stating the situation on Harry’s head in more precise terms, but will identify the propositions expressed by any pair of tautologies, even if one tautology is significantly less obvious than another.\(^1\) Call theories like this, that distinguish between necessary equivalents but not logical equivalents, ‘moderately fine-grained’. To see why a moderately fine-grained theory of propositions is motivated I think it is important to distinguish two problems.

One of these is a problem in the philosophy of language about propositional attitude reports. This is a puzzle about how we succeed in reporting facts about a persons mental states in English using verbs like ‘believes’ and ‘desires’ and so on. In my view this is fundamentally a puzzle about how we use words to ascribe attitudes – relations between people and propositions – and not a puzzle about the attitudes themselves. Indeed many of the most promising solutions to the problem of attitude reports say something distinctive about the kinds of words we use to ascribe attitudes, whilst in principle remaining compat-

\(^1\)It will also plausibly distinguish other necessarily equivalent propositions whose distinctness has nothing to do with vagueness, such as the proposition that Hesperus is Hesperus and the proposition that Hesperus is Phosphorus.
ible with various theses concerning how fine-grained the arguments of these attitudes are.\footnote{See Salmon [109], Soames [117], Crimmins and Perry [27], Richard [106]. Note, however, that although many of these authors also have views about how fine-grained propositions are, these views are theoretically separable from the solution to the attitude ascription. According to the view I favour, people typically stand in many different but related attitudes to many different but related propositions at any given time, and there's a certain amount of context sensitivity about which of these attitudes words like ‘believes’ and ‘desires’ pick out which can be used to explain the linguistic data on attitude reports.}

A contextualist, for example, postulates the existence of a number of distinct but related propositional attitudes, which the word ‘believes’ can pick out in different contexts; when it appears as though we are making conflicting attitude reports about the same proposition the contextualist says we are really attributing different attitudes to the same proposition.

This is how it should be – if we took all of our immediate judgments about differences between propositional attitude reports to show that there was a corresponding difference between the objects of the propositional attitudes, propositions would be as fine grained as sentences, or maybe even more fine grained.\footnote{For example we can judge that the sentence ‘John said that it was hot out’ is true without also judging ‘John said that it was hot out’ as true. In both cases we have the same embedded sentence with different emphasis. These judgments, most will agree, call out for a pragmatic explanation. However, someone who insisted on taking all of these judgments at face value would have to treat propositions as more fine grained than sentences.} According to the theories I just listed, one cannot infer much about the fine grainedness of propositions from judgments about attitude reports. To make this vivid, note that these theories seem to have the explanatory power to reconcile our judgments about attitude reports with the view that propositions are sets of worlds, or even truth values!\footnote{For example, perhaps when we dissent from the attitude report ‘Louis Lane believes that Clark Kent flies’ we are denying that Lane stands in a relation to the true via a certain mode of presentation. Of course, there are plenty of problems with treating propositions as truth values, but I think that the problem of accounting for propositional attitude reports is not one of them.}

The reasons I take to motivate a moderately fine grained theory of propositions have nothing to do with accommodating attitude reports, or self-ascriptions of belief. The linguistic data on belief reports, despite the attention it receives, is only a small aspect of a full account of propositional attitudes. It is important not to forget that a persons propositional attitudes are also important for explaining and evaluating all kinds of behaviour, both verbal and non-verbal. An instructive example is the view that there are only two propositions: the true and the false. According to this view it is impossible to explain a person’s behaviour in terms of their beliefs and desires, for there are only sixteen types of people depending on the combination in which they believe or desire the two propositions, and this is certainly not enough to explain or evaluate the all the possible kinds of behaviour they exhibit. Thus in order to explain or evaluate a person’s behaviour in terms of their propositional attitudes, propositions must be more fine-grained than truth values. This line of reasoning plausibly generalises to rule out sets of worlds as well – two people with necessarily equivalent beliefs and desires can rationally behave very differently. On the other hand the fact that propositional attitudes determine rational behaviour gives us a defeasible reason to think that propositions cannot be so fine-grained as to distinguish logical equivalents. This is because our best theories of rational action – decision theory and probability theory – assume that propositions are moderately fine-grained. Decision and probability theory typically begin by assigning probabilities and utilities to indices of some sort and from this one assigns probabilities and expected values to arbitrary sets of these entities; to treat propositions as isomorphic to sets of entities is just to assume that propositions are structured like a Boolean algebra which in turn guarantees that equivalence in classical propositional logic suffices for identity. (Note that it is possible to do decision theory and probability theory with a very fine-grained notion of proposition by instead assigning values and probabilities to equivalence classes of logically equivalent propositions. Those who insist on such a fine-grained account of propositions can still accept the theory I am about to propose as an account of a kind of theoretical entity: entities that play the theoretical role equivalence classes play in the more fine-grained account.)
A second general point about this project is that, although it rejects possible worlds as ordinarily conceived, it is not supposed to be revisionary to ordinary semantic theorizing. Possible world semantics has enjoyed a great amount of success among linguists and is widely adopted in contemporary semantics, however the success of this style of semantics has nothing to do with the fact that in many philosophical interpretations the objects at which we evaluate sentences for truth are taken to be possible worlds; little would change if the worlds of the theory were interpreted differently. The proposed theory allows us to retain the thought that the meaning of a sentence is given by a non-linguistic entity, a 'proposition', whilst rejecting the claim that these things are individuated modally.

Lastly, a view such as this one does not preclude us from adopting the standard representation of propositions as sets of objects of some sort – things I have neutrally called 'indices'. Note, however, that we cannot think of the indices as maximally specific ways the world could have been. This assumption seems to load the dice in favour of natural language operators like 'could' and 'possibly'. It is no surprise that on this way of constructing indices we run into trouble interpreting attitude operators like 'believes that' or 'desires that' in this framework. The assumption that indices are maximally specific ways the world could have been is not forced on us; we could just as easily interpret the indices as being maximally specific ways the world could be believed to be, or desired to be. One can still think of sets of such things as truth conditions: they are, after all, still conditions which can obtain or fail to obtain. The conditions under which a belief, say, is true could easily be when Harry is bald or when Hesperus is Phospherus, for these kinds of things can be true or not as the case may be, and they can be believed to be the way things are, or the way things are not, and so on and so forth. There is no good reason to think that truth conditions must be individuated coarsely by their relation to verbs like 'necessarily' and 'possibly'.

It's worth noting that even without bringing in considerations to do with propositional attitudes the style of semantics which employs indices and accessibility relations may not always allow one to interpret the indices as possible worlds. For example, a purely modal language can only allow the indices to be interpreted as representing possible worlds if we are assuming a modal logic including at least the $S4$ principle. If we are taking the index semantics at face value then only the indices accessible to the index representing the actual world will be genuine 'possible' worlds, and the other indices needed to represent the semantics cannot be understood as representing possible ways the world could be. Similar points hold for counterfactual logics which allow for non-trivial counterfactuals with impossible antecedents. Neither of these two examples rely on the features of propositional attitudes.

The success of the form of semantics which uses indices and relations to model operators therefore has nothing to do with the indices being interpreted as possible worlds, or with the assumption that propositions are individuated modally. If they are not individuated modally, how are they individuated? This is the question I shall be devoting my attention to here. Roughly speaking, I will argue that propositions are individuated by their role in thought – something analogous to a 'conceptual role', although, of course, something which applies to propositions and not sentences or thoughts. The role that a proposition, $p$, plays in thought is roughly analogous to the conceptual role of a belief, desire (or Ving) that $p$. This is partly given by the objective norms that governs one's propositional attitudes towards that proposition. Since the word 'conceptual role' has such strong linguistic connotations I shall henceforth use the phrase 'role-in-thought' for the role that a proposition plays in a theory of rational propositional attitudes.

\footnote{Where strength is measured by the notion of $L$-implication introduced in chapter 3.}
7.1.1 A theory of vague propositions

The theory of propositions and indices follows from a broadly Stalnakerian way of making sense of the possible world talk. According to the Stalnakerian view outlined in [120] we begin with a set of indices understood as primitive objects playing a special role in a theory of rational agency. Whatever structure indices have is abstracted from this theory, and beyond this their nature is left open.

Stalnaker is not explicit about what this theory is exactly, but he does say the following:

“What is essential to rational action is that the agent be confronted, or conceive of himself as confronted, with a range of alternative possible outcomes of some alternative possible actions. The agent has attitudes, pro and con, towards the different possible outcomes, and beliefs about the contribution which the alternative actions would make to determining the outcome. One explains why an agent tends to act in the way he does in terms of such beliefs and attitudes. And, according to this picture, our conception of belief and of attitudes pro and con are conceptions of states which explain why a rational agent does what he does.” Stalnaker, Inquiry, p5. [120]

On this picture of the metaphysics of indices, they are not things which are deeply tied to the world, as a possible world in Lewis’s sense would be: “they obviously are not concrete objects or situations, but abstract objects whose existence is inferred or abstracted from the activities of rational agents” ([120] p50-51.)

From the above passage you would be forgiven in thinking that Stalnaker individuates the ‘possible outcomes’ according to a rational agents bouletic attitudes (‘attitudes, pro and con’) and doxastic attitudes. It is worth noting, however, that Stalnaker individuates propositions, and possible outcomes, modally, and indices in the sense abstracted above are identified with possible worlds. Stalnaker’s reasons for doing this seem to have little to do with the picture outlined so far, and have more to do with his reductionist ambitions with respect to the problem of intentionality. He even considers the impossible worlds approach, writing ‘could we escape the problem of equivalence by individuating propositions, not by genuine possibilities, but by epistemic possibilities – what the agent takes to be possible?’ but dismisses it on the grounds that although ‘this would avoid imposing implausible identity conditions on propositions [...] it would also introduce intentional notions into the explanation, compromising the strategy for solving the problem of intentionality.’

For those who do not have any reductionist ambitions, there does not seem to be any barrier to individuating these objects epistemically. Indeed, it seems obvious to many that the entities we abstract from a theory of rational decision will be more fine grained than possible worlds. An astronomer who believes that Hesperus is a planet may display very different behaviour, both verbally and non-verbally, from someone who believes that Phosphorus is a planet. It furthermore seems perfectly possible that this astronomer may have very good evidence that Phosphorus is a planet, which is not also evidence that Hesperus is. In which case it seems to me to be perfectly rational for this agent to believe that Hesperus is a planet without believing that Phosphorus is and rational for her to act accordingly.

Since I am arguing for a moderately fine-grained theory – I am claiming that propositions are more fine grained than possible worlds but not as fine-grained as to distinguish logical equivalents – it’s natural to ask at what point do we stop individuating? How fine grained are propositions? Stalnaker’s theory also provides a perfectly principled answer to this

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6 Of course, for Stalnaker indices are metaphysically possible worlds.

7 Note: in the more recent [119] he disavows the project of reduction.

8 Stanley [121], for example, makes the similar point that Stalnaker’s ‘causal pragmatic’ approach to intentionality appears to be compatible with, and even motivates, a theory in which indices are epistemically possible worlds.
question: indices are as fine grained as we need them to be to specify the functional role of the belief that \( p \) in terms of the indices at which \( p \) is true. In other words, indices are whatever they need to be so that functional roles do not distinguish more finely than sets of indices.

The functional role of a belief that \( p \) does not include the role that a belief that \( p \) plays in the cognitive workings of completely erratic and irrational agent (if we could ever determinately attribute a belief that \( p \) to such an agent.) No two belief states have the same cognitive role across every possible agent, rational and irrational; the resulting notion of proposition would be uninteresting and useless for the purposes of giving a theory of objective information that can be communicated among different agents. For example, in order have a theory of communication some degree of rationality must be assumed by both participants in a conversation if we are to be able to infer anything about what they believe from the sentences they are uttering.

Let us now pin down some of the formalities. According to the Stalnakerian picture, indices are to be abstracted from a theory of rational action. The theory I shall adopt for this purpose contains two primitives. Firstly a set of objects, \( P \), to be understood informally as representing the set of propositions. However, although this will be the ultimate interpretation of \( P \) we do not assume that this set has any structure, Boolean or otherwise, from the outset. We shall not even assume that the standard logical operations are defined on \( P \); definitions of conjunction, negation and so on, and their logical properties will arise out of the theory. Secondly, a set, \( Coh \), of binary functions taking two elements of \( P \) to a real number in \([0,1]\). \( Coh \) informally represents the set of conceptually coherent conditional ur-priors (these terms will be explained later), and each element of \( Coh \) can be informally thought of as taking two propositions and telling us how likely one of these propositions is conditional on the other according to that ur-prior. Lastly, one could also augment the theory with a set, \( U \), of conceptually coherent utility functions – I suggest one reason why we might need this later, however for the basic theory that follows we won’t need to appeal to \( U \).

It is important to realize that the following theory axiomatizes these two notions together. The theory simultaneously entails the most important facts about the structure of \( P \), the set of propositions, and also the behavior of the elements of \( Coh \), the rational priors – it is not possible adequately characterise only the propositions without indirectly characterising the conceptually coherent ur-priors, or vice versa. Effectively the theory will entail that \( P \) has the structure of a complete Boolean algebra, and that \( Coh \) consists of ‘normal Popper functions’ – a slight generalisation of the notion of a probability function that allows us to talk about probabilities conditional on zero probability events. They are called ‘normal’ because they assign sensible probabilities conditional on all consistent propositions. The result is a theory that both describes a role in thought, whilst simultaneously postulating the existence of objects that play that role. The following theory is thus not unlike Popper’s own theory (expanded on by Field in [48]), in which the logical properties of conjunction and the other Boolean operators are inferred from the account of probability, rather than simply assumed before we apply probabilistic notions to these entities.

With these primitives at hand we can now introduce the concept of an index that has played a crucial role in our theorising so far. Firstly, we must say what it means for a member of \( P \) to be consistent:

**Definition of Consistency:** A proposition \( p \) is inconsistent if and only if \( \Pr(q | p) = 1 \) for every \( q \) and \( Pr \in Coh \). \( p \) is consistent otherwise.

Intuitively, only the inconsistent proposition makes everything fully probable on its supposition.

**Definition of Index:** A proposition, \( i \), is an index if and only if it is both consistent and \( \Pr(p | i) \in \{0,1\} \) for every proposition \( p \in P \) and every \( Pr \in Coh \).
In other words, conditional on an index every proposition is either certainly true or certainly false according to every ur-prior. Another important relation is the relation of a proposition being true at an index, which can be spelt out in this framework as follows:

**Definition of True-At:** \( p \) is true at an index \( i \) iff \( Pr(p \mid i) = 1 \) for every \( Pr \in Coh. \)

\( p \) is false at \( i \) otherwise.

Note that there is a putative asymmetry between the definition of truth at and false at: a proposition is true at an index iff it has probability 1 on it relative to every coherent prior, whereas it is false at the index iff it has probability 0 for some coherent prior. The leaves open the possibility that there are two coherent priors that both assign \( p \) probability 1 or 0 on the index \( i \), but differ regarding which. This possibility will eventually be ruled out by the axioms: we can show that \( p \) has probability one on an index for some coherent prior iff it has probability 1 on that index for every prior.\(^9\) From this result it also follows that \( p \) has probability 0 at an index for some prior iff it has probability 0 on that index for every prior.

This finally allows us to introduce the logical operations within this framework, as promised:

**Definition of Conjunction:** Say that \( p \) is a conjunction of a set \( X \subseteq P \) iff \( p \) is true at exactly the indices at which every element of \( X \) is true.

**Definition of Negation:** \( p \) is a negation of \( q \) iff \( p \) is true at exactly the indices \( q \) is false at.

Given conjunctions and negations we can also introduce the other logical operations in the usual way. Note that we cannot assume that every set of propositions has a conjunction and we cannot assume that every proposition has a negation, nor can we assume that, if they do exist, they are unique. These will all be derived facts. Finally say that a set of propositions, \( X \), entails another proposition \( p \) if and only if \( p \) is true at an index whenever every element of \( X \) is true at that index.

One of the key elements of this theory will be an axiom that tells us how to individuate propositions. Intuitively, this axiom tells us to individuate propositions no more finely than they need to be in order to satisfy their role in conditional belief.

**Individuation Axiom:** If \( Pr(A \mid C) = Pr(B \mid C) \) for every \( Pr \in Coh \) and \( C \in P \) then \( A = B \)

An important consequence of this axiom is that it rules out a purely linguistic interpretation of the set \( P \): an interpretation in which \( P \) consists of the set of sentences of some language. While the other axioms, to be listed shortly, are compatible with this interpretation, and indeed this was Popper’s own interpretation of his theory, this axiom requires explicitly rules it out. For example, the axioms of my theory will require that for any two propositions \( A \) and \( B \), there be another proposition, it’s conjunction, whose probability is related to the probability of \( A \) and the probability of \( B \) in certain ways across all elements of \( Coh. \)

However these axioms allow that there be several propositions related to \( A \) and \( B \) in this way. Indeed there is a model of the remaining axioms in which \( P \) is represented by a set of sentences in an infinitary language in which the sentences \( A \land B \) and \( B \land A \), and indeed, any sentence logically equivalent to \( A \land B \), are distinct conjunctions of \( A \) and \( B \) and whose probabilities conditional on certain propositions are related to the probabilities of \( A \) and of \( B \) on those propositions in the ways described above. However, since any two conjunctions

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\(^9\)The axioms entail the existence of a negation function, \( \neg \), on propositions, such that (i) \( \neg \neg p = p \) and (ii) \( Pr(\neg p \mid q) = 1 - Pr(p \mid q) \) for any \( q \) and \( Pr \). Then the following things are equivalent: (1) the probability of \( p \) on \( i \) is 1 for every prior, (2) \( p \) is true at \( i \), (3) \( \neg \neg p \) is true at \( i \), (4) \( \neg p \) is false at \( i \), (5) \( \neg p \) has probability 0 on \( i \) for some prior, (6) \( p \) has probability \( 1 - 0 = 1 \) on \( i \) for some prior.
of $A$ and $B$ must have the same probability on any supposition to satisfy the relevant connections, the individuation axiom ensures that the conjunctions of $A$ and $B$ must be identical to one another.

Before we move on let me briefly take up one issue I raised earlier. The theory I am describing is one in which propositions are individuated by their role in thought, yet the individuation axiom above only references the role that propositions play in conditional belief. Might a propositions role in desire not also be a source of fine-grainedness? We could accommodate this by enriching $\text{Coh}$ so that it represents as set of coherent conditional probability-value function pairs, $(Pr, V)$, where $Pr$ and $V$ are related by Jeffrey’s equation. Thus a weakened individuation axiom could go as follows:

**Individuation Axiom**: If $V(AC) = V(BC)$ and $Pr(A \mid C) = Pr(B \mid C)$ for every $(Pr, V) \in \text{Coh}$ and $C \in P$ then $A = B$

This weaker theory might be natural for modelling moral propositions that play the same doxastic role, but differ in their motivational properties – to rationally believe them would require you to care about certain things. Such a theory might be useful for expressivists wishing to have some kind of lightweight theory of propositions. Although this is worth investigating, it goes well beyond my purposes here so I shall focus on the stronger but simpler individuation axiom.

The axioms of the theory can then be stated as follows:

**Bottom**: There is an inconsistent proposition.

**Conjunction**: Every set of propositions has a conjunction.

**Negation**: Every proposition has a negation.

**Individuation**: If $Pr(A \mid C) = Pr(B \mid C)$ for every $Pr \in \text{Coh}$ and $C \in P$ then $A = B$

**Reflexivity**: $Pr(A \mid A) = 1$ for every $Pr \in \text{Coh}$ and $A \in P$

**Conjunction Elimination**: $Pr(X \mid B) \leq Pr(A \mid B)$ for every $Pr \in \text{Coh}$ for every conjunction of $A$ and $C, X$.

**Multiplication Rule**: $Pr(X \mid B) = Pr(A \mid Y)Pr(C \mid B)$ whenever $X$ is a conjunction of $A$ and $C$, and $Y$ a conjunction of $C$ and $B$, for every $Pr \in \text{Coh}$.

**Additivity**: If $C$ is consistent and $B$ is a negation of $A$ then $Pr(A \mid C) = 1 - Pr(B \mid C)$

A notable absence from this theory is any form of countable additivity for Popper functions. This is how Popper initially defined his functions, although others have augmented the theory with countable additivity (van Fraassen [54]). However there is, I think, a decisive reason not to include countable additivity, namely that it is inconsistent with the existence of an infinity of probabilistically independent propositions with probabilities bounded by $(a, b)$ with $0 < a < b < 1$. You couldn’t have, for example, countably many independent coin flips.

**Theorem 7.1.1.** No fully countably additive (i.e., countably additive on any condition) Popper function has countably many mutually independent propositions, $A_n$, with unconditional probability in $(a, b)$ with $0 < a < b < 1$. (Unconditional probability here just means probability conditional on a tautology.)
I put the proof in a footnote.\textsuperscript{10}

While the pure mathematical theory puts important structural constraints on the space of propositions, there are many questions it does not settle. For example, one could simply insist that it is conceptually incoherent to assign necessarily equivalent propositions different conditional credences. Under this assumption, the individuation axiom entails, among other things, that the proposition that Hesperus is Phosphorus is identical to the tautologous proposition. More importantly the proposition that Harry is bald would be identical to a precise proposition about Harry’s hairline that it is necessarily equivalent to. This hypothesis about which priors are coherent therefore makes the theory unsuitable as a theory of vague propositions.

Conversely, one could also insist that it is conceptually coherent to have a different prior credence in the proposition that John is a bachelor than one has in the proposition that John is an unmarried man. So by Leibniz’s law these would be different propositions. On my understanding of ‘conceptually coherent prior’, however, it is simply incoherent to assign these propositions different credences or conditional credences. So on my preferred interpretation of this theory these two propositions would be identified.

Thus the take home message is that, although the formal axioms force us to make some choices – such as identifying logical equivalents – the informal notion of ‘conceptual coherence’ is also doing important work. Although I cannot hope to explicitly define the notion, it can be elucidated by examples which, I hope, should be enough to give the reader a reasonably good grasp on the notion. For example, although I am skeptical of the idea that a sentence can be true purely in virtue of the meanings of its constituents, I suspect that many standard examples of analytic sentences express propositions that get probability 1 according to every conceptually coherent prior. A prior which assigns less than full credence to the proposition that vixens are foxes represents a conceptual confusion of some sort, and according to our theory that proposition’s role-in-thought is the same as the tautologous proposition’s role-in-thought. Vagueness introduces more interesting examples. For instance, I take it that to be conceptually coherent you should be certain that Harry is bald conditional on the proposition that he has no hairs at all. This seems like a fairly straightforward example of a proposition expressed by a conceptual truth, analogous to those mentioned above. However, conditional on the proposition that Harry has N hairs, where N is in the borderline region, I take it that it is conceptually incoherent to have anything other than some intermediate credence that Harry is bald. If you furthermore have a description of all the precise facts about Harry’s head then, as I argued in chapter 5, there is a particular credence which all conceptually coherent priors assign to the proposition that Harry is bald, conditional on Harry satisfying that description.

In what follows I will also apply the notion of conceptual coherence to utility functions, measuring how much people care about certain matters. I believe the notion of conceptual coherence, as it applies to desires, also has some pretheoretic appeal. Its relation and importance to the study of vagueness has already been discussed in literature. For example, in [46], Hartry Field contrasts two examples. One involves a character, Roger, who thinks that if his bank account password has the same last digit as the number of nanoseconds Bertrand Russell was old for, then his life will go better. The other involves Sam, who things that his life will go better if the last digit of his bank account password is the seventeenth significant digit of the Centigrade temperature at the currently hottest point.
in the interior of the sun. According to Field, while Sam’s belief is thoroughly irrational, Roger’s is intuitively even worse as it is conceptually confused. The distinction between being merely irrational and conceptually confused will play an important role in the theory I am endorsing.

### 7.2 Attitudinal Symmetries

Let us now try and integrate our theory of propositions with the various theses about vagueness that we have defended throughout this book. The theory of vagueness defended in this book can effectively be summarized by the four principles listed below:

**Boolean**: The precise propositions form a complete atomic Boolean algebra.

**Plenitude**: For any function from the maximally specific precise propositions to $[0, 1]$, $E$, there is a proposition such that $p$ such that $Pr(p | w) = E(w)$ for every $w$ and conceptually coherent ur-prior $Pr$.

**Rational Supervenience**: If $p$ is any proposition and $w$ any maximally strong precise proposition then $Pr(p | w) = Pr'(p | w)$ for every pair of conceptually coherent ur-priors $Pr$ and $Pr'$.

**Indifference**: If $p$ and $q$ both entail a maximally strong precise proposition, $w$, then you should be indifferent between $p$ and $q$.

Let us try and put these ideas together into a coherent account of vagueness. Boolean ensures that we can divide the space of propositions into a partition of maximally strong precise propositions. Along with our assumptions from the last section it also ensures that we can represent propositions using sets of indices. In this isomorphic representation the partition of maximally strong precise propositions will determine an equivalence relation that clumps the indices into non-overlapping cells, or equivalence classes, of indices that agree about all precise matters.

Although different coherent priors can disagree about the sizes of these cells – in the sense that they can disagree about how probable or improbable they are – Rational Supervenience guarantees that all coherent priors agree about what proportion of each cell is taken up by each proposition. If $p$ takes up half of a cell according to one coherent prior, it takes up half the cell according to all coherent priors. Finally Indifference ensures that indices in the same cell get assigned the same utility; thus each cell gets to be associated with a potentially different utility, although within a cell all the utilities are constant. [Ref] Diagram here: logical space according to two different priors – the cells depicted with different sizes, but the propositions cutting across cells always taking up the same proportion.

This, then, is the structure of rational degrees of belief and rational degrees of desire according to our account so far. The above principles provide us with the beginnings of a theory of vagueness: they relate the notion of precision (and thus vagueness) to other concepts such as the notion of rational belief and rational desire.

However it is natural to ask if we can reverse the order of explanation to give a definition of precision in terms of these concepts. Is it possible to start with some theses purely about the structure of the coherent priors and utilities and arrive at an independent characterisation of the precise propositions?

Our starting point will be the observation that the models of propositional belief and desire described above are closed under certain operations on the space of propositions which leave the structure of belief and desire, and the logical structure of the propositions, completely unchanged.

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7.2.1 Symmetries

The key idea that will be exploited in our account of vagueness is the notion of a symmetry. The notion of a symmetry of a theory is often appealed to in the philosophy of physics. To focus on a classic example, consider Leibniz’s observation that uniform translation in a particular direction in Newtonian physics is a symmetry of that theory. The result of moving every object two meters in a particular direction will take you to a system that also obeys the laws of Newtonian physics, provided the original system obeys those laws. Moreover, this translation appears to preserve all physically significant facts. The distances between objects, their relative velocities, their shapes and sizes, indeed just about any observable property you can think of seems to be preserved by this symmetry of the theory.

The theory has other symmetries as well; spatial rotations and reflections, temporal shifts, time reversal (temporal reflection) and scaling the velocity of every object (temporal rotation) as well as arbitrary combinations of these transformations all preserve physically significant properties. Formally we say that these symmetries form a group: the operation of doing nothing is vacuously a symmetry, the result of performing one symmetry and then another is also a symmetry of the same type, and for every symmetry there is another that ‘undoes’ it, that is, takes you back to where you started. The symmetry group of a theory often gives us an important insight into the structure of the kind of objects theory characterises; one can represent many things in this fashion – from operations on a Rubik’s cube to the symmetries between the roots of a polynomial – and extract important structural insights.

The group in our above example divides the space of possible worlds into equivalence classes, known as orbits, where two worlds belong to the same orbit if they can be related to one another by some symmetry transformation. Thus, for example, two worlds containing only three equidistant colinear particles, stationary relative to one another, might belong to the same orbit because one can get from one world to the other by some combination of translation, rotation and reflection, but a world with three particles in a triangular formation would not be in the same orbit since symmetry operations preserve directly observable properties like colinearity.

The result is a picture according to which the space of worlds is divided into equivalence classes (orbits) of worlds which all agree with one another about the physically significant facts, and differ only over the positions of those objects in absolute space. The relevant symmetries therefore do not preserve all facts – de re facts about which space-time points are occupied are not preserved. However, for those who find such facts suspicious or in some sense less basic, the structure the group imposes gives us a precise way to distinguish between propositions that are straightforwardly factual in the relevant sense. Some propositions are preserved by the symmetries, and are expressible as some union of orbits, and these propositions, because they are fixed by the symmetries, will not entail suspect claims about the de re locations of objects.

In a slightly more abstract setting, we can think of a symmetry as a permutation on a set of possible worlds or indices: a mapping from worlds to worlds such that no two worlds get mapped to the same thing and such that every world gets mapped to by some world. A translation of two meters in a certain direction, for example, determines such a mapping on the set of Newtonian worlds – translation by a given vector never takes distinct worlds to the same world, and every world is translation of some other world by that vector.

This observation will be key to generalising the notion of a symmetry beyond this limited example. We can also raise the notion of a symmetry to the level of propositions. The relevant notion for propositions is that of an automorphism: a permutation of propositions that preserves all the logical relations between propositions. A function, \( \sigma \), from propositions to propositions is an automorphism iff the following things hold:

1. \( \sigma \) is a bijection (a one to one correlation.)

2. \( \sigma(\neg p) = \neg \sigma(p) \) for each proposition \( p \)
3. \( \sigma(\bigvee X) = \bigvee \{ \sigma(p) \mid p \in X \} \) for any set of propositions \( X \).

By the deMorgan laws, 2 and 3 ensure that automorphisms also preserve arbitrary conjunctions – indeed they preserve any logically definable operation. If we are thinking of propositions in terms of their representations as sets of indices, then a permutation of indices induces an automorphism of propositions by mapping a set of indices corresponding to \( p \), to the set of indices each member of \( p \) is mapped to. Conversely, every automorphism determines a unique permutation by looking at it’s action on singleton sets. In what follows I shall talk about permutations on indices and the corresponding automorphism on propositions interchangeably.

In our toy example we saw that it was possible to characterise a particular class of facts in terms of permutations on worlds that preserve certain physical features. This is possible because the theory of Newtonian mechanics has symmetries. As we will see, the theory of rational belief and desire we have sketched above also enjoys symmetries in a somewhat analogous sense. A symmetry of our theory is an automorphism which preserves the preferences and credences of every (possible) rational agent. More precisely:

**Symmetry:** An automorphism of propositions, \( \sigma \), is a rational symmetry if and only if \( V(p) = V(\sigma(p)) \) and \( Pr(p) = Pr(\sigma(p)) \) for every conceptually coherent ur-prior \( Pr \) and coherent value function \( V \) for \( Pr \).

Given the averaging principle, from chapter 6, probabilities are representable by values; so under that assumption the clause stating that ur-priors must be preserved is redundant. Thus \( \sigma \) is a symmetry if and only if every possible rational ur-agent is indifferent between \( p \) and \( \sigma(p) \).

A symmetry not only preserves all the logical relations between propositions, it preserves the relevant rational attitudes. If \( p \) and \( q \) are related by a symmetry then every coherent ur-prior must agree about \( p \) and \( q \), and the conditional expected utility on \( p \) and on \( q \) respectively must be the same for any coherent utility function. The intuitive gloss is that two propositions are related by a symmetry iff one cannot coherently hold a certain kind of propositional attitude towards one proposition but not the other. To employ a more familiar, albeit more contentious way of talking, \( p \) and \( q \) are related by a symmetry if a belief or desire that \( p \) has the same conceptual role as a belief or desire that \( q \).

As before, symmetries form a group: the identity automorphism is a symmetry, applying two symmetries in succession is a symmetry, and every symmetry has an inverse which is also a symmetry. It remains to show that our theory actually has symmetries. Note that the existence of a non-trivial symmetry implies that there will be pairs of propositions, related by the non-trivial symmetry, which have the same value and probability for all ur-priors.

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11This fact is easy to show when the agent cares about whether \( p \) (i.e. \( V(p) \neq V(\neg p) \)). For supposing \( V(p) = V(\sigma(p)) \). Then since it is a consequence of averaging that \( Pr(p) = \frac{1}{V(p)/V(\neg p)} \) and of averaging and the properties of automorphisms that \( \frac{1}{V(\sigma(p))/V(\neg \sigma(p))} = Pr(\sigma(p)) \) we can conclude that \( Pr(p) = Pr(\sigma(p)) \). If the agent doesn’t care about \( p \) then consider the value function you get from the same probability with a utility function that does make \( p \) valuable and run the same argument.

12Although the term ‘conceptual role’ comes with a lot of baggage which I don’t want spend time on, it is certainly a term one can make sense of without the baggage, and one that does a good job of describing what I have in mind here. I take it that the conceptual role of a belief or desire that \( p \) includes at least the role that belief or desire would play in a somewhat idealised psychological description of how a coherent rational agent’s beliefs and desires would evolve over time in response to sensory inputs, and the role these desires and beliefs in turn affect their dispositions to act in certain ways. It seems very plausible, given our preference theoretic definition of symmetry, that a belief or desire that \( p \) has the same conceptual role as a belief or desire that \( \sigma(p) \), for symmetries \( \sigma \). Although not as direct, the notion of conceptual role presumably extends to other propositional attitudes such as hopes, fears, wonderings and so on. Therefore a natural further question of interest is whether symmetries preserve the conceptual role of these attitudes. For example, let \( \sigma \) be an symmetry, whether the following holds:

A hope, fear, wondering that/whether \( p \) has the same conceptual role as a hope, fear, wondering that/whether \( \sigma(p) \).

These additional theses bears further investigation; I shall not do so here.
As a warm up to the existence of symmetries, note that our present theory implies the existence of pairs of propositions like this. Since all priors will agree about what proportion of each cell a proposition takes up, it is possible to talk about the proportion of a cell that a proposition takes up, relative to every prior. Thus take any two propositions, \( p \) and \( q \), that take up the same proportion of each cell. Pairs like this can be generated by **Plenitude** – for example, take a proposition \( p \) that takes up half of every cell. Then its negation, \( q \), will also take up half of every cell. It follows that \( \Pr(p) = \Pr(q) \) for any prior \( \Pr \), and since utility is always uniform within a cell it furthermore follows that \( V(p) = V(q) \) for every value function based on \( \Pr \).

The existence of full-blown symmetries is really just a generalisation of these ideas. Given the partition structure of maximally precise propositions the idea is to find permutations that move indices around within each cell of the partition, and which only maps a subset of a cell to another set of indices in that cell if both the sets take up the same proportion of the cell according to all ur-priors.

It is important to note, at this juncture, that a symmetry is not merely a permutation that preserves the precise facts. Not every permutation that moves indices around within cells satisfies the constraint that the permutation be measure preserving. (A mathematical example demonstrates this point: the \( x^2 \) function is a permutation of the unit interval, yet it maps the interval \([0, \frac{1}{2}]\) to the interval \([0, \frac{1}{4}]\) which is half its size.) The group of permutations that preserve precise facts has the same orbit structure as the group of symmetries, but it is larger. The group of symmetries therefore contains more information than the mere classification of propositions into precise and vague.

What is the relation between precision and symmetries? Given our earlier remarks it is natural to that every precise proposition should be fixed by every symmetry. Indeed, it is possible to prove this fact if we make the assumption that the set of coherent utilities and priors are sufficiently rich. If we assume, roughly, that assignments of utility and probability to maximally strong precise propositions are relatively unconstrained it is possible to show that precise propositions are fixed by all symmetries. Either of the following richness conditions would be sufficient:

**Richness of Priors:** For any probability function over the space of precise propositions there is a conceptually coherent prior over the whole space that extends it. (Which, given **Rational Supervenience**, will in fact be unique.)

**Richness of Utilities:** Any function on indices which assigns constant values through each cell represents a conceptually coherent utility function.

Here is how richness helps. Suppose a symmetry maps an index, \( i \), to another index \( j \). \( i \) and \( j \) must be within the same cell: since they’re related by a symmetry they must have the same value for every utility function, but unless they were in the same cell the second richness condition would allow us to make the values of \( i \) and \( j \) different. A similar argument can be given using the first richness condition. This time suppose there is a third index, \( k \), whose value is greater than both \( i \) and \( j \). If \( i \) and \( j \) belonged to different cells you could choose a prior that made \( i \) more probable than \( j \), and thus makes the news value of the disjunction of \( i \) with \( k \) greater than the value of the disjunction of \( j \) with \( k \). Either way we have shown that every symmetry must take each index in a given cell to another index in the same cell; thus symmetries fix all precise propositions.

What about the converse result? If a proposition is fixed by every symmetry, must it be precise? This is harder to prove, so I shall only sketch the idea. It’s equivalent to showing that for each vague proposition there’s a symmetry that doesn’t fix it. We’ll begin by showing the result for vague propositions confined within a single cell. By applying **Plenitude**\(^{13}\) we can construct another proposition, \( q \), that is part of the same cell as \( p \), is distinct from \( p \) but has the same measure conditional on the cell. Indeed, given **Plenitude**,\(^{13}\)
we can construct a measure preserving bijection, \( \tau \), between \( p \) and \( q \) (thinking of them as sets of indices).\(^{14}\) It follows that the function that fixes everything outside of \( p \lor q \), behaves like \( \tau \) on \( p \) and like \( \tau^{-1} \) on \( q \) is a symmetry that does not fix \( p \) (it maps it to \( q \)). (The result extends to arbitrary vague propositions, since if \( p \) is vague then there’s some cell which it partially but not completely overlaps, and it is then possible to run the above construction.)

In summary, then, a proposition is precise if and only if it is fixed by every symmetry. This result strongly suggests that it might be possible to reverse the order of explanation. Is it possible to start off with a group of symmetries and from this define the notion of a precise proposition? By defining a precise proposition in this way, it may in fact be possible to derive \( \text{Boolean, Rational Supervenience and Indifference} \) from this definition. \( \text{Plenitude} \), because it asserts the existence of certain propositions, cannot be derived from a definition, however it would be interesting to see if our theory of vagueness can simply be reduced to \( \text{Plenitude} \) and a definition of precision in terms of a symmetries. I shall now turn to such a definition.

### 7.2.2 Vagueness and Precision

Recall that indices are permuted within a cell in such a way that no index gets sent out of its cell by a symmetry. Thus any proposition that is either a cell, or a union of cells will be left alone by each symmetry. In other words precise propositions are fixed by symmetries. This forms the basis for a definition of precision:

**Precision:** A proposition \( p \) is precise if and only if \( \sigma p = p \) for every symmetry \( \sigma \).

A vague proposition is defined as a proposition that is not precise.

By contrast with supervaluationism, which only gives us an account of borderlineness and determinacy, a notable feature of this framework is that it instead takes the notion of a proposition being precise as basic. To provide an account of vagueness the supervaluationist must appeal to something like the modal account of vagueness as the possibility of being borderline that we criticised in chapter 3.

If we want a definition of determinacy and borderlineness we can get that from the notion of precision as in the chapter 3; a proposition is determinate iff it is entailed by the strongest true precise proposition. There is also a direct definition in terms of symmetries:

**Determinacy:** It’s determinate that \( p \) if and only if every proposition that \( p \) is mapped to under a symmetry is true.

A borderline proposition is defined as a proposition which is neither determinate nor has a determinate negation.

Our abstract analysis of precision in terms of being fixed by every symmetry entails some desirable structural features. For example, we can infer that the precise propositions form a complete Boolean algebra: negations and arbitrary conjunctions and disjunctions of precise propositions are also precise. For if a proposition is fixed by every symmetry automorphism, so is its negation by the properties of automorphisms. Similarly, if every member of \( X \) is fixed by every symmetry, so is the disjunction and conjunction of \( X \).\(^{15}\)

If we go beyond the abstract definition and invoke the particular notion of symmetry we have been using, the rational supervenience principle and indifference principle (but not plenitude)\(^{16}\) likewise fall out of our definition.

Let me now turn to some points of clarification in our analysis. The first point to observe is that while there are symmetries in the space of conceptually coherent priors and values,

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\(^{14}\)Provided they have the same cardinality: this can be guaranteed by requiring that every proposition with positive measure has the same cardinality. (This is, for example, true of the Lebesgue measure on the real numbers – a consequence of the perfect set theorem.)

\(^{15}\)[REF] What about atomicity?

\(^{16}\)This is hardly suprising since the principle of plenitude has existential import – no definition can guarantee is truth.
these may not represent symmetries in the credences and values of informed people. This fact is due to the possibility, defended in chapter 4, of one’s total evidence being vague, for vague evidence can break symmetries in the space of coherent priors. For example, take a proposition that takes up half of every cell, so that there is a symmetry switching it for its negation. If this propositions was someone’s total evidence then they would assign it full credence and it’s negation no credence, breaking a symmetry that existed among the priors.\textsuperscript{17}

Despite the fact that symmetries can be broken amongst the credences of people whose evidence is vague, it is still natural to think that the space of rational values and credences of informed people are closed under the symmetry in an extended sense. If $V$ is the value function of a possible rational person, so is function $V(\sigma(p))$, which assigns $p$ the value of its image under the symmetry $\sigma$. Clearly when $V$ is value for a prior credence function then these two value functions are identical, but even when $V$ corresponds to the values of an informed person symmetries do take you outside the set of possible rational values.

Another point that requires some clarification is the role that the preservation of rational desires is playing in our notion of symmetry. What would happen if we just worked with a notion of symmetry that preserves initial credences?\textsuperscript{18} Evidently the indifference principle would no longer be a consequence of our definitions, so the principle is needed for my specific project. But would a definition purely in terms of credences give an extensionally adequate characterisation of precision?

In section 7.2.1 we effectively showed that provided we accepted a richness condition on the space of coherent priors, one can characterise the set of precise propositions as those which are fixed by all symmetries that preserve all coherent initial credences. (A similar result applies for symmetries which preserve coherent utilities.) But what exactly is the status of the richness condition?

For a function to be to be a rational prior it is certainly necessary that it be probabilistically and conceptually coherent. But is this sufficient? Perhaps priors ought also to satisfy the principal principle, support sensible inductive hypotheses, respect the principle of indifference, and so on. Once these further constraints are taken into account one might think that there are propositions that all initial priors must agree on. Indeed, at the extreme end of the spectrum there are some who think that there is exactly one prior which it is rational to adopt (see Carnap [24], and more recently Williamson [133].) On such views the richness condition on rational priors fails quite dramatically.

This conception of a rational prior might be thought to pose a problem for my abstract analysis of precision. Let me focus on one potential example like this. Suppose that a principle of indifference regarding purely haecceitic differences is in operation. Imagine, for example, a world exactly our own except that two people have swapped qualitative role: Bob has lead a life qualitatively indiscernable from the life Bill actually has led, and conversely Bill has led a life qualitatively exactly like the one Bob has led. You might think that any rational prior should assign just as much confidence to the former possibility as the latter. This, in some sense, substantiates the idea it is hard to distinguish between such possibilities without possessing any de re evidence. More generally, any permutation of individuals will naturally induce a permutation on qualitatively identical worlds – a general principle of haecceitic indifference would require that these permutations preserve rational prior credence. Thus although the proposition that Bob is exactly 175cm tall seems to be precise, assuming the haecceitic indifference principle, there’s a symmetry preserving rational prior credence that maps it to the distinct proposition that Bill is exactly 175cm

\[\text{This example demonstrates the general phenomenon. However, because this proposition is necessarily borderline, it is not particularly plausible that this proposition could be someone’s total evidence. If we disjoin this proposition and its negation with some contingent precise fact we get a more realistic example.}\]

\[\text{The notion of a symmetry preserving initial credences is naturally defined by the equation } Pr(p) = Pr(\sigma(p)) \text{ holding for every coherent prior } Pr. \text{ However probability 0 propositions can have interesting features according to a Popper function that needn’t get preserved according this definition of symmetry. A better definition would be } Pr(p | p \lor \sigma(p)) = Pr(\sigma(p) | p \lor \sigma(p)) \text{ for every coherent prior } Pr.\]
The account I have developed states things in terms of credences that are conceptually coherent, not in terms of a generic notion of rationality. Conceptual coherence is a fairly weak constraint: a prior which makes no purely conceptual confusions may still be irrational in the wider sense. There is nothing conceptually incoherent about a prior that supports strange inductive inferences, for example, but many would not count such a prior completely rational. Similarly, while it may, in some sense, be unreasonable to have priors that find it more likely that Bob has a certain qualitative role than that Bill does, there is nothing conceptually incoherent about this belief. It is not, for example, like having priors that assign the proposition that Harry is both poor and is a billionaire a high probability.

Provided we are clear about the distinction between rationality in toto and conceptual coherence, then it may indeed be possible to extensionally capture the notion of precision without invoking desires. However, I would imagine that there will be some who, despite my attempts at elucidation, find the distinction between a rational prior and a merely conceptually coherent one too obscure to be bearing the burden of explicating this important philosophical notion alone. For those wishing to theorise only with the notion of rationality in toto it is more important that the constraint that symmetries preserve desires be included. It seems quite evident that I could coherently have haecceitist cares: that I could care about what happens to Bob but not about Bill for example. This is even more striking when it comes to caring about oneself – surely Bob needn’t be completely indifferent between what happens to Bob and what happens to Bill. The inclusion of bouletic notions means that even the neo-Carnapians, who hold that there is only one rational prior, can make sense of this account of precision, provided they accept a moderate kind of permissivism about rational desire.

Let me also point out that even if the definition of symmetry in terms of preserving a certain class of priors is extensionally adequate for characterising vagueness, it might not do the job of a good explanatory theory. For example, if the principle that one should not care intrinsically about the vague is true, it calls out for an explanation. Presumably the explanation ought to have something to do with vagueness. An abstract analysis of vagueness purely in terms of coherent credences does not provide any such explanation, yet a theory that invoked bouletic notions could provide such an explanation.

Let me end this discussion by considering the question of whether our abstract analysis in terms of symmetries can act as a reduction of vagueness to more basic notions. In this section we showed how one could start off with nothing but a certain class of priors and utilities, and from these characterise the class of precise propositions. From the notion of a conceptually coherent prior and utility we could introduce a class of symmetries that preserved values according to every prior and utility, and from this we characterised a precise proposition as something that is fixed by every symmetry. Could this be considered a reduction of the notion of precision to to the normative notion of being conceptually coherent?

There are a couple of reasons to resist this further reductive claim. A common theme in metaethics is the idea that normative facts cannot be taken as primitive as they involve truths that typically call out for explanation in more basic terms themselves. If you were sympathetic to this kind of thought you might think that an analysis of vagueness in terms of normative notions has things back to front.

A more pressing worry, in my view, is the objection that we cannot get a handle on the notion of a conceptually coherent prior without already having the concepts of vagueness and precision at our disposal. A crucial distinction that came up in our earlier discussion was the difference between being rational in toto, and merely being conceptually coherent. To adopt our earlier example from Field, it seems irrational to care whether the last digit of my bank account is the same as the last digit of the temperature of the sun in centigrade, but it is outright conceptually confused to want it to have the same last digit as the number of nanoseconds Bertrand Russell was old for.
To distinguish mere conceptual coherence from general rationality we noted that the former satisfied a richness condition: roughly, although one can have pretty much any opinion or preference about the precise matters without committing a conceptual confusion, not all such opinions are rational in the wider sense. Formally, for any probability function over precise matters there is a conceptually coherent prior over all matters that agrees with it; this may not hold when restricted rational priors. Similarly for any utility over the cells, there’s a conceptually coherent utility that agrees with it and is constant within each cell. To explain what a conceptually coherent prior or utility is to someone by appeal to these richness conditions would require them to already possess the concept of precision and vagueness.

If, like me, you think that completely reductive analyses are rare the above conclusion is hardly surprising; the value of abstract analyses is of an entirely different nature altogether. What, then, have we gained from our analysis if not a reduction? At least one important result is to widen the circle of concepts that vagueness and precision are related to. Even if you don’t have the concept of a coherent prior at your disposal, we have still succeeded in related vagueness and precision to overall rationality: we have discovered that it is simply irrational, for example, to care about the vague, even if we can’t get the converse claim without invoking notions that presuppose the concept of vagueness. Note also that such analyses deliver important structural features of the target concepts. Let me here simply quote Timothy Williamson on safety analyses of knowledge: ‘For comparison, think of David Lewis’s similarity semantics for counterfactual conditionals. Its value is not to enable one to determine whether a counterfactual is true in a given case by applying ones general understanding of similarity to various possible worlds, without reference to counterfactuals themselves. If one tried to do that, one would almost certainly give the wrong comparative weights to the various relevant respects of similarity. Nevertheless, the semantics gives valuable structural information about counterfactuals, in particular about their logic. Likewise, the point of a safety conception of knowing is not to enable one to determine whether a knowledge attribution is true in a given case by applying ones general understanding of safety, without reference to knowing itself. If one tried to do that, one would very likely get it wrong. Nevertheless, the conception gives valuable structural information about knowing.’ ([REF] ‘Probability and Danger’.) In the present case our analysis delivers important structural features of precision and determinacy: that determinacy has a certain normal modal logic, for example, or that the precise propositions form a complete Boolean algebra.

7.3 Beyond Vagueness

There are lots of different ways in which a portion of language can end up being defective or indeterminate. In the case of vagueness I have argued that the source of the indeterminacy is to be found in the kind of proposition borderline sentences express, rather than in the language itself. There are other phenomena within this broad category of ‘defective language’ that cannot be treated in a similar way. Presupposition failure and sentences containing failed demonstratives, for example, bear some faint similarity to vagueness, yet I think it is clear that the source of the defectiveness is at least partly to do with the language itself. Both the phenomena and diagnosis call for consideration of the kind of language that gives rise to it.

Other phenomena bear a less tenuous relation to vagueness. Indeterminacy that arises due to the liar paradox, in certain counterfactuals, in mathematically indeterminate statements such as, perhaps, the continuum hypothesis, in the open future (and so on) all bear a close family resemblance to vagueness. If it is conceded that these examples are indeterminate at all, a natural extension of the project I have been engaged in attempts to

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19For the liar paradox see Field [45], or in the classical setting Bacon [3], for counterfactuals see Stalnaker [118], for the continuum hypothesis see Field [50] and for the open future Belnap [11].
treat these examples within a broadly non-linguistic framework analogous to the one we have advocated for analysing vagueness.

Problematic for this project is the existence of putative examples of what I shall call ‘genuine semantic indecision’: examples of indeterminacy that seem to call out for a linguistic explanation. Let me first stress that nothing I have said so far precludes the possibility that there is genuine semantic indecision. Indeed, I suspect that some of the examples we are about to discuss are examples of genuine semantic decision. At any rate, the main point is that the primary commitment of my view is that the kind of indeterminacy arising from vagueness – the phenomena associated with the sorites paradox and related puzzles – can be explained in non-linguistic terms. If it is possible to extend this explanation to other phenomena the appropriateness will have to be determined on a case by case basis.

What I want to do here is firstly consider a few putative examples of genuine semantic indecision and see whether they can, despite appearances, be treated within the non-linguistic framework I have advocated so far. The second aim of this section is to give a theory of genuine semantic indecision – one that situates it within the theory of vagueness I have been developing without compromising it.

7.3.1 Semantic Indecision

The first putative example of semantic indecision is adapted from Brandom [17], although I shall be following Field’s presentation in [43]. Field begins by inviting us to imagine that at some point before the 16th century a community of English speakers had been separated from the rest of us and that they had then been reintegrated with us at the present day. Upon reintegration, we find out that their mathematicians had also discovered the complex numbers. We also discover that their dialect of English is much like ours, except that in introducing names for the two square roots of \(-1\) they used the symbols ‘/’ and ‘\’, instead of ‘\(i\)’ and ‘\(-i\)’. We may suppose that the two dialects of English become integrated, but the four names for the two square roots of \(-1\) remain. Now it is easy to show that \(i\) and \(-i\) are the only two square roots of \(-1\), and since / refers to a square root of \(-1\), so we can prove that either ‘/’ refers to \(i\) or ‘/’ refers to \(-i\). But which of these two possibilities obtains? The dominant view seems to be that there is no fact of the matter; the names ‘/’ and ‘\’ referentially indeterminate (symmetrical reasoning suggests a similar conclusion for ‘\(i\)’ and ‘\(-i\)’). This is because there is an automorphism of the complex numbers which maps \(i\) to \(-i\), so there is no ‘/’ free sentence the isolated community could utter that would distinguish / from \(i\) without it also distinguishing it from \(-i\).

To turn this example of referential indeterminacy into an example of a semantically undecided sentence, consider the sentence:

\[ / \text{ is positive} \]

where an imaginary number is taken to be positive if it is a positive real multiple of \(i\), and negative if it is a negative real multiple of \(i\). We assume that ‘positive’ will not be a notion the separated community will have until they are reintegrated. This sentence presumably either means that \(i\) is positive (making it true) or that \(-i\) is positive (making it false), but there is nothing settles which of these two things it might mean.

The second putative example of genuine semantic indecision arises from consideration of incomplete definitions. Here is an example taken from Fine [51]. Suppose that I stipulate that a natural number is nice* if it is less than 15 and that it’s not nice* if it is larger than 15, and that is all that I stipulate about the meaning of the word nice*. Assuming that this stipulation is good we can go on to say many contentful things about being nice*. For example, we may say that there are nice* primes, or that there are at least 10 nice* numbers. It seems that being nice* is a perfectly legitimate property, which some numbers have and others don’t. The difficulty concerns cases like the following:

15 is nice*
What is the status of this sentence? Of course, one could question whether the stipulation succeeded at all, and that our uses of ‘nice∗’ ever express a property. However it is common practice to make use of a word without there being explicit necessary and sufficient conditions for its application in all cases, and the stipulation we made seems similar enough to this phenomenon to make it hard to see how it might differ. For example, even in mathematics, we freely make use of exponentiation even though there are many functions, \( f \), which agree with our use of exponentiation but differ over the value of \( f(0,0) \). As before, this looks like a case of semantic indecision. The candidate properties ‘nice∗’ might denote appear to be the property of being at most 15 and the property of being at most 14. Both properties are determinate, and the indeterminacy is just a matter of which of the two properties the word ‘nice∗’ picks out; the indeterminacy appears to arise from semantic indecision stemming from our incomplete definition of the word ‘nice∗’.

The last putative example of semantic indecision arises in relation to questions about what to make of scientific terms after our scientific theories have undergone some kind of radical change. In [47] Field considers the example of the word ‘mass’ before and after Newtonian mechanics was replaced by special relativity. The theory of special relativity reveals that there are in fact two closely related properties, relativistic mass and proper mass, that at ordinary speeds play pretty much exactly the same role. Before the discovery of special relativity, however, it seemed as though it was indeterminate which property people were using the word ‘mass’ to refer to – as Field puts it ‘there are two physical quantities that each satisfy the normal criteria for being the denotation of the term.’ Assertions like ‘mass is conserved in all interactions’ appear to be indeterminate depending on which of the properties we take ‘mass’ to mean.

In each case the phenomena bears a family resemblance with vagueness, yet it seems obvious to many that these phenomena are to be explained by the way that language is used.

### 7.3.2 Can we get by without semantic indecision?

Let us begin by asking what would happen if we attempted to assimilate these cases to our theory of vagueness. The relevant move, in each case, would be to insist that the linguistic items in question are not semantically undecided about which of a number of determinate propositions to express, it is rather semantically decided that they express a single indeterminate proposition. In each case, then, this requires positing a further fine grained proposition over and above the relevant precise propositions. This move is not entirely ad hoc, for many of the arguments we have adduced in favour of the non-linguistic theory of vagueness extends to the apparent examples of semantic indecision. Since, for example, it appears as if nobody knows whether 15 is nice∗ or not, despite knowing that it is at most as big as 15 and knowing that it is not at most as big as 14, there needs to be a more fine grained proposition to represent our uncertainty and ignorance about this fact. Moreover, just as we argued in chapter 2, we need some explanation for this ignorance.

How might we apply our framework to each of the above examples? The first example looks like it lends itself naturally to an analysis in terms of symmetries. The fact that there is an automorphism of the complex numbers that maps \( i \) to \(-i\) is effectively a symmetry of the complex numbers – one that preserves all mathematical relations between complex numbers.

If we think of propositions as being determined by sets of indices, and that the indeterminacy described above is a source of fine-grainedness then the picture will be as follows. All the indices will agree that there are two square roots of \(-1\), although according to some \( i \) and \( / \) will be identified with the same root (and thus \(-i\) and \( \backslash \) will be identified with the other root), and according to others \( i \) and \( \backslash \) will be identified with the same root (and \(-i\) and \( / \) to the other).

Given this picture you can define an automorphism of propositions that is determined
by the mapping that maps an index in which $i$ and $\setminus$ are identified to the index where $i$ and $\setminus$ are identified but otherwise agrees about the other facts. We can think of these kinds of automorphisms either as switching $/$ for $\setminus$ while keeping $i$ and $-i$ fixed, or as switching $i$ and $-i$ while keeping $/$ and $\setminus$ fixed. Either way, there is an extremely natural analogy with the automorphism of complex numbers defined earlier, and like that automorphism, all mathematical relations will be preserved by this permutation. Although all mathematical relations are preserved, non-mathematical relations need not be – for example being positive, being identical to $i$ and similar properties are not preserved. Nonetheless, it’s natural to think that these automorphisms preserve the things you can reasonably care about, and preserves rational credences: there is something incoherent about caring about things like whether $/$ and $i$ are the same or not, and it also seems that it would me irrational to me more confident that $i$ is $/$ than that it is $\setminus$. It is therefore not too far fetched to think that this permutation determines a rational symmetry of the kind defined in section [REF].

Thus anything of importance to communication, decision making and so on is preserved if we uniformly replace the concept of $i$ with $-i$ in everything we think and say.

In our second example we stipulated that a number was nice$^*$ if it is less than 15 and not nice$^*$ if it is greater. To see how this can modelled using symmetries, let us suppose that these stipulations allowed us to refer to an indeterminate property – the property of being nice$^*$ – which is neither identical to the property of being less than 15 nor the property of being greater than 15. If I can do this then it seems that I could now introduce, via exactly the same method, a name for another vague property, being nice$\_i$, as follows:

A number is nice$\_i$ if it is either less than 15, or equal to 15 and not nice$^*$. Notice that a number is nice$\_i$ if it is less than 15, it is not nice$\_i$ if it is greater than 15 and 15 is nice$\_i$ if and only if 15 is not nice$^*$. There is a symmetry in these definitions which isn’t immediately apparent. For if we already had the notion of being nice$\_i$, we could define nice$^*$ as follows:

A number is nice$^*$ iff it is either less than 15, or equal to 15 and not nice$\_i$.

Now we can easily see that the proposition that 15 is nice$^*$ is distinct from the proposition that 15 is nice$\_i$: the truth of one proposition implies the falsity of the other and at least one of them must be true, so they cannot be identical. Similarly neither proposition is identical to the proposition that 15 is less than or equal 15, or the proposition that 15 is less than or equal to 14, since we presumably don’t know whether 15 is nice$^*$ or nice$\_i$, but we surely know that 15 is less than or equal to 15 but not less than or equal to 14. Nevertheless, that each proposition implies the negation of the other seems to be the only important difference between them. Anything else that we might think or say to distinguish them would involve the new propositions in some way or another. The proposition that 15 is nice$^*$ and the proposition that 15 is nice$\_i$ can’t be distinguished in terms of the role they play in our mental lives. They bear exactly the same evidential relations to other propositions, and cannot be treated asymmetrically by a rational agent within her system of beliefs and preferences.

In this case as well it looks as though there will be a natural way to model this situation in terms of symmetries. Indeed it is not hard at all to model the relevant space of propositions and equip it with an automorphism that switches the proposition that 15 is nice$^*$ with the proposition that 15 is nice$\_i$, but leaves logically independent propositions alone.

Applying our definitions of precision and vagueness we get that neither the proposition that 15 is nice$^*$ or the proposition that $/ is positive are precise. Both of these can be mapped, via a symmetry, to a distinct proposition and so are not fixed by all symmetries. Moreover both propositions are indeterminate. The claim that it’s determinate that $p$ is just the conjunction of proposition $p$ is mapped to under symmetries. Thus the claim that it’s determinate that 15 is nice$^*$ is just the conjunctive proposition that 15 is both nice$^*$ and nice$\_i$, which is always false. Analogously, the claim that it’s determinate that $i = /$ ends up entailing both $i = /$ and $i = \setminus$ and so must be false since $/ \neq \setminus$. 

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At any rate, insofar as our formal framework goes, the first two examples of semantic indecision lend themselves quite naturally to our model theory that is spelt out in terms of symmetries. Unfortunately I think that there are philosophical reasons to be sceptical of this account (although I have by no means made up my mind on this point).

The first problem is that it seems that examples analogous to the above examples suggest that there are actually a large number of indeterminate propositions mapped by symmetries to the proposition that 15 is nice$^*$ or that $\sqrt{1}$ is positive. For instance, suppose that instead of two splintered mathematical communities there were 20 who each introduce their own names for the roots of $-1$. Are we to think that once the 20 communities reintegrate there will be $2^{20}$ propositions expressed by the various combinations of identity statements between the newly introduced names? If so how is it that these propositions came about?

Note that this situation is unlike the evidential roles that we took to be sufficient for the postulation of a vague proposition in chapter 4 – these were determined by the kinds of effects inexact evidence can have on our credences in the precise, and so our reasons for postulating vague propositions can’t simply be multiplied in the same way.

Anyway, the crucial question for this picture is to explain how propositions can be multiplied so easily. One could say that the propositions only existed because of the linguistic practices of the 20 different communities, and had there only been two communities there would have only been 2 relevant propositions. On this story the existence of propositions is highly dependent on the contingent practices of linguistic communities. An account of propositions along these lines, however, would not be friendly to the non-linguistic theory of propositions I have been endorsing here – propositions would be language dependent in a way that strips the theory of its primary advantages.

A better thing to say, then, is that the propositions would have existed whether or not the communities had introduced names for the roots of $-1$ in the way they did. But then, presumably, there will be much more than $2^{20}$ propositions – there will be at least as many propositions as there could be possible mathematical communities that diverge and reconverge in the way discussed. This will presumably be some large infinite cardinal, or perhaps something too large to be assigned a set theoretic cardinality. Although messy, I do not take this view to be completely hopeless. There is the charge of arbitrariness to contend with: whatever cardinality we do assign to the number of propositions corresponding under symmetry to the proposition that $\sqrt{1}$ is positive, we can always ask why that cardinality and not another. However the problem of arbitrariness with respect to cardinality questions is, I think, something we must make our peace with in other areas.\footnote{See, for example, Sider [REF], Hawthorne and Uzquiano [63].}

At any rate, while a non-linguistic account of these examples is not hopeless, both the examples discussed above find a natural analysis in terms of genuine semantic indecision which I shall sketch later. What of examples involving scientific terms? In broad outline a non-linguistic account of this example would posit, in addition to the properties of proper mass and relativistic mass, a third vague property, Newtonian mass. The value of an object’s Newtonian mass is always either its proper mass or its relativistic mass, but when these differ it is always indeterminate which.

A couple of other features of Newtonian mass seem reasonable. Firstly, it is natural to think that there is a penumbral connection between the Newtonian mass of any two objects: determinately, if the Newtonian mass of the first object is its proper mass then the Newtonian mass of the second is its proper mass, and similarly for relativistic masses. Secondly, it is also natural to think, given that Newtonian mass turned out not to be a fundamental quantity, that the values of objects Newtonian masses supervene on the values of the more fundamental quantities of proper and relativistic mass: either it’s necessary that the Newtonian mass of every object is identical to its proper mass or it’s necessary that the Newtonian mass of every object is its relativistic mass. Although Newtonian mass is necessarily coextensive with another more fundamental quantity, it is indeterminate which of these quantities it is coextensive with, and there is therefore ignorance about
which of these quantities Newtonian mass coincides with. Thus Newtonian mass might have a distinctive conceptual role which neither proper mass nor relativistic mass has: perhaps someone who was pretty, if not fully, confident that Newtonian physics is the correct description of reality, and therefore also pretty confident that Newtonian, proper and relativistic mass are coextensive, might nonetheless treat the first differently from either of other two conditional on the supposition that the theory of relativity is true.

At any rate, on this way of understanding the example, it was determinate that Newton was neither giving a theory of proper mass nor of relativistic mass – he was giving a theory of Newtonian mass. As we later discovered, some of the consequences of his theory are determinately false: they neither hold of proper nor relativistic mass and thus do not hold of Newtonian mass. Other consequences are indeterminate: they would have been true, or approximately true, if he had been talking about proper mass and not relativistic mass, or vice versa. When we eventually did discover the nature of relativity, our use of the word ‘mass’ changed. It no longer referred to Newtonian mass, and we introduced disambiguations of the word ‘mass’ to refer to the two properties which we believe did exist.

The picture described seems at least coherent, but is it plausible? One source of implausibility is the idea that physicists frequently find themselves theorising about vague properties, instead of the perfectly natural and fundamental properties we typically take them to be talking about. Presumably this issue isn’t confined just to previous physical theories we know to be false – it also applies to our present physical theories which, for all we know, are false. Presumably the most we know about our present theories is that they are approximately true, but this allows for the possibility that our present physical concepts refer to indeterminate concepts much like the notion of ‘mass’ did amongst physicists before the theory of relativity. Note, however, that if physicists working with false theories are theorizing about properties at all, it is independently natural to think that they are not theorizing about fundamental properties. Note also that we have already cast doubt on the hypothesis that physicists usually find themselves theorizing about precise properties. In section ??, for example, I argued that even physical properties like being an electron are vague.

The story here is also subject to the multiplicity worry raised in connection with the first two examples. To prevent the number of vague properties depending on the number of incorrect physical theories proposed throughout history we would have to posit an abundance of vague properties to account for all of the possible incorrect physical concepts that could introduced by false physical theories. Perhaps in this case there is more hope of delineating a fixed class of conceptual roles, and arguing that there is a property for each of those conceptual roles, thus answering the arbitrariness worry. I do not know, however the mechanism by which we posit such properties seems different enough from the case of vagueness to warrant treating this example in a different way.

7.3.3 An Account of Semantic Indecision

Each of the examples mentioned look as though they ought to be amenable to an analysis in terms of semantic indecision. Semantic indecision, unlike my account of vagueness, is something that only makes sense when it is attributed to a linguistic item, such as a sentence or a predicate. But what exactly is semantic indecision? According to McGee and McLaughlin, prominent defenders of the semantic indecision account of vagueness, it is semantically decided that an object falls under a predicate when ‘the thoughts, experiences, and practices of the speakers of the a language determine the conditions of application if its predicates, and a predicate definitely applies to an object just in case the facts about the object determine that these conditions are met’.

Let us consider how we might apply this to the word ‘bald’. According to the proposal we are to suppose that there is no property – or ‘condition’ in McGee and McLaughlin’s lingo – which is determined by the linguistic practices of the speakers to be the meaning of
‘bald’. Despite this, there might well be several properties that are candidate meanings for the word "bald": a property which is not determined to be not meant by the word ‘bald’ by the linguistic practices of English speakers. With this in place, McGee and McLaughlin go on to develop a broadly supervaluationist account of semantic indecision.

But what exactly is the notion of ‘determining’ that is being invoked here? Perhaps it can be spelt out in terms of metaphysical necessity by some kind of supervenience thesis. $P$ is determined by present linguistic practices to be the meaning of the word ‘bald’ if and only if it’s necessary that when the linguistic practices of English are as they in fact are, the word ‘bald’ means $P$. On the other hand, if no meaning for ‘bald’ is determined by linguistic practices, $P$ is a candidate meaning iff it’s metaphysically possible that linguistic practices be as they in fact are and for ‘bald’ to mean $P$.

This account of ‘determining’ will not do, for McGee and McLaughlin want their theory to be consistent with the supervenience of meaning on use, yet such a supervenience thesis would ensure that the meaning of ‘bald’ was always determined by the usage facts. Moreover, even if meaning does not supervene purely on use – perhaps the existence of reference magnetism, or something similar, prevents this – it would not be in the spirit of their view to think that the precise meanings of vague predicates are determined once you add these extra facts to the supervenience base. Presumably meaning ought to supervene on the totality of physical facts, for example, but in the relevant sense of ‘determine’ the extension of the ‘bald’ is surely not determined by the totality of physical facts for otherwise its extension would be determined by completely precise matters.

Perhaps the notion of determining at hand is an epistemic one. Although the linguistic usage facts necessarily imply that the word ‘bald’ means what it means, and thus ‘determines’ in this same sense what its cut-off point is at each world, neither the cut-off nor the necessary implication between the usage facts and meaning facts are knowable to us. The sense in which the meaning of ‘bald’ is undetermined thus might be an epistemic one: there are several properties that, for all we know, are the meanings of the word ‘bald’. We don’t know which because, although the meaning of ‘bald’ supervenes on its use, we do not know how the meaning of ‘bald’ supervenes on its use – thus we are just as ignorant about what ‘bald’ means as we are about how this meaning gets fixed by usage.

This is, of course, just a version of the epistemic theory of vagueness – a theory that McGee and McLaughlin, and other sympathetic writers, take care to distinguish from semantic indecision.\(^{21}\) If we are to explicate the notion of semantic indecision in terms of how linguistic practices fail to determine meanings of vague words, we need a non-epistemic account of this ‘determining’ relation. Moreover, the determining relation cannot be spelt out in terms of semantic decideness itself. For not only would this be circular, the determining relation is something that holds between two types of fact, whereas semantic decideness is a feature of linguistic items. Luckily, if we accept the theory of vagueness that I have been endorsing so far, then there is a perfectly suited non-linguistic notion of borderlineness, which is neither contingency or ignorance, that can be used to explicate the sense in which the usage facts can fail to determine the meanings of certain words: when the usage facts leave it indeterminate or borderline what a word means.

On this account, the linguistic facts determine that ‘$F$’ means $P$ when it is necessarily determinate that if the linguistic facts are as they in fact are, ‘$F$’ means that $P$.\(^{22}\) The linguistic facts leave it open whether ‘$F$’ means that $P$ if it’s possibly not determinate that ‘$F$’ means something else while the linguistic facts are as they in fact are.

Assuming the supervenience of meaning on use, the ‘possibly’s and ‘necessary’s can be eliminated from these definitions. Thus we can give simple definitions of semantic definite-ness and undecidedness: say that a sentence is semantically definite iff every proposition it

\(^{21}\) Indeed, Williamson turns these kinds of observations into an objection to McGee and McLaughlin’s theory in [135].

\(^{22}\) The definition in terms of ‘necessary determinacy’ rather than ‘determinate necessity’ seems to be the more natural to me; although given their potential distinctness it is worth noting that variant accounts of ‘determining’ could be formulated in terms of ‘determinate necessity’ or both.
doesn’t determinately not express is true, and semantically undecided if neither it nor it’s negation is semantically definite. Given fairly uncontroversial assumptions, a sentence is semantically undecided if there’s both a true and a false proposition it doesn’t determinately not express.23

Two things about this account are worth pointing out at this juncture. Firstly, this account of semantic indecision clearly cannot be accepted by someone who does not already accept the ideology of a non-linguistic determinacy operator. On this account the borderlineness operator must be taken to be more basic than semantic indecision, and so the latter cannot be used to explicate the former, pace McGee and McLaughlin. Secondly, I do not claim to be analysing some pretheoretic notion of ‘semantic indecision’ that McGee and McLaughlin, or anyone else has in mind. It suffices for my purposes to note that if we have a non-linguistic borderlineness operator, then the above definitions are perfectly sound. Thus anyone accepting a borderlineness operator can make sense of the concepts – I am simply calling the concept ‘semantic indecision’ due to its resemblance to the notion McGee and McLaughlin are trying to capture.24

Of course, it is controversial whether the examples I listed in the last subsection are semantically undecided according to this account of semantic indecision. Nonetheless, semantic indecision so defined undoubtedly exists – semantic properties and relations, like most non-fundamental properties and relations, are vague. In particular the linguistic meaning relation is vague and has borderline cases. To see this note that words clearly change their meanings over time. Consider, for example, a sorites in which a persons use of ‘water’ switches from referring to H2O to another water-like substance, XYZ: perhaps I have moved to twin earth and my uses of the word ‘water’ are slowly changing their meaning to refer to the local colourless, odorless watery stuff. I start off clearly referring to water, and end up clearly referring to XYZ, but presumably there will be some point inbetween where it’s borderline whether my use of ‘water’ refers to water or XYZ. It will therefore be borderline which proposition sentences involving the word ‘water’ express. Sentences involving the word ‘water’ can be semantically undecided according to my view.

It is important to distinguish semantic indecision from linguistic borderlineness. As I argued in chapter 2, even if one accepts a non-linguistic account of vagueness there is quite clearly a distinction between sentences to be captured, even if it is not taken to be basic: the difference between the sentence ‘Harry is bald’ and ‘Ghandi is bald’, for example. The former is linguistically borderline, I argued, because it expresses a borderline proposition, whereas the latter is not linguistically borderline, because it expressed a determinately false proposition.

Note however that a sentence can be linguistically borderline without being semantically undecided. Suppose that there is a proposition, \( P \), which the sentence ‘Harry is bald’ determinately expresses. Presumably \( P \) would be the proposition that Harry is bald, and since this is borderline it follows that the sentence ‘Harry is bald’ is linguistically borderline. However it is not semantically undecided: if Harry is bald then every proposition the sentence ‘Harry is bald’ doesn’t determinately not express is true – there’s only one such proposition and that’s the proposition that Harry is bald. If Harry is not bald then by analogous reasoning there’s a proposition the sentence ‘Harry is bald’ determinately expresses and it’s false: thus ‘Harry is bald’ is either semantically definite, or its negation is semantically definite (although it is borderline which).

Conversely, a sentence can be semantically undecided without being linguistically borderline. For example, suppose that it is determinate that XYZ does not contain hydrogen.

23The latter way of expressing semantic undecidedness is perhaps preferable because it can be applied to languages that do not contain a negation operator.
24Michael Caie [21], for example, has recently argued for a similar account of semantic indecision in terms of a non-linguistic notion of indeterminacy. However unlike me, he takes this to be what vagueness consists in. Although Caie is explicit about his commitment to a non-linguistic notion of indeterminacy, many supervaluationists implicitly seem to commit themselves to this kind of view by talking about vagueness as though there were many candidate meanings which a vague word indeterminately refers to.
as a chemical component. H2O, on the other hand, determinately does contain hydrogen as a chemical component. Now even if it is borderline whether ‘water’ refers to H2O or XYZ, the sentence ‘water contains hydrogen’ is not linguistically borderline. In fact, it’s determinate that ‘water contains hydrogen’ expresses a non-borderline proposition, because it either expresses the determinately true proposition that H2O contains hydrogen or the determinately false proposition that XYZ contains hydrogen. However ‘water contains hydrogen’ is semantically undecided because there’s a true and a false proposition it doesn’t determinately not express.

The concept I have introduced under the name ‘semantic indecision’, I think, does a good job of accounting for the examples introduced in this section. In subsection [REF] I considered the suggestion that sentences involving the words ‘nice∗’, ‘/’ and ‘mass’ often expressed vague propositions, and that this could be used to explain the apparent indeterminacy of these examples. In other words I was suggesting that sentences like ‘15 is nice∗’ are linguistically borderline: they express borderline propositions. However, there are some puzzles for this view and I think a much more natural way to model the above examples would be in terms of semantic indecision.

Let us focus on the example involving incomplete definitions. According to this analysis, the word ‘nice∗’ expresses either the property of being at most 14 or the property of being at most 15, although it is borderline which. Thus the sentence ‘15 is nice∗’ is semantically undecided: it’s doesn’t determinately fail to express the true proposition that 15 is at most 15, and it doesn’t determinately fail to express the false proposition that 15 is at most 14. Note also that neither of the candidate propositions for ‘15 is nice∗’ are borderline, thus even though this sentence semantically undecided, it is not linguistically borderline. Presumably on this view there is also a penumbral connection between what ‘nice∗’ refers to and what ‘nice∗’ refers to, namely, the former refers to whichever property the latter doesn’t.

It is independently plausible that it is borderline which property the word ‘nice∗’ picks out. If there was a sorites sequence starting with ways of introducing the word ‘nice∗’ so that it refers to the property of being at most 15 and ending with ways of introducing the word ‘nice∗’ so that it ends up referring to the property of being at most 14, you might expect the incomplete definition of ‘nice∗’ given in section [REF] to be one of the cases in the middle of the sorites sequences – one of the cases where it’s borderline which of the two properties it refers to.

Presumably in this case I can still talk about the property of being nice∗, but when I do I end up referring either to the property of being at most 15 or the property of being at most 14. That is to say, the expression ‘the property of being nice∗’ inherits semantic indecision from the subexpression ‘nice∗’. The crucial thing about this view is that, although there is the property of being nice∗, it is a precise property for it is either the property of being at most 15 and the property of being at most 14. Both of these properties are precise – the property of being nice∗ isn’t a third vague property distinct from either of these two properties. This is the crucial difference between the semantic indecision account and the account in which ‘15 is nice∗’ is linguistically borderline. There is therefore no problem of property multiplication to be levelled against this account in the way that there was according to the latter account.

A similar story can be told about the other two examples. According to the semantic indecision account, the prerelativistic uses of ‘mass’ refers either to proper mass or relativistic mass. However, the practices of the people using the word ‘mass’ leave it borderline whether they meant proper mass by it, or relativistic mass for they never really had to apply the word in the cases where it came apart. Again, one can justify the ascription of borderline in meaning by considering a sorites starting off with a community of people who are disposed, after learning about relativity, to apply the word ‘mass’ to the proper mass of objects, and ending with a community disposed to apply the word ‘mass’ to relativistic mass upon learning about relativity. One would assume that the way people actually were
disposed to apply the word ‘mass’, once the special theory of relativity was discovered, is one where it’s pretty much borderline whether we would want to apply the word ‘mass’ to proper mass or relativistic mass. On this account, then, we do not need to posit this strange third quantity, ‘Newtonian mass’. It was simply borderline whether ‘mass’ as it was used then referred to proper mass or relativistic mass.

As for the example involving the two mathematical communities, the semantic indecision account would say that the sentence ‘\(i = /\)’ either expresses a necessarily false proposition saying of one of the square roots of \(-1\) that it is identical to the other, or a necessarily true proposition saying of one of square roots that it is identical to itself. That is to say, letting \(x\) and \(y\) be the square roots of \(-1\), that the candidate propositions would be the singular propositions that \(x = x\), that \(y = y\), that \(x = y\) and that \(y = x\). There are thus between two and four candidate propositions depending on how finely we individuate, and they are all perfectly precise. Again, we can contrast this with the earlier view in which ‘\(i = /\)’ did not express a singular proposition at all. On that view the contribution of ‘\(i\)’ and ‘\(/\)’ was roughly the same as a vague name such as ‘Everest’ to the sentence ‘Everest is 11,000 ft’ (see chapter 2.) The proposition that ‘\(i = /\)’ expressed was a borderline proposition due to names like ‘\(i\)’ and ‘\(/\)’ introducing vague individual concepts to the proposition.

The account of semantic indecision I have outlined here assumes that it is sometimes borderline which proposition I have expressed with a particular sentence. There is, however, a contrary argument that it’s never borderline what a sentence expresses which I therefore need to address. Taking the example of the sentence ‘15 is nice∗’ the argument starts with a disquotational premise:

1. It’s determinate that ‘15 is nice∗’ expresses the proposition that 15 is nice∗ and nothing else.

2. Therefore it’s not borderline what ‘15 is nice∗’ expresses.

The argument is indeed valid, and the conclusion excludes semantic indecision for the sentence ‘15 is nice∗’.

A little thought, however, demonstrates that this argument overgenerates quite radically. For if the above argument is fine, then one can argue quite generally that, given that ‘\(P\)’ determinately expresses the proposition that \(P\) and nothing else, it is not borderline what ‘\(P\)’ expresses. But this conclusion is absurd for arbitrary sentences for it shows that expressing relation has no borderline cases. The relation of semantic expressing is clearly as vague as any other non-fundamental relation, and therefore is just as prone to having borderline cases. It is a matter of routine to describe a sorites in which an expression begins with a particular meaning but over time changes its meaning as people begin to use it in incrementally different ways. As with any sorites we should conclude that there will be borderline cases in the middle of this sequence.

The solution is to reject the determinacy of the disquotational principle we began with. The idea, in the present case, is that it is borderline whether ‘15 is nice∗’ expresses the proposition that 15 is nice∗ or the proposition that 15 is nice∗, but determinate that it expresses one of them. What, then, is to be made of our intuitions in favour of the determinacy of the disquotational principle? What is it that seems to be special about disquotational sentences like “15 is nice∗” expresses the proposition that 15 is nice∗”?

In chapter 2 I cast some doubt on the determinacy of the disquotational principles in our discussion of Williamson’s metalinguistic safety account of vagueness. However there is a status that they enjoy which could easily be taken to explain the intuitions we have in favour of them: they are determinately true. This point requires distinguishing sharply between the following two principles:

It’s determinate that ‘15 is nice∗’ expresses the proposition that 15 is nice∗

This is not entirely uncontroversial. John Earman, for example, argues that it’s not actually borderline how we use the word ‘mass’.

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25This is not entirely uncontroversial. John Earman, for example, argues that it’s not actually borderline how we use the word ‘mass’.
The sentence “15 is nice∗” expresses the proposition that 15 is nice∗ is determinately true.

The former principle I am denying. The latter however is true, which can be reasoned out from the following principles.

1. It’s determinate that either ‘15 is nice∗’ expresses the proposition that 15 is nice∗ or the proposition that 15 is nice∗.

2. It’s determinate that the word ‘expresses’ expresses the expressing relation.

3. The semantic principle of compositionality is determinate.

The first premise presents us with two cases to consider. Suppose that ‘15 is nice∗’ expresses the proposition that 15 is nice∗. Then the sentence “15 is nice∗” expresses the proposition that 15 is nice∗ by the second premise and compositionality (I am using square brackets here simply to indicate scope). The proposition expressed is true by hypothesis. For the second, non-disquotational, case suppose that ‘15 is nice∗’ expresses the proposition that 15 is nice∗. This time the sentence “15 is nice∗” expresses the proposition that 15 is nice∗ expresses the proposition that ‘15 is nice∗’ expresses the proposition that 15 is nice∗] by the second premise and compositionality. Again, by the description of the case, this proposition is true. So either way the proposition ‘15 is nice∗’ expresses is true. Since each of the premises of this argument are determinate we can determinize our conclusion: thus ‘15 is nice∗’ is determinately true.26

What is interesting about this case is that the two propositions that the disquotational principle is associated with are both borderline. As I argued earlier, the proposition that ‘15 is nice∗’ expresses the proposition that 15 is nice∗ is borderline, as is the proposition that ‘15 is nice∗’ expresses the proposition that 15 is nice∗]. Nonetheless, the indices at which the disquotational principle expresses the former proposition happen to be indices at which that proposition is true, and similarly for the indices at which it expresses the latter proposition. There is a penumbral connection between what the sentence expresses and the truth value of the proposition it expresses. This is why it is possible for it to be determinate that the disquotational principle expresses some true proposition, even though everything it doesn’t determinately not express is borderline.

This situation might seem pathological, but can be constructed easily in other cases. Let’s suppose I stipulatively introduce a name, ‘Fred’, for the tallest short person.27 There is plausibly a penumbral connection between the individual ‘Fred’ refers to and the cut-off point for being short. The sentence ‘Fred is short’ is thus surely determinately true, since we know what whoever ‘Fred’ refers to, they’re going to be a short person (the tallest one, in fact). But all of these candidate referents of ‘Fred’ are people who are borderline short, thus we can also be sure that whoever ‘Fred’ refers to, the proposition ‘Fred is short’ expresses is borderline.

26This argument superficially used reasoning by cases, but it is easy to reformulated so that it can be proved just using the logic KT.

27I am treating ‘Fred’ here as a descriptive name, much like well-known example of ‘Julius’ for the inventor of the zip.
Chapter 8

Vagueness At Every Order

It is an upshot of classical logic that if there are any small numbers at all then there is a last small number. It is compatible with this result that it is a vague matter which number that is. The boundary between the small and non-small isn’t precise. There is a boundary, but it’s *vague* where it lies, and it is the existence of precise boundaries, not vague ones, that we should be worried about.

This is, in a very schematic form, the classical response to the Sorites paradox. To illustrate why precise boundaries (but not vague ones) are problematic consider the following example. There is something very bad about asserting that the total length of your childhood was 378432178928476829 nanoseconds. Vagueness prevents you from ever discovering this, and similar precise facts about the length of your childhood. If the boundary between your childhood and the rest of your life was not vague, however, there would have been no reason you couldn’t have discovered the length of your childhood in nanoseconds, just as, perhaps, one could find out the number of nanoseconds in a year, and no reason to refrain from going about asserting it. Everyone, regardless of her preferred account of vagueness, must agree that the above assertion is bad and that this badness is due to its being at best vague whether your childhood lasted for this length of time.

This reasoning extends. Is there a precise boundary between the determinate children, in other words, the children that are not borderline children, and everyone else? It seems we should not be any happier about assigning sharp numbers to the length of one’s determinate childhood than to one’s childhood. There is a completely analogous Sorites for ‘determinate child’ as there is for ‘child’. To be sure, there is a last child and a last determinate child in any Sorites sequence, but it is always vague which person that last child or determinate child is. Similar comments apply to the further iterations: it’s vague which the last determinately determinate child is in the sequence, and so on and so forth through the finite orders.

Indeed, there are some people who are determinately* children for any amount of iterations, *n*, and some which are not (I shall write ‘determinate*’ as shorthand for *n* successive ‘determinately’s.) Surely it is vague where that boundary lies as well? In other words, there are some children such that it’s neither vague nor higher order vague (vaguely vague or vaguely vaguely vague or ...), whether they’re children, and others such that it is either vague or higher order vague whether they are children, and it’s borderline where the boundary between the two lies. To see this note that:

> The period of my childhood during which it was neither vague nor higher
> order vague whether I was a child was 378432178928476829 nanoseconds
> in length.

(8.1)

sounds just as terrible, and would sound terrible no matter what number one used. However, if this sentence was not borderline and was furthermore true, what possible reason could

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1. Although note that you want these claims to assuage the initial intuition that there can’t be a last small number, know that you let the epistemicist off the hook too.

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prevent us from finding out whether or not (1) was true? If we knew (1) it would be hard
to explain the inappropriateness of asserting (1).\(^2\)

It should be noted that the only resources one needs to establish the existence of higher
order vagueness is (i) the expression ‘it’s borderline whether’ (a linguistic borderline
ness predicate would do just as well), giving one the means to express when someone is a child
without it being borderline whether they’re a child (that is, when someone is a determinate
child, in my nomenclature), (ii) the acknowledgement that a Sorites sequence for ‘\(x\) is a
child’ is typically also a Sorites sequence for ‘\(x\) is a determinate child and (iii) that any
Sorites susceptible predicate has vague instances. Since all these considerations can be
made independently of ones preferred analysis of vagueness this is a problem for everybody –
epistemicism, for instance, offers no respite here.\(^3\)

Some philosophers deny the phenomenon of even second order vagueness (see, for in-
stance, [138] and [101].) According to these theorists, the duration of your determinate
childhood is a precise length. However these philosophers do not typically offer concrete
hypotheses about the exact number of seconds at which it begins to be borderline whether
you’re a child (and at which you stop being a determinate child.) If this is due to some kind
of inability on their part, some explanation of this inability is required. The explanation
cannot be that the boundary is vague, because by assumption the distinction between being
determinately bald and being vaguely bald is a precise one. One might reasonably wonder
what this new kind of obstacle to knowledge is if it is not vagueness. Equally puzzling issues
arise for those who think higher order vagueness cuts out at some finite level larger than 2
(see Burgess [20].)

The subject of this chapter concerns a number of arguments that purport to show
that for any predicate, \(F\), there must be a precise boundary between the things which
are determinately \(F\) at every order (henceforth “determinately* \(F\)”) and the rest. That
being vaguely \(F\) at some order or other and not being vague at any are precise distinctions.
If this argument succeeds we should expect to be seeing exact numbers associated with
vague predicates all over the place. Indeed numbers that are in principle discoverable;
thus one should not be surprised to hear things like ‘my determinate* childhood lasted
exactly 378432178928476829 nanoseconds’ or ‘I became determinately* bald after I lost my
1451st hair’ and so on. It is tempting to sweep the infinitary version of the Sorites paradox
under the carpet - to say that predicates of the form “determinately*-\(F\)” are indeed precise
but they’re so esoteric we shouldn’t worry about them. I think that recognising that this
response involves the possibility of finding out propositions like (1) is a cost many would
not be willing to pay.

Let me introduce some notation. As usual I shall write \(\nabla p\) to mean it’s borderline
whether \(p\), \(\Delta p\) to mean that \(p\) and it’s not borderline whether \(p\), and \(\Delta^\ast p\) to mean \(p\)
and it’s neither vague nor higher order vague whether \(p\), which is to say \(p\), it’s not borderline
whether \(p\), it’s not borderline whether it’s borderline whether \(p\), and so on. The problems
considered here can be generated without iterating into the transfinite ordinals, so by ‘higher
order vague’ I shall just mean nth-order vague for some finite order \(n\). I shall at various
points appeal to some fairly weak principles governing the behaviour of the determinacy
operator: factivity (if it’s determinate that \(p\) then \(p\)), closure (if both a material conditional
and its antecedent are determinate so is its consequent) and a principle of necessitation that
allows us to infer that any classical consequence of factivity, closure and necessitation is
determinate. When these principles are assumed one can show that \(\Delta^\ast p\) is equivalent to
the infinite conjunction of \(p\), \(\Delta p\), \(\Delta \Delta p\), and so on. (As usual, one can also show, assuming
our definition of \(\Delta\) in terms of \(\nabla\), that \(\nabla p\) is equivalent to \(\neg \Delta p \land \neg \Delta \neg p\).)

The structure of the chapter is as follows. In §1 I show that two frequently cited informal

\(^2\)Although see [31] for a possible explanation.

\(^3\)Eklund [36], for example, claims that his particular analysis of vagueness fairs better with respect to
the problem of higher order vagueness. However, since he can make sense of the property of being bald
without being vaguely bald, and he believes that Sorites susceptibility involves vagueness it is hard to see
how this claim stands up to this version of the problem.

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presentations of the paradoxes higher order vagueness, due to Sainsbury and Graff Fara, are not valid. In sections 2 and 3 I present two valid paradoxes of higher order vagueness, the first is novel but is based on some observations due to Timothy Williamson, the latter due to Graff Fara. In each case I identify the problematic premise, and I provide a natural class of models that demonstrate that there is no incoherence involved in the notion of vagueness at every order.

8.1 The problem of higher order vagueness

Many authors have thought that precise cut-off points re-emerge once considerations involving higher order vagueness are taken into account. The following passage by Mark Sainsbury is often cited in favour of this conclusion:

Suppose we have a finished account of a [vague] predicate, associating it with some possibly infinite number of boundaries, and some possibly infinite number of sets. Given the aims of the description, we must be able to organize the sets in the following threefold way: one of them is the set supposedly corresponding to the things of which the predicate is absolutely definitely and unimpugnably true, the things to which the predicate’s application is untainted by the shadow of vagueness; one of them is the set supposedly corresponding to the things of which the predicate is absolutely definitely and unimpugnably false, the things to which the predicate’s non-application is untainted by the shadow of vagueness; the union of the remaining sets would supposedly correspond to one or another kind of borderline case. So the old problem re-emerges: no sharp cut-off to the shadow of vagueness is marked in our linguistic practice, so to attribute it to the predicate is to misdescribe it. [108]

Raffman, for example, describes this reasoning as ‘decisive’ ([101]). However, as we have seen, its conclusion is paradoxical. If the distinction between the people who are ‘absolutely definitely and unimpugnably’ bald is a precise one, we ought (at least in principle) to be able to find out and say where it does and doesn’t apply much like we can in principle find out if someone has less than 1000 hairs, or we ought to be able to give an explanation for why we cannot find this out that does not appeal to vagueness. This much is characteristic of precise distinctions.

Much turns on whether you accept classical logic or a non-classical logic. Sainsbury’s conclusion that a given object either falls under a predicates realm of application, absolutely definitely and unimpugnably, or it fails to do so in some way, is equivalent to an instance of the principle of excluded middle. Perhaps there is some reason why this particular instance of excluded middle must be true, but Sainsbury’s argument has done nothing to establish that.

For the classical logician, on the other hand, Sainsbury’s conclusion has no bite; what distinguishes a precise from a vague predicate is not whether it obeys the principle of excluded middle. The question then remains: could it be vague whether a sentence is absolutely definitely and unimpugnably true, without a shadow of vagueness? Let us grant, for the time being, this talk of claims being ‘absolutely definitely and unimpugnably true, without a shadow of vagueness’ (for short: ‘true without a shadow of vagueness’). Let us also grant that no claim that is true without a shadow of vagueness is such that it’s vague whether it’s true without a shadow of vagueness. Let us just take it for granted that vagueness concerning whether a claim is true without a shadow of vagueness is sufficient for the claim to count as having a shadow of vagueness.

The argument could proceed as follows. Suppose it could be vague whether a sentence is true without a shadow of vagueness. It then follows that the claim isn’t true without a shadow of vagueness. If it’s vague whether a sentence is true without a shadow of vagueness,
it’s not true without a shadow of vagueness. Thus the assumption of vagueness about what is true without a shadow of vagueness apparently leads to absurdity, the argument goes, so we may conclude that it is always a precise matter whether something is absolutely definitely true. In argument form we have, for any proposition \( p \):

1. If it is vague whether \( p \) is true without a shadow of vagueness, then \( p \) is not true without a shadow of vagueness.

2. Therefore, it is not vague whether \( p \) is true without a shadow of vagueness.

Generalising this argument to all propositions it follows that for no proposition is it vague whether it is true without a shadow of vagueness. Looking at the above argument, however, one quickly realizes that this conclusion is does not follow validly from the premise; we have not reduced the claim that it could be vague whether a sentence was absolutely definitely and unimpugnably true to a logical absurdity. (Indeed, the inference from \( \neg A \rightarrow \neg A \) to \( \neg \neg A \) cannot be in general valid, or one could prove that everything is determinate.) This argument is consistent with the existence of cases where it’s vague whether \( p \) is true without a shadow of vagueness, so long as these are cases in which the sentence is not true without a shadow of vagueness.

This possibility might initially sound absurd, since it is a claim of the form ‘there are things such that it is vague whether they are \( F \) and which are furthermore not \( F \).’ Be that as it may, it is a simple consequence of classical logic and the existence of vagueness that there are many truths of this form. For example, suppose it is vague whether Harry is bald. By excluded middle either Harry is bald or he isn’t. If he isn’t bald then it’s both the case that it’s vague whether he’s bald (by hypothesis) and that he isn’t bald. Symmetrically, if Harry is bald then it’s vague whether he’s not bald (a consequence of our hypothesis) and he’s not not bald. In either case we have a property, \( F \), such that Harry vaguely instantiates it, but doesn’t in fact instantiate it (\( F \) is ‘bald’ in the first case and ‘not bald’ in the latter.)

This appearance of absurdity, if it is indeed one, is something which the classical logician must already countenance.

Of course, there are no determinate examples of things which are vaguely \( F \) without being \( F \), so there are no determinate examples of things such that it is vague whether they have a shadow of vagueness. If it’s vague whether a claim has a shadow of vagueness, it’s also second order vague (and by analogous reasoning, it’s vague at all orders.) But it is also compatible with a classical treatment of vagueness that it can be determinate that there are \( G \)’s without there being any determinate \( G \)’s; it is no objection to this approach that we cannot find any determinate examples of the phenomenon we are interested in. To conclude that every predicate is precise at higher orders requires further consideration.

8.1.1 The problem of higher order vagueness: take two

There is much in Sainsbury’s argument which is left unexplained. What do the adjectives ‘absolutely’, ‘definitely’ and ‘unimpugnably’ add? What does he mean by truth without a ‘shadow of vagueness.’ It is very natural to interpret ‘having shadow of vagueness’ simply as having vagueness at some order. In this section I shall therefore consider a more specific version of this argument, based around this interpretation, which is due to Graff Fara [41] (we treat a more rigorous argument due to Graff Fara in section [REF].)

Imagine that we are talking about the natural numbers less than 10,000 and we want to know which ones are small. Obviously, there’s no sharp boundary between small and non-small numbers. So there will be the numbers which are definitely small, the numbers which are definitely not small, and the borderline cases in between. There’s also no sharp boundary between the definitely small numbers and the rest either: there are numbers for which it’s vague whether they’re definitely small or borderline small. To put it another way, there are numbers which are definitely definitely small, and those which are definitely not definitely small, but there’s a range of borderline cases between the two in this case as
well. Similar points apply to the boundary between the definitely definitely small numbers and the numbers which are not definitely definitely small, and so on and so forth.

One can see that the set of small numbers, the definitely small numbers, the definitely definitely small numbers, and so on, gradually shrinks as the number of ‘definitely’ increases; after all, being definitely F is generally a more stringent condition than being F. But this set can’t shrink forever! The first set in this sequence clearly starts off with less than 10,000 members, so in this most generous case it can shrink at most 10,000 times before it becomes empty. For some N being definitely-N small is the same as being definitely^m small for any m \geq N.

Does this result mean there is some kind of sharp boundary between the determinately N small numbers and the rest? Fortunately this doesn’t follow from what we have said so far. We may, and indeed must, accept that there is a set of numbers which are determinately^m small for every n, and a largest such set, but we may also maintain that it’s vague which set that is. The starting point of this shrinking process - the set of small numbers - was vague. There is a boundary between the small and non-small numbers, but it’s vague where it lies. So there is no reason to think that it won’t be vague which set you end up with after the shrinking process is complete, even if for each candidate set corresponding to smallness (i.e. set each that is not determinately not coextensive with the small numbers) determines a unique end to the shrinking process, there is no reason this end point will be the same. Thus it is not at all obvious, from what we’ve said so far, that it can’t be vague which number is the last determinately N small number. The most we can conclude is that if a number is determinately N small it is determinately^k small at all orders k > N.

In conclusion, then, common presentations of the problem of higher order vagueness do not obviously pose any kind of problem for someone endorsing a classical account of vagueness. In the following sections I shall consider two further paradoxes; once the premises are made absolutely explicit it becomes clearer what must be denied in order to deny the conclusion.

8.2 Sharp Boundaries from B

The first paradox I shall consider purports to show that whether something is determinate at all orders or not is always a determinate matter. The particular paradox I present here a strengthening of Williamson’s paradox (see chapter 5 of [131]): an argument that establishes, to my mind fairly conclusively, that whatever is determinate at all orders is determinately determinate at all orders (alternatively, if it is indeterminate whether something is determinate at all orders it isn’t determinate at all orders.) The result that I have argued to be problematic, however, is the result that it can never be vague whether a claim is determinate at all orders; only this stronger claim would deliver sharp cut-off points of the kind objected to in the opening paragraphs. Williamson’s paradox does not deliver this result; in order to show the further claim one must also show that whatever is not determinate at all orders is determinately not determinate at all orders.

In order to show this one must go marginally beyond classical logic. The notion of a proposition being determinate at all orders is defined using infinite conjunctions, thus one must assume a minimal logic of infinite conjunctions. In order to represent the infinite conjunction of a countable sequences of sentences φ_0, φ_1, . . . I shall write \( \bigwedge_{i<\omega} \phi_i \).

Beyond these background logical assumptions the only remaining principles that are needed are as follows:

C1. \( \bigwedge_{i<\omega} \phi_i \rightarrow \phi_n \) for each n < \( \omega \).

C2. \( \bigwedge_{i<\omega} (\phi \rightarrow \psi_i) \rightarrow (\phi \rightarrow \bigwedge_{i<\omega} \psi_i) \).

C3. If \( \vdash \phi_i \) for each i < \( \omega \), \( \vdash \bigwedge_{i<\omega} \phi \).

In order to show this one must go marginally beyond classical logic. The notion of a proposition being determinate at all orders is defined using infinite conjunctions, thus one must assume a minimal logic of infinite conjunctions. In order to represent the infinite conjunction of a countable sequences of sentences φ_0, φ_1, . . . I shall write \( \bigwedge_{i<\omega} \phi_i \).
Nec If \( \vdash \phi \) then \( \vdash \Delta \phi \)

\[
K \Delta(\phi \rightarrow \psi) \rightarrow (\Delta \phi \rightarrow \Delta \psi)
\]

\[
B \phi \rightarrow \Delta \neg \Delta \neg \phi
\]

Call this \( \text{KB}_\infty \).

Within this system we can prove the following useful lemma: that a conjunction of determinate truths is determinate, which is formally rendered as the principle (D): \( \bigwedge_{i<\omega} \Delta \phi_i \rightarrow \Delta \bigwedge_{i<\omega} \phi_i \).

**Lemma 8.2.1.** In \( \text{KB}_\infty \) one can prove the principle (D) that states that if each member of a countable list of propositions, \( \phi_i \), is determinate, then the conjunction of all those propositions is determinate.

The proof is delegated to appendix [REF]. It is important to note that the proof makes essential use of the principle \( B \).

The principle (D) is sufficient to derive Williamson’s paradox: that if \( p \) is determinate at all orders, it’s determinate that \( p \) is determinate at all orders.

\[
(+) \Delta^* \phi \rightarrow \Delta \Delta^* \phi.
\]

The reason is quite straightforward: if \( p \) is determinate, determinately determinate, and so on, then \( p \), the proposition that \( p \) is determinate, and so on, are all determinate so by (D) it follows that their conjunction is determinate. That is, if \( p \) is determinate at all orders, the conjunction stating that \( p \) is determinate at all orders is itself determinate.

From \( (+\Delta) \) one can also prove

\[
(-) \neg \Delta^* \phi \rightarrow \Delta \neg \Delta^* \phi.
\]

This proof, again, makes essential use of the principle \( B \) (see appendix [REF].)

The fact that principles \( (+\Delta) \) and \( (-\Delta) \) are derivable in our system delivers, with a moments thought, our main result:

**Theorem 8.2.2.** \( \Delta \Delta^* \phi \vee \Delta \neg \Delta^* \phi \) is a theorem of \( \text{KB}_\infty \).\(^4\)

4

The details of the above proofs are not that important, since the take home message of this theorem is simply this: \( \text{KB}_\infty \) entails there is no vagueness concerning what is determinate at all orders, so either we accept the conclusion or some principle in \( \text{KB}_\infty \) has to go. The question is, setting aside the logical principles, which out of \( K \), \( B \) and \( \text{Nec} \) must we reject to avoid the conclusion? I will investigate the hypothesis that \( B \) is the culprit, but let us first briefly survey the alternatives.

8.2.1 Is a conjunction of determinate truths determinate?

Williamson’s original argument ([131]) was for the conclusion \( (+\Delta) \), which I called ‘Williamson’s paradox’, and was established directly with the principle (D): the principle that a conjunction of determinate truths is determinate. A natural place to block Williamson’s argument is to deny (D), and this is exactly Field’s strategy in [45]. Notice, however, that in our argument we did not appeal to (D) directly – it is derived from other principles including \( B \). Thus in order to reject (D) one must already reject the principle \( B \) or some other principle of \( \text{KB}_\infty \).

If we drop \( B \), however, one can consistently accept or reject (D) (see appendix [REF].) Thus a natural question arises for those who, like me, reject \( B \): should we drop both \( B \) and (D) or should we drop only \( B \) and keep (D)? Are there any reasons to accept or reject (D) that are independent of the \( B \) axiom.

\(^4\)It should be noted that this result could also be proved if the axiom \( B \) were weakened to the pair of rules: \( \neg \Delta \neg \Delta \phi \vdash \phi \) and \( \phi \vdash \Delta \neg \Delta \neg \phi \).
On the one hand, if we accept (D) we are committed to Williamson’s paradox (+Δ): that whatever is determinate at all orders is determinately determinate. Note, however, that +Δ does not suffice to prove theorem [REF] – that it is never borderline what is determinate*. In particular, to get from (+Δ) to (−Δ) one must use B again. Thus it is consistent with Williamson’s paradox that there is vagueness concerning what is determinate*. However, one might argue, Williamson’s paradox is puzzling in its own right, giving us reason to reconsider (D).

With that in mind: is Field’s denial of (D) at all plausible? According to linguistic conceptions of vagueness, the following principle seems incredibly attractive:

If each constituent of a sentence is precise then the sentence itself is precise.

For if a sentence is vague where could the vagueness come from if not one of the constituents?

Although perhaps not as straightforward, a variant of this principle for non-linguistic theories also seems defensible. Unfortunately, however, the principle seems to contradict Field’s suggestion. Suppose for reductio that \( \bigwedge_{i \leq \omega} \Delta \phi_i \) but that \( \neg \Delta \bigwedge_{i \leq \omega} \phi_i \). Each \( \phi_i \) is precise because we have \( \bigwedge_{i \leq \omega} \Delta \phi_i \). Furthermore, infinitary conjunction is surely precise, so it follows that \( \bigwedge_{i \leq \omega} \phi_i \) is precise by (3), i.e. \( \Delta \bigwedge_{i \leq \omega} \phi_i \) or \( \neg \Delta \bigwedge_{i \leq \omega} \phi_i \). By assumption it’s not true that \( \Delta \bigwedge_{i \leq \omega} \phi_i \) so \( \neg \bigwedge_{i \leq \omega} \phi_i \) must hold. By factivity we have \( \neg \bigwedge_{i \leq \omega} \phi_i \), but \( \bigwedge_{i \leq \omega} \Delta \phi_i \) also commits us to \( \phi_i \) for each \( i \) - this is a contradiction by C3.

I think, then, that we are indeed committed to Williamson’s paradox: if \( p \) is determinate* then it’s determinate that \( p \) is determinate*. As noted above, however, we are not committed to paradoxical claim that it is always determinate whether \( p \) is determinate* or not unless we also accept B.

8.2.2 Closure and necessitation

My purpose is to explore the prospects of a classical theory of higher order vagueness. Thus I shall retain the assumption of classical logic (by which I also mean to include the logic of infinite conjunctions.) It thus follows that the only way to avoid the conclusion is to drop necessitation, the closure principle \( K \) or Brouwer’s axiom B. I think it is fair to say that B is the least obvious of these principles. However some theorists have considered giving up necessitation (see e.g. Dorr [32]) and closure (Bobzien [14]) so it would be worth saying a bit more about this.

The discussion of (D) above generalises straightforwardly to the finitary case. Since conjunction appears to be precise a conjunction of two determinate truths should be determinate. The converse of this principle seems to be on good standing too: a conjunction involving one or more borderline propositions clearly cannot be determinately true. Thus it seems as though we have good grounds for a finitary distribution principle:

\[ \Delta(\phi \land \psi) \leftrightarrow (\Delta \phi \land \Delta \psi) \]

These conditionals were not motivated by closure but by independently plausible thoughts. However, if we assume that logical equivalents are intersubstitutable we can quickly prove closure.\(^5\)

---

\(^5\)The argument proceeds as follows:
1. \( \Delta \phi \land \Delta (\phi \rightarrow \psi) \rightarrow \Delta (\phi \land (\phi \rightarrow \psi)) \) by finitary distribution
2. \( \Delta (\phi \land \psi) \rightarrow \Delta (\phi \land \psi) \) tautology
3. \( \Delta (\phi \land (\phi \rightarrow \psi)) \rightarrow \Delta (\phi \land \psi) \) by substitution of logical equivalents in antecedent.
4. \( \Delta (\phi \land \psi) \rightarrow \Delta \phi \land \Delta \psi \) finitary distribution
5. \( \Delta \phi \land \Delta \psi \rightarrow \Delta \psi \) tautology
6. \( \Delta \phi \land \Delta (\phi \rightarrow \psi) \rightarrow \Delta \psi \) by 1, 3, 4 and 5.
Necessitation, on the other hand, has only very limited application in this proof. It can, in fact, be dispensed with altogether if each of the premises are strengthened to the claim that they are determinately determinately determinate (that is to say, no more than three successive applications of necessitation are needed in this proof.) While it is not strictly speaking incoherent to accept something while denying that it is determinately determinately determinate it is certainly an odd epistemic situation to be in, and, furthermore, all of the principles I have appealed to are of a very general kind and are surely determinately true if true at all. The prospects of avoiding the theorem by denying necessitation are therefore dim.

8.2.3 Accepting the conclusion: nihilism

A final response to theorem [REF] would be to simply accept its conclusion. In the opening paragraphs I urged against this response as it would leave open the possibility of being able to discover very precise numbers associated with vague predicates; for example it seems absurd to suggest that one could know how long one was a determinate child. Yet such knowledge might less puzzling if you knew that you were never a determinate child. One way to respond to a Sorites paradox for a predicate of the form ΔFx is simply to deny that anything satisfies ΔFx. I shall call this view ‘nihilism’ since it mimics the nihilist response to the ordinary Sorites paradox.

It should be noted that the necessitation principle for Δ, a principle we have appealed to throughout this book, already supplies counterexamples to nihilism since, with the principle C3 of infinitary conjunction introduction, we can easily show Δ*(Fx ∨ ¬Fx). Not only can we show that purely logical truths are determinate, but we can also show things like Δ*(ΔFx → Fx). Although certain logical and conceptual truths are among the determinate propositions, some conceptual truths aren’t. For example, if it is vague whether any foetus of a certain age is a person, then presumably it is is a conceptual truth one way or the other without being a determinate, and hence a determinate, truth one way or the other.

Any who accepts necessitation for Δ has to admit a distinction between certain determinate truths and others. I would be very skeptical that such a distinction would be a precise distinction. And without a motivated precise distinction between the Δ* truths and the rest, a response to the paradoxes of higher order vagueness is needed.

A more radical approach is to deny necessitation altogether. This is the strategy that Dorr endorses in [32]. Dorr’s motivation for this denial is explicitly tied to a linguistic conception of vagueness: roughly for a sentence to be definite at the actual world it must be true as it is used at worlds similar to it. For the sentence ‘S is definite’ to be definite, S must true as it is used at worlds similar to worlds similar to the actual world. However such worlds needn’t be similar to the actual world. Indeed as the number of ‘definitely’s increases we might find ourselves looking at worlds at which English is used in ever more deviant ways; perhaps even worlds where the connectives aren’t used in the same way they are actually used. Since we can connect these two types of worlds by a sorites sequence of intermediate worlds, even logical laws needn’t be definite at all orders.

This type of view, however, seems harder to motivate on a non-linguistic conception of vagueness. It is easy to imagine a state of affairs in which, say, the sentence ‘Hesperus is Hesperus’ is used in such a way that it expresses a falsehood in English. However, to run an analogous argument on a non-linguistic view would require a sorites sequence connecting the actual way things are to one in which Hesperus itself is not self-identical. This is inconsistent even in the fine grained theory of vague propositions outlined in chapter 7.

To endorse this view one also has to relinquish other nice theoretical principles, like the principle that everything supervenes on the precise discussed in chapter 3. For a non-

6At least, there isn’t any obvious precise criteria for distinguishing the two such as ‘is a tautology’ and so on.
linguistic theorist this route seems like a last resort; thus I think we have good enough reason to explore the other options.

8.2.4 Sharp boundaries from $B$  

By now, I hope, you are feeling uneasy about the principle $B$. What reasons might we have, then, to accept $B$? The best argument for $B$ that I know of is a theoretical one: there is a particularly simple and natural semantics for making claims about vagueness, due to Williamson [131], according to which $B$ is valid. Thus Williamson writes “Although these considerations in favour of $B$ are by no means decisive, we will provisionally include it as contributing to a particularly simple conception of the semantics, and call it into question again if it proves to have dubious consequences” [131] p130.

Williamson’s semantics is of a familiar kind, deriving from the semantics of modal operators, given by a set of indices, $W$, and a binary relation of ‘accessibility’ over the indices $R$. According to this semantics sentences are evaluated relative to indices and, in particular, a sentence of the form $\Delta \phi$ is evaluated true at an index $i$ just in case $\phi$ is true according to all indices accessible from $i$. This abstract semantics is consistent with a number of interpretations of the indices (they could be, for example, epistemic possibilities or world precisification pairs.) However all these interpretations, according to Williamson, share a common conception of accessibility: $y$ is accessible to $x$ just in case $y$ is sufficiently similar to $x$ with regards to how it assigns cut-off points to vague predicates. Two indices which state that ‘small number’ ends at 15 and 16 respectively (but otherwise agree) count as closer together than two indices which state that ‘small number’ cuts out at 15 and 25 respectively (and otherwise agree.)

This idea suggests that there should be some measure of distance between indices. We can represent this by a metric $d(\cdot, \cdot)$, on the set of indices $W$, which given two indices outputs a real number saying how close they are. $x$ is accessible to $y$ iff they are sufficiently close. We may represent this by saying that their distance is less than some fixed number $\alpha$: if $R$ is the accessibility relation then $Rxy$ iff $d(x, y) \leq \alpha$. Thus we have a class of Kripke frames $C$ which contains frames, $(W, R)$, for which there is some metric over $W$, $d(\cdot, \cdot)$, and $\alpha \in \mathbb{R}$ such that $Rxy$ iff $d(x, y) \leq \alpha$.

It is clear that the accessibility relation of each such frame is symmetric: if the distance between $x$ and $y$ is less than $\alpha$ then, the distance between $y$ and $x$ is less than $\alpha$. This is tantamount to saying that $B$ is validated: if $\phi$ is true at $x$ then, by symmetricity, $\phi$ is possibly true at every index accessible to $x$.

This argument for $B$ relies essentially on the adequacy of this particular semantics. However, there are good reasons to think that this particular semantics is too restrictive: in these models an index bears $R$ to another index if it is sufficiently close to it in some relevant sense. However, in [86], Mahtani notes that ‘sufficiently close’ as it is being used here is surely a vague concept. Yet at every index what counts as sufficiently close is being within a radius of $\alpha$ from that index; the cut-off point for being ‘sufficiently close’ is the same at every index. So what counts as sufficiently close is completely precise at every index $i$: the cut-off point is the same for each index accessible from $i$. In Mahtani’s terminology the ‘accessibility range’ does not vary, when it should.

If each point, $x$, has its own accessibility range, $f(x)$, symmetry is no longer guaranteed. The distance between $x$ and $y$ may be less than $f(x)$ but not less than $f(y)$ (see figure 1.) Mahtani’s semantics, which I develop in section [REF], is well motivated and not particularly complicated; this seems to me to undermine the theoretical argument for $B$.

7For example, all kinds of factors contribute to how we classify people as bald – not just hair number, but distribution, colour and so on. Thus closeness of indices must be in part determined, in a weightet way, by how the indices match these various factors. How we weight these different factors is surely a vague matter.
8.2.5 Sharp boundaries from other principles

We have been focussing on $B$ as it is the most natural principle of this form, however it actually unnecessarily strong. It is in fact possible to strengthen theorem [REF] by weakening $B$ while keeping the other background assumptions. There is an infinite chain of weaker principles, all statable in the finitary language, that could be substituted for $B$ in theorem [REF]. Readers not interested in the technical details of these further principles should skip ahead.

$$B^n: p \rightarrow \Delta (q \rightarrow \phi_n) \quad (8.3)$$

where $\phi_1 := \neg \Delta \neg p; \phi_{n+1} := \neg \Delta (q \land \phi_n)$. And the principle

$$B^*: \Delta (p \rightarrow \Delta p) \rightarrow (\neg p \rightarrow \Delta \neg p) \quad (8.4)$$

I omit the proofs for reasons of space. However it is quite to simple establish that these principles semantically entail our conclusion that it is always determinate whether something is determinate$^\ast$.

In terms of frame conditions, $B^n$ characterises the the property that if $Rxy$ then there are $n$ points, $z_1, \ldots, z_n$ such that $Ryz_n, Rz_nz_{n-1}, \ldots, Rz_2z_1$ and $Rxz_i$ for each $i$ - i.e. if $x$ sees $y$ then you can get back from $y$ to $x$ in $n$ steps which $x$ can see. $B^*$ characterises the the property that if $Rxy$ then for some $n$, there are $z_1, \ldots, z_n$ such that $Ryz_n, Rz_nz_{n-1}, \ldots, Rz_2z_1$ and $Rxz_i$ for each $i$ - i.e. if $x$ sees $y$ then you can get back from $y$ to $x$ in finitely many steps which $x$ can see. Call this latter property the ‘backtrack’ property. In the lattice of modal logics, $KTB^*$ is the infimum of $\{KTB^n \mid n \in \omega\}$.

As stated, any one of these axioms is sufficient to close the gap between $K_{\infty}$ and the existence of sharp higher order cut-off points. I shall show that each frame validating $KTB^n$ or $KTB^*$ also validates $\neg \Delta \neg p \rightarrow \Delta \neg \Delta \neg p$ and hence $\Delta \Delta \neg p \lor \Delta \neg \Delta \neg p$. Suppose the frame $F := \langle W, R \rangle$ validates $KTB^n$ or $KTB^*$. For any model based on $F$, if $\neg \Delta \neg p$ is true at $x$ and $Rxy$ then (a) for some $n$ you can get from $x$ to a $\neg p$ world in $n$ steps which $x$ can see. (b) for some $m$ you can get back to $x$ from $y$ in $m$ steps. Thus you may get from $y$ to a $\neg p$ world in $n + m$ steps, so $y \not\vDash \Delta^{n+m}p$ and thus $y \vDash \neg \Delta^p$. But $y$ was an arbitrary point accessible from $x$, thus $x \vDash \Delta \neg \Delta^p$. So $x \vDash \neg \Delta^p \rightarrow \Delta \neg \Delta^p$ for every $x$. On the other hand the fact that $x \vDash \Delta^p \rightarrow \Delta \Delta^p$ falls out automatically (if $p$ is true at any world you can get to from $x$, then at any world, $y$, you can get to from $x$ $p$ is true at any world you can get to from $y$.)

What to make of all this? These paradoxes all rely on the principle $B$ or weakenings of it such as $B^n$ and $B^*$. None of these principles appear to have any direct intuitive appeal. Mahtani’s observation demonstrates furthermore that the semantic framework
Williamson uses to motivate the principle $B$ (at least) rests on contentious assumptions. The paradox in question is thus deniable. However, several questions remain. Firstly, while we have seen that a particular argument for higher order precision fails, it remains open whether the denial of higher order precision is consistent. Secondly, even if it is consistent to deny higher order precision, it would be good to know what things looks like when there is vagueness at every order: are there any natural looking models, perhaps simple modifications of Williamson’s models, which permit vagueness at every order?

8.3 Sharp Boundaries from $\Delta$-introduction

A reasonable hypothesis, then, is that $B$ (or one of its weakenings) is the culprit for the above paradox; we will return to the business of showing that the remaining principles of $KB_\infty$ are consistent with vagueness at all orders in due course.

Let me turn now to another paradox of higher order vagueness in the literature which does not rely on the principle $B$. This is a variation on an argument originally due to Wright [136]. Here the blame clearly does not lie with $B$; we shall see instead that these arguments rely on a contentious view about the connection between vagueness, permissible assertion and knowledge.

Let us begin by supposing that we are presented with a Sorites sequence for baldness. We can write $B(n)$ to denote the proposition that the $n$th person in this sequence is bald and we shall use $N$ to denote the place of the last person in the sequence (the $N$th person is thus someone who is clearly not bald.) The argument, due to Delia Graff Fara [41], makes use of only two principles. The first are frequently called ‘gap principles’:

\[(G) \; \Delta \Delta^n B(k) \to \neg \Delta \neg \Delta^n B(k + 1)\]

The gap principles effectively state that there is vagueness at each order in this Sorites sequence. For each of the predicates of the form $\Delta^n B(\cdot)$ one cannot find successive people in the sequence such that one determinately falls under the predicate and the next determinately does not; for that is just what it means for the predicate to have a sharp cut-off point.

The other assumption is the rule of proof called $\Delta$-introduction:

If $\Gamma \vdash \phi$ then $\Gamma \vdash \Delta \phi$

A trivial consequence of $\Delta$-introduction is the rule of inference $\phi \vdash \Delta \phi$ which follows from the validity of $\phi \vdash \phi$. This allows us to formulate Fara’s result:

**Theorem 8.3.1.** (Fara’s paradox) $(G)$, $B(0)$ and $\neg B(N)$ are inconsistent assuming $\Delta$-introduction as a rule of proof.

The proof is relatively simple. From $B(0)$ and $\Delta$-introduction one can infer $\Delta B(0)$. From $(G)$ we can infer $\neg \Delta \neg B(1)$, then $\Delta \neg \Delta \neg B(1)$ by $\Delta$-introduction, then $\neg \Delta \Delta \neg B(2)$ by $(G)$, and so on to eventually conclude $\neg \Delta^N \neg B(N)$. But by repeated application of $\Delta$-introduction to $\neg B(N)$ one can infer $\Delta^N \neg B(N)$ which is a contradiction.

8.3.1 The Forced March Sorites and Assertion

There are much fewer moving parts in this argument so the upshot is clear: either give up $\Delta$-introduction or admit precise higher order boundaries. Before we go on to assess $\Delta$-introduction, however, I want to consider another issue that should help clarify the matter.

One of the more puzzling issues relating to vagueness and higher order vagueness is the so-called ‘forced Sorites march.’ One is to imagine that you are to be presented with the elements of a Sorites sequence for $F$ in succession, and in each case you are required

\[\text{[REF]} \text{ Explain why } \Delta\text{-intro is stronger than necessitation.}\]
to say to the best of your ability whether the element is \( F \) or not. The puzzle is that there must surely be a first element at which you stop saying ‘yes, it’s \( F \)’ and switch to doing something else. Perhaps that is saying ‘no, it’s not \( F \)’, or saying ‘I don’t know’ or perhaps it is not saying anything at all. The point is, whatever one does, it seems one is committed to a sharp boundary. (For simplicity I shall confine attention to the responses: ‘yes’, ‘no’, ‘I don’t know’, ‘it’s indeterminate’ and saying nothing at all. Other responses are available: ‘I’m not sure whether I know or not’, ‘I don’t know whether it’s determinate’, ‘it’s indeterminate whether it’s indeterminate’, and so on, but since these iterations get less and less relevant to the question you’re being asked to answer, silence is often a better response even if these answers are known.)

The notion of commitment here is a pragmatic one: which propositions are assertable, i.e. which answers are appropriate in a forced march, depends on the truth of the commitments of an assertion of that proposition. As I am using the term an assertion cannot be appropriate if its commitments are false. A natural theory of assertion and commitment, in the context of a forced Sorites march, might include principles such as

\[ \text{Assert } p \text{ only if it’s determinate that } p. \]

In asserting that \( p \) you commit yourself to it being determinate that \( p \).

What these principles encode, in effect, is the thought that one shouldn’t assert indeterminate propositions. It might be possible to derive the above norms by appealing to more basic principles; perhaps the principle that one shouldn’t assert what one doesn’t know, and the commonplace assumption that one cannot know \( p \) if it is vague whether \( p \).

The question I’m primarily interested in here is not whether these norms are true, but whether they are complete. Are there any other reasons to refrain from asserting \( p \) in the context of a forced Sorites march? I want to argue that the answer to this question is ‘no’ – the strongest proposition an assertion that \( p \) commits you to is the proposition that it is determinate that \( p \). Provided you’re as knowledgeable as you can be about the relevant facts, \( p \) is assertable (ignoring other pragmatic factors) when \( p \) is determinate. Thus, for example, saying ‘yes, \( a \) is \( F \)’ to a given question commits you to \( a \) being determinately \( F \). You therefore should not say this if it’s vague whether \( a \) is \( F \). However asserting that \( a \) is \( F \) does not commit you to \( a \) being determinately determinately \( F \). Analogously, if you say ‘it’s vague’ then you are committed to it’s being determinate that it’s vague whether \( a \) is \( F \), and so on. Not saying anything is importantly different from saying ‘I don’t know’, for the latter commits you to the determinacy of your not knowing, which plausibly means that you could come to know that you don’t know. If you don’t know whether you know something you are better of staying quiet than saying ‘I don’t know.’

My case for the completeness of these principles revolves around the claims like the following

\[ \text{If it’s not vague whether Harry is bald (i.e. if it’s a determinate matter whether Harry is bald) then one can in principle know whether Harry is bald.} \tag{8.5} \]

Of course, the principle is not true in all scenarios – if Harry was not spatiotemporally related to us, for example, we couldn’t know whether he was bald. However in the particular context described in the forced march it’s clear that these considerations aren’t important. In this context, provided there are no further barriers to knowledge, there is no further barriers to assertion. Why, then, might someone object to this principle? Of course, the general principle that all precise propositions have knowable truth values might fail for reasons not relating to vagueness at all. But I take it that if the above instance fails at all it fails for reasons relating to the vagueness of the property of being bald.

I espouse the principle. According to an opposing view it might be impossible to find out whether Harry is bald, even if it is a determinate matter whether he’s bald. According
to classical treatments of vagueness it is possible for a proposition to be both determinately true, and for it to be vague whether the proposition is determinately true just as someone can be bald but not determinately bald. In these cases, the opposing view claims, it is not possible to know whether $p$, even though $p$ is determinately true. In these cases the obstacle to our knowing is not vagueness (the proposition in question is determinate) but second order vagueness – vagueness concerning whether the propositions is determinate. According to this view, then, higher orders of vagueness are sources of ignorance in the same way that vagueness is.  

The view that vagueness and only vagueness is responsible for ignorance gives a more intuitive explanation of the phenomenon. The easiest way to explain to a non-philosopher the philosophical notion of vagueness is by saying it’s whatever prevents us from knowing whether a given person is bald when we know all the relevant facts about hair number and distribution. Clearly the kinds of reasons we can’t know whether a given person is bald or not, even if we know all the facts about hair number and distribution, forms some sort of natural kind. The simplest theory simply identifies vagueness with this obstacle to knowledge.  

As we proceed, we shall see that the simple view does a much better job at explaining the puzzles of higher order vagueness.

The converse of [REF] seems to be relatively uncontroversial, so given what I have said so far it is reasonable to assume that both [REF] and it’s converse are determinate – in other words the following principle is determinate: one can in principle know whether someone is bald (in this scenario) if and only if it’s a precise matter whether he is bald. It is a consequence of this, in the standard logic of determinacy (which includes the $K$ principle), that if it’s vague whether $Harry$ is determinately bald, it’s vague whether we can know that he’s bald. Assuming that the respondent knows as much as she can about Harry’s head, it follows that it’s vague whether she in fact knows that he’s bald, and it thus also presumably vague whether it’s permissible to assert that Harry is bald in such circumstances. None of this should be intrinsically surprising given the inherent vagueness of the relevant notions of possibility, knowledge, permission and assertion.  

In summary: in a forced march, if you know as much as you can about the situation then ‘it’s determinate that $p$’, ‘you know that $p$’ and ‘it’s assertable that $p$’ are all coextensive operators.

The companion to this view about knowledge is the view that the strongest proposition an assertion that $p$ commits you to is the proposition that $p$ is determinate. If one is as knowledgeable as possible, then one knows $p$ iff $p$ is determinate. Since, ignoring other pragmatic factors, an assertion is appropriate iff it’s knowledgeable, it follows that an assertion that $p$ commits you to no more than the claim that $p$ is determinate in a forced march. In contrast, the companion to the opposing view about knowledge has it that the strongest proposition an assertion that $p$ commits you to is the proposition that $p$ is determinate$^\star$.

Let us apply this theory of commitment to the forced march Sorites. We may consider the possible responses in turn. Certainly saying ‘yes’ up to a certain point, and then saying ‘no’ commits one to sharp boundaries, for that is to commit one to the elements being determinate cases up to a certain point, and determinate non-cases from there on. This is just what it is to say that $F$ is sharp. To respond by saying ‘yes’ up to a certain point, $a_n$, and then continue by saying ‘it’s indeterminate’ or ‘I don’t know’, is slightly more complicated.

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9An explicit example of this view can be found in Fine [52].

10One might prefer to simply introduce a separate name for whatever this obstacle to knowledge is: schmagueness. Arguments analogous to those in this chapter would demonstrate that schmagueness iterates non-trivially, and strictly analogous puzzles for higher order schmagueness arise as for vagueness.

11I am using ‘can’ here in a way that guarantees that you cannot know that $p$ if $p$ is false. ‘can’ cannot simply mean metaphysical possibility here because many false propositions are true, and even known, in some metaphysically possible world.

12I take it that the same Sorites sequence that shows that ‘$x$ is bald’ is vague, shows that ‘it’s possible to know that $x$ is bald’ and ‘it’s appropriate to assert that $x$ is bald’ are vague. I am further claiming that the vague instances of the latter two predicates are precisely the second order vague instances of the former, and the determinate instances of the latter are the determinately determinate instances of the former.
This commits one to the determinacy of \( Fa_1 \ldots Fa_n \). Asserting that it’s indeterminate that each of \( a_{n+1} \ldots a_m \) is \( F \) commits one to the determinacy of \( \nabla Fa_{n+1} \ldots \nabla Fa_m \). Thus we are committed to the following: \( \Delta Fa_n \) and \( \Delta \neg \Delta Fa_{n+1} \). This is not a commitment to sharp boundaries, but in a standard Sorites (without gaps) one would not expect there to be a determinate \( F \) adjacent to a determinately not determinate \( F \). It is thus plausible to assume that this response pattern commits you to some falsehoods; this this pattern of assertions is inappropriate. Finally, if we assume the respondent knows as much as she can about the relevant background precise facts, then her knowledge is coextensive with what’s determinate, so if she answers ‘I don’t know’, we may reason as above.

On the other hand, saying ‘yes’ up to a certain point, and then saying nothing for a stretch is a different matter altogether. This response pattern does not commit one to sharpness of any kind. Not saying anything is not the same as asserting that you don’t know \( Fa_n \), since the latter is not inappropriate when you do not know that you don’t know that \( Fa_n \). Saying nothing does not commit you to any claim about the vagueness or nth order vagueness of \( Fa_n \). It is completely compatible with this response pattern that every predicate of the form \( \Delta^aFx \) is vague. In other words, asserting that the cases are \( F \) up to a certain point, remaining silent for a while, asserting the cases are indeterminate, sliding back into silence for a bit, and then asserting the cases are not \( F \) from then on, does not commit you to any thing inconsistent with the thesis that \( \Delta^aFx \) is vague for each \( n \).

To demonstrate this, suppose that \( Fa_1 \ldots Fa_n \) are determinate, and that \( Fa_{n+1} \ldots Fa_m \) aren’t. Since there is second order vagueness, it is vague which number \( n \) is in our example, but we may be certain there is some such \( n \) by classical logic. If I happen to say ‘yes’ from cases \( a_1 \ldots a_n \) and remain silent for cases \( a_{n+1} \ldots a_m \) I have (a) asserted correctly, in the sense that I have asserted \( p \) when \( p \) is determinate and (b) have committed my self to nothing incompatible with vagueness at all orders. Furthermore, despite the fact that \( a_n \) is the last determinate \( F \), it is presumably vague that \( a_n \) is the last determinate \( F \), so my assertions, despite being correct, fail to be determinately correct. If a perfect assserter is someone who asserts \( p \) just in case it’s determinate that \( p \), there can be perfect asserters but it will always be at best vague whether you’re a perfect assserter (provided we assume that it is always a determinate matter whether you have asserted \( p \) or not.\(^1\))

It is worth remarking that if you knew you were a perfect assserter, i.e. if you knew that you asserted \( p \) just in case it’s determinate that \( p \), you would be able to infer from your having not asserted \( Fa_{n+1} \) that \( Fa_{n+1} \) was not determinate. Since in a typical forced march Sorites, it is at best vague whether you are a perfect assserter, such knowledge would not be available to you. So while there always is a correct response to the forced march Sorites, second order vagueness makes it impossible to know if you’ve made the correct responses.

### 8.3.2 Fara’s Paradox

There has been some debate concerning whether a supervaluationist should accept classical logic, where ‘classical logic’ is construed broadly to include classical rules of inference and rules of proof. The rule Fara appeals to is incompatible with certain classical rules of proof. For example one could not apply conditional proof to \( p \vdash \Delta p \) (which we saw to follow from \( \Delta \)-introduction) to obtain \( \vdash p \rightarrow \Delta p \), since this would imply, as a matter of logic, that everything is precise.\(^2\)

There are some tricky questions in this area concerning the nature of logic, whether logic should regulate rational credences and beliefs, and so on. The significance of Fara’s result does not depend on what we classify as ‘logic’ however; if the result is significant

\(^{13}\)This assumption may not be unassailable. For example, I might falter or hesitate as I say ‘yes’ in such a way as to make it vague whether I actually committed myself to the \( F \)ness of the case in question. This might be one way to be a determinate perfect assserter.

\(^{14}\)We have, by hypothesis, \( p \rightarrow \Delta p \) and \( \neg p \rightarrow \Delta \neg p \). From an instance of excluded middle and reasoning by cases we can infer \( \Delta p \lor \Delta \neg p \).
it must entail that in some sense it is incoherent to believe or assert \( B(0), \neg B(N) \) and the gap principles simultaneously. (If being inconsistent in Fara’s logic does not have this upshot then I can safely ignore the result and go about my business asserting that there’s vagueness at every order.) Given this fact it would be nice to talk directly about the normative conclusions without the detour through ‘consequence’ talk which is quite obscure. Recall our two notions of a ‘good inference’ from chapter 5: \( p_1 \ldots p_n \) to \( q \) that are formulated only in terms of the notion of a rational credence:

1. \( Cr(q) = 1 \) if \( Cr(p_i) = 1 \) for each \( i \leq n \) and \( Cr \in E \).

2. \( Cr(q \mid r) = 1 \) if \( Cr(p_i \mid r) = 1 \) for each \( i \leq n \), proposition \( r \), and \( Cr \in E \).

Here \( E \) is a set of credence functions that you would be justified in having in some possible epistemic situation. One notion governs what can be inferred given what you are already fully justified in believing, whereas the other constrains your beliefs when you are less than certain in the premises. Sometimes people talk about ‘global’ and ‘local’ consequence.

I wish to avoid these terms if possible, however one can certainly draw the distinction between the notion of the definiteness of the premises strictly implying the definiteness of the conclusion (i.e. ‘preservation of supertruth’) and the premises simply strictly implying their conclusion (i.e. ‘preservation of disquotational truth’.) We could represent these notions in the object language as follows \( \Box (\Delta p_1 \land \ldots \land \Delta p_n \rightarrow \Delta q) \) and \( \Box (p_1 \land \ldots \land p_n \rightarrow q) \) where \( \Box \) represents some suitable notion of logical necessity.

Since one could never find oneself in an object language which fully supported \( p \land \neg \Delta p \), but one could quite easily have evidence for \( \neg p \lor \Delta p \) the former notion of good inference invalidates reductio: we have \( p \land \neg \Delta p \vdash \) but not \( \vdash (p \land \neg \Delta p) \). The the former notion of a good inference behaves in some ways like a supervaluationist consequence relation.

I am happy to engage in either talk provided it is clear what one means and one is careful which normative conclusions one draws. However, neither notion permits the inference from \( p \) to \( \Delta p \). Crucially the first notion, 1., does not permit this inference. Observe first that this rule does not preserve determinate truth, for example if \( p \) is determinate but not determinately determinate, the premise of \( p \vdash \Delta p \) is determinate and it’s conclusion isn’t. Similarly, since \( p \) is precise (although not determinately so), one could in principle have evidence which justifies certainty in \( p \), yet be uncertain in \( \Delta p \) due to the vagueness in \( \Delta p \). To be sure this counterexample relied on higher order vagueness, but this is clearly not the place to beg that question.

Note that the consequence relation that instead preserves determinacy\(^*\) does appear to validate \( \Delta \)-introduction. If \( p \) is determinate at all orders then so is \( \Delta p \). However, as I have argued in the previous section, if your beliefs are ‘inconsistent’ according to this consequence relation you are not necessarily being incoherent by committing yourself to a contradiction. The most any belief or assertion commits you to is the determinacy, not the determinacy\(^*\), of what is believed or asserted.

In this vein Zardini presents an argument that does not rely on \( \Delta \)-intro [139]. His argument operates directly with the notion of determinacy\(^*\). His argument shows that if we assume the determinacy\(^*\) of (a) the vagueness of \( \Delta a \) for each \( n \), (b) the \( F \)ness of \( a_0 \) and (c) the non-\( F \)ness of \( a_{1,000,000} \) we can derive a contradiction. More formally he assumes the following: \( \Delta^* \exists x \neg \Delta^* a_n, \Delta^* F a_0, \Delta^* \neg F a_{1,000,000} \).

What is surprising about Zardini’s argument is that although I have throughout been arguing for the vagueness of predicates of the form \( \Delta^* a \), I cannot maintain that this claim is determinate\(^*\). This is interesting as it is a concrete example of something I would count as a permissible assertion which is not determinate\(^*\).

To check that these assertions, including \( \exists x \neg \Delta^* a_n \) and \( \exists x \neg \Delta^* F a_n \), do not commit us to any contradictions we need to show that not only are there models in which they are all true, but that there are models in which they are all determinately true. In fact, one can show for any \( n \in \omega \) that there is a model in which these claims are determinate\(^n\) true.
Let’s start with an example in which $\exists x \nabla \Delta^* Fx$ is simply true. Note also that all these examples apply also to $\exists x \nabla \Delta^n Fx$. In the models that follow the nodes represent indices (world precisification pairs, epistemic possibilities, or whatever objects suit your preferred theory of vagueness), and the number at an index represents the cut-off point for the predicate ‘small’ according to that index. An arrow going from an index $x$ to an index $y$ means that everything determinate at $x$ is true at $y$; in this case we say that $y$ is ‘accessible’ from $x$. Thus something can be determinate at $x$ only if it is true at every index accessible from $x$. Something is determinately determinate at $x$, if it is determinate at every index accessible from $x$, which given the former observation means that it is true at everything index accessible from an index accessible from $x$. Similar observations can be made about higher iterations; in particular, a proposition is determinate‘ at an index, only if it is true at every index you can get to from it by following the arrows.

Note, then, that a number is determinately* small at a point iff you can’t get to a smaller number by following the arrows. So, for example, the left node sees a node (itself) in which 14 is not determinately* small, and can see a node in which it is (the right node.) Thus, at the left node it is vague whether 14 is determinately* small, so ‘determinately* small’ has a borderline case. However the vagueness of ‘determinately* small’ is not determinate because the left node sees a node in which ‘determinately* small’ is completely precise: the right node.

Here at the leftmost node it is vague whether 13 is determinately* small: it sees a world where it isn’t (itself) and a world where it is (the middle node.) At the middle node it’s vague whether 14 is determinately* small (see above). So at the leftmost node we have $\Delta(\nabla \Delta^* S(13) \lor \nabla \Delta^* S(14))$. Thus at every node the leftmost node sees ‘determinately* small’ has a borderline case, witnessed by 13 and 14 respectively. This gives a model for $\Delta \exists x \nabla \Delta^* Fx$ (and also $\Delta \exists x \nabla \Delta^n Fx$, for each $n$.)

Just as before, it is vague at the leftmost node whether 12 is determinately* small. At every world it sees either 12 or 13 is a borderline case of determinate* smallness (see above), and at every node seen by a node seen by the leftmost either 12, 13 or 14 is a borderline case of determinate* smallness. So we have $\Delta \Delta(\nabla \Delta^* S(12) \lor \nabla \Delta^* S(13) \lor \nabla \Delta^* S(14))$. Thus this is a model for $\Delta \Delta \exists x \nabla \Delta^* Fx$ (and also $\Delta \Delta \exists x \nabla \Delta^n Fx$, for each $n$.) It should be clear how to carry on this series.

8.4 Vagueness at every order

To deny sharp cut-off points at all levels we must reject the principles $\Delta$-intro, $\mathcal{B}$, $\mathcal{B}^*$ and $\mathcal{B}^*$. The rest of this chapter is an evaluation of the prospects of this proposal. The general strategy is to find models in which the claims ‘there are borderline cases of being determinately* (determinately*n) small’ ($\exists x \nabla \Delta^* Sx$, $\exists x \nabla \Delta^n Sx$) come out true. These models serve the purpose of showing that no contradiction can be derived from these assertions and plausible background assumptions. I attempt also to construct more realistic models, taking Williamson’s fixed margin models as a starting point, to show that these principles are compatible with more realistic structural assumptions.

Below is a model which demonstrates that it is at least possible to deny sharp cut-off points at every level. Each node represents an interpretation of the language, with the number at each node representing the greatest number which satisfies ‘small for a number
less than 100' according to that interpretation. The truth value of each atomic statement of the form ‘$n$ is small’ at a node is thus determined by whether number $n$ is less than or equal to the number at that node. The truth values of extensional combinations of formulae are calculated as usual relative to a node, and a claim of the form ‘$\Delta \phi$’ is evaluated true at a node $x$ iff $\phi$ is true at every node accessible via an arrow from $x$.

No point can see a point which differs from it by more than one. The converse fails, however, since interpretations may differ radically in the interpretation of other expressions and might thus be inaccessible to one another. It is tacitly assumed that every point sees itself.

![Diagram of a v-frame](image)

Remember that ‘the last small number’ according to a point is the number written beside it, so ‘the last determinately$^*$ small number’ at a point is the smallest number you can get to from that point by following the arrows. Note that the bottom node can see a node where the last determinately$^*$ small number is 7: follow the arrow to the right. But it can also see two nodes where the last determinately$^*$ small number is 8: follow the arrow up or left. Thus it is vague, at this point, whether the last determinately$^*$ small number is 7 or 8. Indeed, it’s vague whether it’s determinate$^*$ that 8 is small.

Of course we tried to make this model look realistic by having several points (we could have gotten away with two) and making sure that adjacent points didn’t disagree substantially over the interpretation of ‘small for a number less than 100’. However, it would be nice to have a general class of models that includes such models as a special case, but are also constrained by facts about vagueness in the same way Williamson’s semantics was. In fact we can modify Williamson’s fixed margin models in just the way Mahtani suggests to allow for variation in accessibility range. We must also make sure that close points don’t interpret ‘determinately’ drastically differently, i.e. we must make sure close points have similar accessibility ranges. This motivates the following definition.

**Definition 8.4.0.1.** A v-frame is a triple $\langle W,d,(\cdot,\cdot),f(\cdot) \rangle$ where $\langle W,d \rangle$ is a metric space, and $f: W \to \mathbb{R}$ obeys the following:

(A) $\forall w,v \in W, |f(w) - f(v)| \leq d(w,v)$

A formula of propositional modal logic is valid on a v-frame $\langle W,d,f \rangle$ iff it is valid on the Kripke frame $\langle W,R \rangle$ where $Rxy$ iff $d(x,y) \leq f(x)$. We shall talk about a v-frame and it’s associated Kripke frame interchangeably from now on.

In a supervaluationist framework, for example, the elements of $W$ can be thought of as interpretations or precisifications of the language, with the metric $d$ representing how close they are to one another. A formula is determinately true at an interpretation if it is true at all nearby interpretations. What counts as nearby the interpretation $w$ is determined by $f(w)$: $v$ is nearby $w$ when the distance between them according to $d$ is less than $f(w)$. Note that what counts as ‘nearby’ depends on the precisification - the constraint (A) says, roughly, that the closer two interpretations are, the less they can differ over their interpretation of ‘nearby’.

A useful fact is that there is a natural way to assign a metric over a (generated) Kripke frame. Simply assign a length to each arrow and define the distance between $x$ and $y$ to be
the length, ignoring the direction of the arrows, of the shortest path between $x$ and $y$. With a bit of fiddling one can show that the model above is generated by a v-frame in this way. The fact that one can refute $B^n$ and $B^*$ over v-frames follows also from the more general fact that the logic of v-frames is $KT$ (see the appendix.)

8.4.1 Realistic frames

Let us consider a toy propositional language whose only atomic sentences are English sentences of the form: ‘$a$ is red’, where $a$ ranges over names for colours in a fixed colour spectrum. It is natural to suppose the interpretations of this language are completely specified by the cutoff point for ‘red’ along this spectrum of colours. Each colour can be represented by a real number, and the distance between two interpretations can be modelled as the difference between the two numbers representing the cutoff points for those interpretations. Thus the metric of our v-frame is the standard notion of distance on $\mathbb{R}$.

This suggests a very natural class of v-frames for modelling vague languages: those based on Euclidean space, $\mathbb{R}^n$, where the dimension $n$ is the number of vague predicates in the language. The good news about these v-frames is that each of the problematic principles $B^n$ are refutable. To refute $B^n$ we consider the standard metric over $\mathbb{R}$. Let $\epsilon = \frac{1}{2n+1}$. Stipulate that $f(0) = 1$, that $f(x) = f(-x) = \epsilon$ for $x \in \mathbb{R}\setminus[-1+\epsilon,1-\epsilon]$ and $f(x) = 1-x$ for $x \in (0,1-\epsilon]$ and $x-1$ for $x \in [\epsilon-1,0)$. It is easy to check this satisfies condition (A) and is a v-frame. Now 0 can see 1, yet the shortest path back from 1 takes $n+1$ steps, thus $B^n$ does not hold. Note that in this model there are no points which can only see themselves, i.e. no points, $x$, such that $f(x) = 0$.

The counterexamples traded on the idea that for any $n$ one can find a v-frame based on an $\epsilon$ small enough to ensure that the longest path from 1 to 0 is longer than $n$. There is, however, no single model which refutes all the $B^n$ simultaneously. Indeed it is not hard to show that v-frames based on $\mathbb{R}^n$ where no points have a 0 accessibility range has the backtrack property: if $x$ sees $y$, then there is a finite path $z_0, \ldots, z_n$ such that $y$ sees $z_0$, $z_0$ sees $z_1$, $\ldots$, $z_n$ sees $x$ and $x$ sees each $z_i$. Thus it follows that $B^*$ holds in these frames (see figure 2.) Intuitively, the closer a point is to $x$, the closer in diameter it’s accessibility range must be to $x$’s thus one always can find a path leading back to $x$:

For our purposes this result would be devastating. It would entail, for example, that being determinately $\ast$ red had completely precise boundaries, and this brings along with it
all the problematic consequences already mentioned.

An obvious place to resist the result is to deny that the accessibility range of any point must be non-zero. Relaxing this constraint is independently motivated. Suppose we are working with the toy language described above, and the ordering of the relevant colour spectrum is not only dense but complete in the sense of containing a limit for any converging sequence of points (thus, for example, it’s structure is like that of \( \mathbb{R} \), but not of \( \mathbb{Q} \)).\(^\text{15}\) If one made the assumption that \( f(x) > 0 \) for any interpretation \( x \), it follows that no colours in the spectrum are determinately* red. It is sufficient to show that any two interpretations, represented as real numbers, \( x \) and \( y \), can be connected by a path. Without loss of generality we may assume that \( x < y \). Let \( (a_n)_{n \in \omega} \) denote the sequence \( x, x + f(x), x + f(x) + f(x + f(x)), \ldots \), i.e. \( a_0 := x \) and \( a_{n+1} = a_n + f(a_n) \). If \( a_n < y \) for each \( n \) then \( (a_n)_{n \in \omega} \) must clearly converge as it is a bounded monotonic sequence. Let \( a_\infty \) be the point it converges to. I claim that \( f(a_\infty) = 0 \), contradicting the assumption that \( f(x) > 0 \) for any \( x \). By condition (A) on v-frames we know that \( |f(a_\infty) - f(a_n)| \leq a_\infty - a_n = d(a_\infty, a_n) \) for each \( n \). However, since the right hand side converges to 0 as \( n \) increases, and \( f(a_n) \) converges to 0, it follows that \( f(a_\infty) = 0 \).

Once one moves away from the simple toy example, v-frames based on \( \mathbb{R}^n \) become implausible for other reasons. For example, suppose now we are considering a language in which the only atomic sentences are of the form ‘\( a \) is red’ and ‘\( a \) is orange’ for \( a \) a colour in a fixed spectrum. Modelling this language using \( \mathbb{R}^2 \) would be overly simplistic because the interpretation of ‘red’ and ‘orange’ are not independent. Any assignment of cutoff points that allowed ‘red’ and ‘orange’ to overlap should be intuitively very far away from the intended interpretation. Thus an interpretation that says that the red colours end at the colour represented by 10 and orange starts at the colour represented by 9 should be very far away from the sensible interpretation that says red ends at 9 and orange starts at 10. However their distance according to the standard metric on \( \mathbb{R}^2 \) is relatively small: \( \sqrt{2} = \sqrt{(10 - 9)^2 + (9 - 10)^2} \).

A final worry in this ballpark is that even if \( \mathbb{R}^n \) v-frames aren’t suitable for modelling vagueness, the correct models might still have enough \( \mathbb{R}^n \)-like properties to guarantee that the backtrack properties hold. Let me finish by considering two such properties we might quite plausibly expect to hold in any realistic model:

- **Density:** for any \( x \) and \( y \) there is a \( z \) such that \( d(x, z), d(y, z) < d(x, y) \).
- **Closeness:** Whenever \( d(x, y) \leq f(x) \) there is a \( z \) such that \( d(y, z) \leq f(y) \) and \( d(x, z) < d(x, y) \).

However, neither of these principles, even in tandem, ensure that the relevant v-frame has the backtrack property. A simple example would be to let \( W := [0, 1) \cup (2, 3] \), \( d(x, y) = |x - y| \), \( f(x) = 1.5 \) for \( x \in [0, 1) \) and \( f(x) = 1 \) for \( x \in (2, 3] \). Anything in the range \( (\frac{1}{2}, 1) \) can see points in \( [2, 3] \), but there is no path from a point in \( (2, 3] \) to a point in \( [0, 1) \). Furthermore this v-frame is both dense and close. These examples are **gappy:** for a given point \( x \), there may be a range of real numbers \( [\alpha, \beta] \), such that the distance between \( x \) and another point is never in \( [\alpha, \beta] \).

### 8.4.2 Vagueness at every order

There is one puzzling aspect of the view defended here deserves some discussion. The puzzling phenomenon is this. Given that we accept Williamson’s paradox we know that the formula \( A \leftrightarrow \Delta A \) is provable (and thus determinate*) whenever \( A \) is of the form \( \Delta^* B \). Thus \( A \) is intersubstitutable with \( \Delta A \) in all formulae constructed from the logical connectives and \( \Delta \). Substituting \( A \) for \( \Delta A \) twice in succession shows that \( A \) is intersubstitutable with

\(^{15}\)If one were to object that some fact about colours prevents the existence of such a spectrum, we could modify the example to be about the vague predicate ‘small’ as applied to real numbers.
\( \Delta \Delta A \) and, by iterating this reasoning, is furthermore intersubstitutable with \( \Delta^n A \) for any \( n \). Thus we can infer from the tautology \( \nabla A \rightarrow \nabla A \) for any \( n \). The following is also a consequence of Williamson’s paradox:

\[ \nabla A \rightarrow \nabla \nabla \nabla A \]

provided \( A \) is of the form \( \Delta^* B \). This takes a slightly more complicated argument.\(^{16}\)

According to the theory I have been endorsing, in a Sorites sequence for baldness (say) it will be vague which the last determinately* bald person is. Suppose that \( x \) such a person, i.e. suppose that it’s vague whether \( x \) is determinately* bald – using obvious formalism:

\[ \nabla \Delta^* B x \]

Writing \( A \) for \( \Delta^* B x \), we can immediately infer from our second observation that \( \nabla A, \nabla \nabla A, \nabla \nabla \nabla A \) and so on.

In other words, the theory I endorse predicts that there are propositions or sentences that are vague at every order (say that \( A \) is vague at every order iff the \( \nabla A, \nabla \nabla A, \nabla \nabla \nabla A \) and so on all hold.) These cases have very curious properties. We cannot assert \( A \), because this claim is vague (we have \( \nabla A \).) But we cannot outright assert this claim either (i.e. that we cannot assert \( A \) because it’s vague whether \( A \).) It’s vague whether it’s vague whether \( A \) (\( \nabla \nabla A \)) so one cannot permissibly assert that it’s vague whether \( A \). Unfortunately the preceding sentence too would be impermissible – I asserted that it was vague whether it’s vague whether \( A \), and this is itself vague (\( \nabla \nabla \nabla A \).) And this assertion would be impermissible because it too is vague, and so on and so forth.\(^ {17}\)

Similar points apply to the structure of knowledge. We cannot know of any \( x \) that is indeterminately determinately* bald, whether it is determinately* bald. But we also could not know that \( x \) is one of those individuals whose determinate* baldness was unknowable for vagueness related reasons, since the unknowability fact would itself too be unknowable. As would the fact that it was unknowably unknowable and so on.

How much should this curious consequence worry us? Nothing we have said undermines the claim that it is vague who the last determinately* bald person is (a claim we know to be consistent by the models we have considered.) We cannot know, of any individual, that they are the last determinately* bald person. We also cannot know if it’s vague whether they’re the last determinately* bald person, or even if it’s vague whether it’s vague, and so on and so forth. This is certainly a puzzling feature of any account which accepts Williamson’s paradox but rejects the conclusion of theorem [REF], however it is perhaps exactly this feature that explains why higher order vagueness is itself so puzzling. If there are examples of claims that are vague at every order, it is not possible to say anything informative about these claims: it is not possible to say that they are vague, second order vague, third order vague, or even vague at some order or other – such examples thus elude us completely.

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\(^ {16}\)Suppose, for induction, that \( \nabla A \rightarrow \nabla \nabla A \). On the one hand we have \( \nabla \nabla A \rightarrow \neg \Delta \nabla \nabla A \), a simple consequence of the factivity axiom, so \( \nabla A \rightarrow \neg \Delta \nabla \nabla A \). On the other hand we must appeal to the obvious (but fiddly to prove) principle \( \Delta^n A \rightarrow \neg \nabla^n A \), the determinacy of which permits the conditional \( \neg \Delta \Delta \nabla A \rightarrow \neg \Delta \nabla \nabla A \). Then we have the conditionals: \( \nabla A \rightarrow \nabla \nabla A \), which we already proved, \( \nabla \nabla A \rightarrow \neg \Delta \nabla \nabla A \) by definition, and the conditional we just appealed to \( \neg \Delta \Delta \nabla A \rightarrow \neg \Delta \nabla \nabla A \). Thus by transitivity of \( \rightarrow \): \( \nabla A \rightarrow \nabla \nabla \nabla A \).

\(^ {17}\)I say these assertions would be impermissible if made outright. I am not making them outright, I am only making these assertions conditional on the assumption that it’s vague whether \( A \); what I am actually doing is no more problematic than asserting the theorems of the form \( \nabla A \rightarrow \nabla \nabla \nabla A \).
Chapter 9

Appendices

9.1 Appendix A

Here we prove that CS and PMP commit you to sharp cut-off points. We shall work in a propositional language consisting of \(\land\), \(\lor\), \(\neg\) and \(\rightarrow\). For readability I shall be lax about parantheses. Let \(S_n\) denote the proposition that the \(n\)th person is rich. According to our definition in chapter [REF], we can say what it means for \(n\) to be a cutoff point by the sentence \(\neg(S_n \rightarrow S_{n+1})\). In addition to CS we assume the following background logic:

MP \(A, A \rightarrow B \vdash B\).

MT \(\neg B, A \rightarrow B \vdash \neg A\).

CC \((A_1 \land \ldots \land A_n) \rightarrow A_i, 1 \leq i \leq n\).

CE \((A_1 \land \ldots \land A_n) \rightarrow (A_1 \land \ldots \land \neg A_n)\).

DM \((A_1 \land \ldots \land A_n) \vdash (\neg A_1 \lor \ldots \lor \neg A_n)\).

As we shall see, the last axiom, DM, is not that important for the main upshot of the argument.

The argument then proceeds as follows:

1. \(((S_0 \rightarrow S_1) \land \ldots \land (S_{999999} \rightarrow S_{1000000})) \rightarrow ((S_0 \rightarrow S_1) \land (S_1 \rightarrow S_2))\) by CE.

2. \(((S_0 \rightarrow S_1) \land (S_1 \rightarrow S_2)) \vdash (S_0 \rightarrow S_2)\) an instance of CS.

3. \(((S_0 \rightarrow S_1) \land \ldots \land (S_{999999} \rightarrow S_{1000000})) \rightarrow (S_0 \rightarrow S_2)\) by the transitivity of \(\rightarrow\). (A consequence of CS, MP and CE.)

4. \(((S_0 \rightarrow S_1) \land \ldots \land (S_{999999} \rightarrow S_{1000000})) \rightarrow (S_2 \rightarrow S_3)\) by CE.

5. \(((S_0 \rightarrow S_1) \land \ldots \land (S_{999999} \rightarrow S_{1000000})) \rightarrow ((S_0 \rightarrow S_2) \land (S_2 \rightarrow S_3))\) by CC from 3 and 4.

6. \(((S_0 \rightarrow S_3) \land (S_3 \rightarrow S_4)) \vdash (S_0 \rightarrow S_4)\) CS.

7. \(((S_0 \rightarrow S_1) \land \ldots \land (S_{999999} \rightarrow S_{1000000})) \rightarrow (S_0 \rightarrow S_3)\) by transitivity of \(\rightarrow\).

... And so on.

8. \(((S_0 \rightarrow S_1) \land \ldots \land (S_{999999} \rightarrow S_{1000000})) \rightarrow (S_0 \rightarrow S_{1000000})\)

9. \(\neg(S_0 \rightarrow S_{1000000})\) (premise)

10. \(\neg((S_0 \rightarrow S_1) \land \ldots \land (S_{999999} \rightarrow S_{1000000}))\) by MT.
11. \( \neg(S_0 \rightarrow S_1) \lor \ldots \lor \neg(S_{999999} \rightarrow S_{1000000}) \) by DM.

The conclusion is just the claim that either 0 is a cut-off point, or 1 is, or 2 is or ... or 100000. Thus we have proved that there is a cut-off point. Notice that the deMorgan law, DM, is applied only at the last step. Thus even without DM one must accept the penultimate step of this argument, which many might think is just as bad as accepting of a sharp cutoff.

The argument against PMP proceeds similarly, although this time we must assume the transitivity of the conditional instead of modus ponens:

\[
\text{TR } A \rightarrow B, B \rightarrow C \vdash A \rightarrow C.
\]

(Indeed MP could have been substituted for TR in the last argument as well. Both of these arguments therefore work for the logic LP which does not have modus ponens.)

Here is the argument:

1. \( S_0 \land \bigwedge (S_n \rightarrow S_{n+1}) \rightarrow (S_0 \land (S_0 \rightarrow S_1)) \) by CE.
2. \( (S_0 \land (S_0 \rightarrow S_1)) \rightarrow S_1 \) instance of PMP.
3. \( S_0 \land \bigwedge (S_n \rightarrow S_{n+1}) \rightarrow S_1 \) from 1 and 2 by TR.
4. \( S_0 \land \bigwedge (S_n \rightarrow S_{n+1}) \rightarrow (S_1 \rightarrow S_2) \) by CE.
5. \( S_0 \land \bigwedge (S_n \rightarrow S_{n+1}) \rightarrow (S_1 \land (S_1 \rightarrow S_2)) \) by 3 and 4 and CC.
6. ... 
7. \( S_0 \land \bigwedge (S_n \rightarrow S_{n+1}) \rightarrow S_{1000000} \)
8. \( \neg S_{1000000} \)
9. \( \neg(S_0 \land \bigwedge (S_n \rightarrow S_{n+1})) \) by modus tollens
10. \( \neg S_0 \lor \neg(S_0 \rightarrow S_1) \lor \ldots \lor \neg(S_{999999} \rightarrow S_{1000000}) \) by DM.

Here \( \neg S_0 \) is also a disjunct in our conclusion, but since this theorist accepts \( S_0 \) this is of little solace.

### 9.2 Appendix B

The principle of plenitude says that, for any function \( t \) from the set of maximally specific precise propositions to \([0, 1]\) there is a proposition \( p_t \) such that for every conceptually coherent prior \( Pr \), and maximally specific precise proposition, \( w \), \( Pr(p_t \mid w) = t(w) \) provided \( Pr(w) > 0 \).

As with any existence postulate, such as Lewis’s plenitude principle for possible worlds or the naïve comprehension principle in set theory, there is a question about its consistency. Here we show that it is in fact consistent.

**Theorem 9.2.1.** Suppose that \( \mathbb{B} \) is a complete atomic Boolean algebra with countably many atoms. Then there is an extension of \( \mathbb{B} \), \( \mathbb{B}' \supseteq \mathbb{B} \), such that for every function \( t : \text{Atom}(\mathbb{B}) \rightarrow [0, 1] \) there is a proposition in \( \mathbb{B}' \), \( p_t \), with the following property.

Any countably additive probability function \( Cr \) over \( \mathbb{B} \) can be extended a probability function \( Cr' \) over \( \mathbb{B}' \) in a way such that:

1. \( Cr'(p) = Cr(p) \) for each \( p \in \mathbb{B} \)
2. \( Cr'(p_t \mid w) = t(w) \) for each \( w \in \text{Atom}(\mathbb{B}) \), provided \( Cr(w) > 0 \).
Proof. Let $W := \text{Atom}(\mathcal{B})$. Without loss of generality we may identify $\mathcal{B}$ with $\mathcal{P}(W)$. Given $p \subseteq W \times [0, 1]$ let $p_w := \{x \mid \langle w, x \rangle \in p\}$. We define the expansion as follows

- $\mathcal{B}' := \mathcal{P}(W \times [0, 1])$
- $Cr'(p) = \sum_{w \in W} \lambda(p_w).Cr(w)$ where $\lambda$ is the Lebesgue measure.
- $p_i := \bigcup_w \{w \times [0, t(w)]\}$

$\mathcal{B} \leq \mathcal{B}'$ via the map $h : p \rightarrow p \times [0, 1]$. Note that $Cr'(p \times [0, 1]) = \sum_{w \in W} \chi_p(w).Cr(w) = \sum_{w \in p} Cr(w) = Cr(p)$, where $\chi_p$ is the characteristic function of $p$. This shows that 1. is satisfied.

Also $Cr'(p_i \mid w) = \frac{Cr'((\bigcup_{w \in W} \{w \times [0, t(w)]\}) \cap \{w \times [0, 1]\})}{Cr'((\{w \times [0, 1]\})} = \frac{t(w).Cr(w)}{Cr'(\{w \times [0, 1]\})} = t(w)$. So 2. is satisfied.

The consistency of Plenitude is a simple corollary of this result. We simply identify the set of conceptually coherent priors with the set of probability measures on $\mathcal{B}'$ that extend regular probability measures over $\mathcal{B}$ and satisfy $Pr(p_i \mid w) = t(w)$ for each atom $w$ of $\mathcal{B}$.

**The principle of plenitude in action**

If our evidence in the scenario where we have seen a tree from a distance is a vague proposition, as I have argued it is, there ought to be a proposition such that the result of updating on it results in the kind of credences you’d expect to be in; something like the credence distribution in figure 4.3. At the end of section 4.3 we argued that the principle of plenitude provides us with such a proposition.

Suppose, as in section 4.3, that my prior credence over the possible heights of the tree less than 1000cm is uniform, and is given by $Cr(w_i) = \alpha$, where $w_i$ is the proposition that the tree is $i$ cm, and my posterior credence function after seeing the tree is given by $Cr'(w_i)$. By the principle of plenitude there is a proposition $p_i$ such that $Cr(p_i \mid w_i) = Cr'(w_i)$ for each $i$, which can be gotten from the principle of plenitude by choosing a function that maps each maximally strong precise proposition consistent with $w_i$ to $Cr'(w_i)$. We’ll show that $p_i$ is the required update to get our credences from a uniform distribution to a distribution looking like figure 4.3. I.e. $Cr(\cdot \mid p_i) = Cr'(\cdot)$.

1. $Cr(w_i \mid p_i) = Cr(p_i \mid w_i).\frac{Cr(w_i)}{Cr'(p_i)}$ by Bayes’ theorem.
2. $Cr(w_i \mid p_i) = t(w_i).\frac{Cr(w_i)}{\sum_j Cr(p_j \mid w_j).Cr(w_j)}$
3. $Cr(w_i \mid p_i) = t(w_i).\frac{\alpha}{\sum_j t(w_j) \alpha}$
4. $Cr(w_i \mid p_i) = t(w_i).\frac{1}{\sum_j t(w_j)}$
5. $Cr(w_i \mid p_i) = t(w_i)$ since $\sum_j t(w_j) = \sum_j Cr'(w_j) = 1$
6. Thus $Cr(\cdot \mid p_i) = Cr'(\cdot)$ since they coincide on the worlds.

**9.3 Appendix C**

Here we demonstrate some relevant facts about the logic of higher order vagueness. Recall that:

**Definition 9.3.0.1.** A v-frame is a triple $\langle W, d(\cdot, \cdot), f(\cdot) \rangle$ where $\langle W, d \rangle$ is a metric space, and $f : W \rightarrow \mathbb{R}^+$ obeys the following:

(A) $\forall w, v \in W, |f(w) - f(v)| \leq d(w, v)$
A formula of propositional modal logic is valid on a v-frame \( \langle W, d, f \rangle \) iff it is valid on the Kripke frame \( \langle W, R \rangle \) where \( Rxy \) iff \( d(x, y) \leq f(x) \).

Dorr [30] shows, translating into the terminology of v-frames, that \( B \) is not valid over the v-frame \( \langle (1, 2), |x - y|, \frac{1}{2} \rangle \) although the weaker principles \( p \to \Delta \Delta \Delta \neg p \) and \( B^2 \) are valid in this frame. It is possible, however, to construct v-frames in which \( p \to \Delta \Delta \Delta \neg p \) is valid for no \( n \in \mathbb{N} \). For example, let \( W := \{0, 1\}, d(x, y) = |x - y|, f(0) = 1 \) and \( f(1) = \frac{1}{2} \).

What is the logic of v-frames? Clearly every v-frame generates a corresponding reflexive Kripke frame, so the logic of v-frames contains KT. One might have hoped that every reflexive Kripke frame could be generated from a v-frame this way ensuring a logic of exactly KT. This reduces to the question of whether every reflexive digraph can be embedded into a metric space in such a way that there is a a closed ball around each node that contains all and only those nodes it can see. Unfortunately this does not hold:

**Fact:** Suppose \( \mathcal{F} \) is a Kripke frame based on a v-frame. If \( \mathcal{F} \) contains a cycle, it contains a 2-cycle.

To see this suppose that \( \langle a_0, \ldots, a_n \rangle \) is a cycle in \( \mathcal{F} = \langle W, R \rangle \) where \( n > 2 \). For convenience let \( a_i = a_j \) where \( j = i \mod (n + 1) \) for \( i > n \). Now suppose that \( \sim Ra_{i+1}a_i \) for every \( i \). Since for each \( i \) \( Ra_{i+1}a_i \) we know that \( d(a_i, a_{i+1}) \leq f(a_i) \) in the corresponding v-frame. We also know that \( f(a_i) < d(a_{i-1}, a_i) \) since \( \neg Ra_i a_{i-1} \). Thus for each \( i \), \( d(a_i, a_{i+1}) \leq f(a_i) < d(a_{i-1}, a_i) \), so \( f(a_n) < d(a_{n-1}, a_n) \leq f(a_{n-1}) < \cdots \leq f(a_1) = f(a_n) \), i.e. \( f(a_n) < f(a_n) \) which is a contradiction. So for some \( i \), \( Ra_i a_{i+1} \) and \( Ra_{i+1}a_i \).

v-frames thus have more structure than reflexive frames. However, it turns out this does not make a difference to the logic:

**Theorem 9.3.1. Completeness.** A set \( \Sigma \) is valid on every v-frame iff it’s members are theorem’s of KT.

**Proof.** Suppose that \( \Sigma \) is a KT-consistent set of formulae. Then \( \Sigma \) is satisfiable on the canonical frame \( \mathcal{F} \). \( \mathcal{F} \) may contain cycles without 2-cycles, so we cannot yet infer that \( \Sigma \) is satisfiable on some v-frame. However we may construct a frame from \( \mathcal{F} \), with all the cycles ironed out, that is equivalent to a v-frame.

Let \( a_0 \) be a maximal KT-consistent set containing \( \Sigma \). We may assume that \( a_0 \) is a root of \( \mathcal{F} \) (if it isn’t take the generated subframe around \( a_0 \) and work with that instead.) Define \( \mathcal{F}^+ := \langle W^+, R^+ \rangle \) as follows

- \( W^+ := \{ s \mid s \text{ a path in } \mathcal{F} \text{ such that } s_0 = a_0 \} \)
- \( R^+ := \{ \langle s, t \rangle \mid |t| = |s|+1 \text{ and } s_i = t_i \text{ for } i \leq |s| \text{ or } s = t \} \)

**Claim:** \( f(a_0, \ldots, a_n) = a_n \) is a bounded morphism from \( \mathcal{F}^+ \) to \( \mathcal{F} \).

1. Suppose \( R^+st \). If \( s = t \) then \( f(s) = f(t) \) so \( Rf(s)f(t) \) since \( R \) is reflexive. If \( |t| = |s|+1 \) then \( Rf(s)f(t) \) since \( t \) and \( s \) are paths.
2. Suppose \( Rf(s)f(t) \). We want to find a \( u \) such that \( R^+su \) and \( f(u) = f(t) \). If \( f(t) = f(s) \) let \( u = s \). Otherwise, let \( u = \langle s, f(t) \rangle \).

Since anything valid on \( \mathcal{F}^+ \) is valid on every bounded morphic image of \( \mathcal{F}^+ \) (see for example [13]) it follows that \( \Sigma \) is satisfiable on \( \mathcal{F}^+ \). Now we construct our v-frame as follows:

- We begin by defining distance between adjacent points. If \( R^+st \) then \( e(s, t) = e(t, s) = \frac{1}{2^n} \). Always fix \( e(s, s) = 0 \)
- \( d(s, t) := \inf \{ \sum_{i=0}^n e(p_i, p_{i+1}) \mid p_0 = s, p_n = t, p \text{ a path in the symmetric closure of } \mathcal{F}^+ \} \)
- \( f(s) := \frac{1}{2^n} \)

It is now easy to check that \( \langle W^+, d, f \rangle \) is a v-frame and that \( R^+st \) iff \( d(s, t) \leq f(s) \).
Cian Dorr has pointed out to me that the constraint (A) on v-frames does not play much of a role in the proof of Theorem 4.1. This allows us to prove a slightly more general result:

**Definition 9.3.1.1.** A difference measure is a function $g : \mathbb{R}^2 \to \mathbb{R}$ such that:

- $g(x, x) = 0$
- $g(x, y) = g(y, x)$ (this constraint is optional in what follows.)

For a given difference measure, $g$, a $g$-frame is a triple $\langle W, d(\cdot, \cdot), f(\cdot) \rangle$ where $\langle W, d \rangle$ is a metric space, and $f : W \to \mathbb{R}$ such that:

$$(A') \forall w, v \in W, g(f(w), f(v)) \leq d(w, v)$$

**Corollary 9.3.2.** For any difference measure $g$, the logic of $g$-frames is KT.

**Proof.** Note that for any positive $a$ there is a $b < a$ such that $g(a, b) \leq a$ since $g(a, a) = 0$ and $g$ is continuous in both arguments. For any $a$ pick a unique such $b$, $a_g$ (choice.)

Now modify the construction in Theorem 4.1 as follows.

- Fix $e(s, s) = 0$ for every $s$.
- Let $e((a_0), t) = e(t, (a_0)) := 1$ for $t \neq (a_0)$ such that $R^+(a_0), t$.
- Suppose that $e(s, t) = e(t, s) = a$ has already been defined for $R^+ st$, and suppose that $R^+ tu t \neq u$. Define $e(t, u) = e(u, t) = a_g$.
- $d(s, t) := \inf \{ \sum_{i=0}^{n} e(p_i, p_{i+1}) \mid p_0 = s, p_n = t, p$ a path in the symmetric closure of $\mathcal{F}^+ \}$
- $f(s) := \sup \{ e(s, t) \mid R^+ st \}$.

\[\square\]

The interest in this generalization is that one might think that it becomes much harder for two points to differ on the interpretation of ‘determinately’ the closer together they are. Perhaps it is not the difference between $f(w)$ and $f(v)$ that must be less than $d(w, v)$ but the difference between their ratios, or some other such $g$.

### 9.3.1 Further restrictions

The proof of completeness and the counterexamples to the $B^n$ principles relied heavily on our considering slightly artificial frames that were based on metric spaces that either aren’t dense, or have points with zero accessibility range. A natural class of v-frames to consider are those based on metric spaces of the form $\mathbb{R}^n$ where $f(a) > 0$ for all $a \in \mathbb{R}^n$. In these frames whenever $x$ can see $y$, there is a path back from $y$ to $x$, even though there are frames invalidating $B^n$ for each $n \in \omega$ (i.e. there is no upperbound on how long these paths might be.) This is worrying since this means that $\Delta^* p \lor \Delta \neg\Delta^* p$ is valid over these frames.

We can express something like this principle in modal logic. I’ll call it $B^*$.

$$B^* : \Delta(p \to \Delta p) \to (\neg p \to \Delta \neg p) \tag{9.1}$$

$B^*$ is valid in the class of v-frames just described. In the presence of KT, $B^*$ defines what I shall call ‘the backtrack principle’.

Whenever $Rx$ there exists $z_1, \ldots, z_n$ such that (a) $z_1 = y$, $z_n = x$ and $Rz_iz_{i+1}$ for $1 \leq i < n$ and (b) $Rxz_i$ for $1 \leq i \leq n$.  

\[\square\]
Proof. We shall show that \((\Delta(p \rightarrow \Delta p) \land \neg \Delta \neg p) \rightarrow p\) defines the requisite property. Suppose the reflexive frame \(\mathcal{F} = \langle W, R \rangle\) has the backtrack property. Now suppose \(x \vDash (\Delta(p \rightarrow \Delta p) \land \neg \Delta \neg p)\). The second conjunct ensures that there is a \(y\) such that \(Rxy\) and \(y \vDash p\). Since \(\mathcal{F}\) has the backtrack property there is a finite path back from \(y\) to \(x, z_1, \ldots, z_n\), which \(x\) can see. Since \(x \vDash \Delta(p \rightarrow \Delta p)\) each \(z_i \vDash p \rightarrow \Delta p\). Since \(z_1 = y\) and \(y \vDash p, y \vDash \Delta p\) - by induction we can see that \(z_i \vDash p\) for each \(i\) which means \(z_n = x \vDash p\) as required.

For the other direction suppose, for contradiction, that \(\mathcal{F} \vDash (\Delta(p \rightarrow \Delta p) \land \neg \Delta \neg p) \rightarrow p\) but \(\mathcal{F}\) lacks the backtrack property. This means that for some \(x\) and \(y, Rxy\) but there is no path back from \(y\) to \(x\) which \(x\) can see. Define the following valuation on \(\mathcal{F}\): \(w \vDash p\) iff there are \(z_1, \ldots, z_n\) such that (1) \(z_1 = y, Rz_n w\) and \(Rz_i z_{i+1}\) for \(1 \leq i < n\) and (2) \(Rxz_i\) for \(1 \leq i \leq n\). Certainly if \(x\) had this property then \(z_1, \ldots, z_n, x\) would be a path back to \(x\) which \(x\) can see, so \(x \not\vDash p\). However \(x \vDash \Delta(p \rightarrow \Delta p)\) since if \(Rxw\) and \(w \vDash p\) then there is a path from \(y\) to \(w\) satisfying (1) and (2): \(z_1, \ldots, z_n\). Furthermore, for any world that \(w\) sees, \(w', z_1, \ldots, z_n, w\) will be a path from \(y\) to \(w'\) satisfying (1) and (2), since \(Rxw\).

\[\Box\]

Is KTB* the modal logic of these v-frames? We start with a negative result: KTB* is not sound and strongly complete with respect to any class of frames. I.e. there is no class of frames, \(\mathcal{C}\), such that a set is KTB* consistent iff it’s satisfiable on a frame in \(\mathcal{C}\). The following also shows it is neither canonical nor compact.

Proof. To show this we shall show there is a KTB*-consistent set of sentences which is unsatisfiable on every frame validating KTB*.

Let \(\Sigma := \{p, \neg \Delta \neg q\} \cup \{\Delta(q \rightarrow \Delta^n \neg p) \mid n \in \omega\}\). If \(\Sigma\) were KTB*-inconsistent some finite subset would be KTB*-inconsistent (since proofs are finite.) We shall show that for every \(m \in \omega, \Sigma_m := \{p, \neg \Delta \neg q\} \cup \{\Delta(q \rightarrow \Delta^n \neg p) \mid n \in m\}\) is KTB*-consistent. \(\Sigma_m\) has a KTB*-model: \(\langle m + 1, R \rangle\) where \(Rxy\) iff \(x = 0\) or \(x > 0\) and \(|x - y| \leq 1\). 0 can see \(m\) and there is a finite \(m\) length path back from \(m\) to 0 that can see but no shorter path. Let \(q\) be true only at \(m\) and \(p\) only at 0.

However, if \(\mathcal{F}\) validates KTB* then \(\mathcal{F}\) has the backtrack property so at no point of \(\mathcal{F}\) is every member of \(\Sigma\) true: if \(x \vDash \neg \Delta \neg q\) then \(x\) sees some \(y \vDash q\). By the backtrack property there is a path \(z_1, \ldots, z_n\) back to \(x\) which \(x\) can see, so \(\Delta(q \rightarrow \Delta^{n+1} \neg p)\) cannot be true at \(x\) if \(x \vDash p\).

\[\Box\]

However there is a positive result, namely that KTB* is sound and complete over the class of reflexive frames with the backtrack property. For this result I refer the reader to [12], who shows that KTB* has the finite model property.

**Theorem 9.3.3.** If \(\phi\) is KTB*-consistent then it is satisfiable on a finite reflexive frame with the backtrack property.

The question whether KTB* the logic of v-frames over \(\mathbb{R}^n\) in which \(f(x) > 0\) remains open.

### 9.3.2 B entails that a conjunction of determinate truths is determinate

The aim is to show distributivity within the modal logic KB with infinitary conjunction (C1-C3 below.) For convenience I shall introduce an operator \(\Diamond p := \neg \Delta \neg p\)

\[\text{D. } \bigwedge_{i < \omega} \Delta \phi_i \rightarrow \Delta \bigwedge_{i < \omega} \phi_i,\]

\[\text{KB}\]

\[K \Delta(\phi \rightarrow \psi) \rightarrow (\Delta \phi \rightarrow \Delta \psi)\]

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$B \phi \rightarrow \Delta \Diamond \phi$

Nec. if $\vdash \phi$ then $\vdash \Delta \phi$

C1. $\bigwedge_{i<\omega} \phi_i \rightarrow \phi_n$ for each $n<\omega$.

C2. $\bigwedge_{i<\omega}(\phi_i \rightarrow \psi_i) \rightarrow (\bigwedge_{i<\omega} \phi_i \rightarrow \bigwedge_{i<\omega} \psi_i)$.

C3. If $\vdash \phi_i$ for each $i<\omega$, $\vdash \bigwedge_{i<\omega} \phi_i$.

Claim: D is independent of K (and KT) + C1-C3.

Construct a Montague-Scott frame as follows (see [25]): Let $W := N$ and for each world $w \in W$ let the necessary propositions at $w$, $N(w)$, be the cofinite subsets of $W$ (if we are trying to model T as well we let $N(w) := \{X \cup \{w\} \mid X \text{ is cofinite}\}$.) Then $(W, N)$ satisfies:

1. $W \in N(w)$ for all $w \in W$
2. If $X, Y \in N(w)$ then $X \cap Y \in N(w)$
3. (For T) If $X \in N(w)$ then $w \in X$.

Thus our frame models K (/KT) including C1-C3. However it does not model D, as can be seen by letting $[p_i] := \{n \in N \mid n > i\}$ (for KT: $\{n \in N \mid n > i\} \cup \{0\}$, allowing D to fail at 0.) On the other hand any Kripke frame (reflexive Kripke frame) will validate K (KT) along with D.

Although the distributivity of infinite conjunctions over $\Delta$ is independent of K, the distributivity of $\Diamond$ over infinite conjunctions, perhaps surprisingly, is not independent in this way and can be show given just some relatively uncontroversial principles governing infinite conjunction.

**Lemma 9.3.4.** $\Diamond \bigwedge_{i<\omega} p_i \rightarrow \bigwedge_{i<\omega} \Diamond p_i$

*Proof.* First note that $\Diamond \bigwedge_{i<\omega} p_i \rightarrow \Diamond p_j$ for each $j$, by C1 and the background modal logic of K. Then by C3 and then C2 we can infer $\Diamond \bigwedge_{i<\omega} p_i \rightarrow \bigwedge_{i<\omega} \Diamond p_i$. □

We may also infer from our lemma that $\Delta \Diamond \bigwedge_{i<\omega} p_i \rightarrow \Delta \bigwedge_{i<\omega} \phi_i$.

In contrast to the fact that (D) is independent of K (and KT) it is not independent of, and is in fact entailed by, KB (and thus KTB.)

*Proof.* B directly gives us:

$$\bigwedge_{i<\omega} \Delta p_i \rightarrow \Delta \bigwedge_{i<\omega} \Delta p_i \tag{9.3}$$

We may also infer from our lemma that

$$\Delta \Diamond \bigwedge_{i<\omega} p_i \rightarrow \Delta \bigwedge_{i<\omega} \Diamond p_i \tag{9.4}$$

by applying necessitation and the K principle. Finally we have

$$\Delta \bigwedge_{i<\omega} \Diamond p_i \rightarrow \Delta \bigwedge_{i<\omega} p_i \tag{9.5}$$

because we have $\Diamond \Delta p_i \rightarrow p_i$ for each $i$ by B. So by C3 and then C2 we get $\bigwedge_{i<\omega} \Diamond \Delta p_i \rightarrow \bigwedge_{i<\omega} p_i$, and by necessitation and K that gives (12).

But (10), (11) and (12) give distributivity. □
It should be noted that this argument does not appeal to any characteristically classical principles. Indeed this argument can be carried out provided one has the following rules of inference as primitive or derived:

\[\begin{align*}
&1. \phi, \phi \rightarrow \psi \vdash \psi \\
&2. \phi \rightarrow \psi, \psi \rightarrow \chi \vdash \phi \rightarrow \chi \\
&3. \phi \rightarrow \psi \vdash \neg \psi \rightarrow \neg \phi
\end{align*}\]

and where KB is understood to contain K and the axioms \(\phi \rightarrow \Delta \Diamond \phi\) and \(\Diamond \Delta \phi \rightarrow \phi\).\(^1\)

To prove theorem [REF]

We then note the logical truth: \(\vdash \bigwedge_{n<\omega} \Delta^n \phi \rightarrow \bigwedge_{n<\omega} \Delta^{n+1} \phi\), which is provable from C1-C3 (it is just an instance of conjunction elimination.\(^2\)) From (D) we can immediately infer \(\vdash \bigwedge_{n<\omega} \Delta^n \phi \rightarrow \Delta \bigwedge_{n<\omega} \Delta^n \phi\), i.e.

\((+\Delta)\) \(\Delta^* \phi \rightarrow \Delta \Delta^* \phi\).

This is ‘Williamson’s paradox’: that from (D) one can prove \((+\Delta)\). We can then continue as follows

1. \(\neg \Delta \Delta^* \phi \rightarrow \neg \Delta^* \phi\) contraposing \((+\Delta)\)
2. \(\Delta (\neg \Delta \Delta^* \phi \rightarrow \neg \Delta^* \phi)\) by necessitation
3. \(\Delta \neg \Delta \Delta^* \phi \rightarrow \Delta \neg \Delta^* \phi\) from 2 by K
4. \(\neg \Delta^* \phi \rightarrow \Delta \neg \Delta \neg \neg \Delta^* \phi\) an instance of B
5. \(\neg \Delta^* \phi \rightarrow \Delta \neg \Delta \Delta^* \phi\) substituting logical equivalents.
6. \(\neg \Delta^* \phi \rightarrow \Delta \neg \Delta^* \phi\) by 3 and 5 and transitivity of the conditional.

Note that step 5 relied on the substitution of logical equivalents which is a derived rule in KB\(_\infty\). Thus we have

\((-\Delta)\) \(\neg \Delta^* \phi \rightarrow \Delta \neg \Delta^* \phi\).

Thus, by \((+\Delta)\) and \((-\Delta)\) and an instance of excluded middle we have the desired result \(\Delta \Delta^* \phi \lor \Delta \neg \Delta^* \phi\). It should be noted that, until this last step, none of this reasoning is characteristically classical.

\(^1\)Without the second version of B the most we could prove without double negation elimination would be things of the form \(\Diamond \Delta \neg \phi \rightarrow \neg \phi\).

\(^2\)\(\vdash \bigwedge_{n<\omega} \Delta^n \phi \rightarrow \Delta^i \phi\) for each \(0 < i < \omega\) by C1. Thus \(\vdash \bigwedge_{i<\omega} (\bigwedge_{n<\omega} \Delta^n \phi \rightarrow \Delta^{i+1} \phi)\) by C3. So finally \(\vdash \bigwedge_{n<\omega} \Delta^n \phi \rightarrow \bigwedge_{i<\omega} \Delta^{i+1} \phi\) by C2.
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